CSM 16A Designing Information Devices and Systems I
Spring 2021 Week 1

1. Gaussian Elimination

Learning Goal: The goal of this problem is to use Gaussian Elimination to describe solutions to systems, both qualitatively and quantitatively. Please review **Note 1B** to understand this problem better.

Write each system as an augmented matrix, and then solve using Gaussian Elimination. Also determine whether each system has no solution, a unique solution, or a set of infinitely many solutions.

(a) Solve the following system of equations:

$$\begin{aligned} x_1 - x_2 + 2x_3 &= 15 \\ 3x_2 - x_3 &= 8 \\ x_1 + 2x_3 &= 21 \end{aligned}$$

(b) [WALK-THROUGH] Solve the following system of equations:

$$\begin{aligned} x_1 + 2x_2 + 3x_3 &= 4 \\ x_1 + x_3 &= 0 \\ -2x_1 + 2x_2 + x3 &= 5 \\ x_1 + x_2 + 2x_3 &= 2 \end{aligned}$$

(c) [WALK-THROUGH]
(i) Now let us just change the third equation from the last problem to

$$-2x_1 + 2x_2 = 4.$$

The other three equations are unchanged. Do you still have a unique solution?
(ii) What if you change the third equation to

$$-2x_1 + 2x_2 = 5?$$

(d) (PRACTICE) Solve the following system of equations:

$$\begin{aligned} 2x_2 + 4x_3 &= -2 \\ -5x_3 &= 10 \\ x_1 + x_2 - 3x_3 &= 8 \end{aligned}$$

(e) (PRACTICE) Solve the following system of equations:

$$\begin{aligned} x_1 + 3x_2 - 2x_3 &= -3 \\ 2x_1 + 6x_2 - 4x_3 &= -5 \end{aligned}$$

2. Computations: Matrix-Vector Operations [WALKTHROUGH]

Learning Goal: The goal of this problem is to present various cases of matrix-vector operations such as addition, multiplication, and transpose. Please review **Note 2A: Section 2.3 and Note 2B: Section 2.1** to understand this problem better.

Consider the following matrices and vectors. Complete the parts below.

$$A = \begin{bmatrix} 2 & 4 \\ 5 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 3 \\ 2 & -4 \end{bmatrix} \quad \vec{u_1} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \vec{u_2} = \begin{bmatrix} 3 \\ -4 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \vec{w} = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

(a) What is the transpose of v?

(b) What is $(\vec{v}+\vec{w})^T$? Find $\vec{v}^T + \vec{w}^T$ too. Compare the results.

(c) What is $2\vec{v}$ - $4\vec{w}$?

(d) What is $\vec{v}^T\vec{w}$?

(e) What is $A\vec{u_1}$? What is $A\vec{u_2}$?

(f) What is **AB**? (Do the columns of **AB** look familiar?)

(g) Find $\mathbf{B}^T$. Then express $\mathbf{B}^T$ in terms of $\vec{u_1}$ and $\vec{u_2}$?

3. Linear or Nonlinear

Learning Goal: The goal of this problem is to draw a distinction between linear and non-linear functions. Please review **Section 1.4 of Note 1** to understand this problem better.

Determine whether the following functions are linear or nonlinear.

(a) [WALK-THROUGH]

$$f(x_1,x_2) = 3x_1 + 4x_2$$

(b)

$$f(x_1,x_2) = x_1^2 + e^{x_2}$$

(c)

$$f(x_1,x_2) = \sin(a)x_1 + e^b x_2,$$

where a and b are constants.

(d)

$$f(x_1,x_2) = x_2 - x_1 + 3$$

4. Spanning Set

Learning Goal: The goal of this problem is to connect Gaussian Elimination and linear (in)dependence to the concept of span. Another goal is to be comfortable with the geometric representation of span.

(a) For what values of b_1, b_2, b_3 is the following system of linear equations consistent? ("Consistent" means there is at least one solution. Please see **Note 1B: Subsection 1.2.4.2** for more details on consistency of a system.)

$$\mathbf{A}\vec{x} = \vec{b}$$

$$\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

(b) What is the geometry represented by span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$?

(c) Find out if $\vec{v}_1 = \begin{bmatrix} -3 \\ 5 \\ 0 \end{bmatrix}$ is in span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$. What about $\vec{v}_2 = \begin{bmatrix} -3 \\ 5 \\ 2 \end{bmatrix}$?

(d) Reflect on your answer from part(b) and find out span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ 0 \end{bmatrix} \right\}$.

(e) What is a possible choice for $\vec{v}$ that would make span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \vec{v} \right\} = \mathbb{R}^3$?