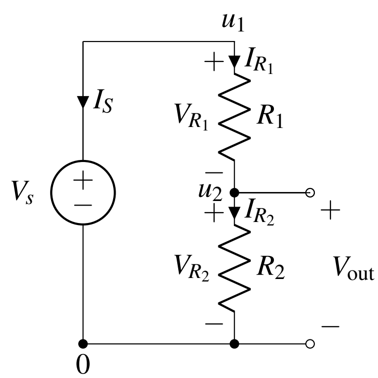
EECS 16A Designing Information Devices and Systems I
Spring 2022 Discussion 6B

1. Voltage Divider

For the circuit below, your goal will be to find the voltage V_{out} in terms of the resistances R_1, R_2, and V_s, using NVA (Node Voltage Analysis). The labeling steps (steps 1-4) have already been done for you.



Here is a reminder of the labeling steps followed to get the circuit diagram above:

- **Step 1:** Select a reference node and label it 0 (ground). Any node can be chosen for this purpose. We will measure all of the voltages in the rest of the circuit relative to this point.
- **Step 2:** Label all remaining nodes.
- **Step 3:** Label the current through every non-wire element in the circuit.
- **Step 4:** Label element voltages following **Passive Sign Convention**.

Our goal is to ***find*** V_{out}. In order to do this, we can use NVA to find equations describing our circuit, and solve the system of linear equations.

Step 5: Write KCL equations for all nodes with unknown voltages.

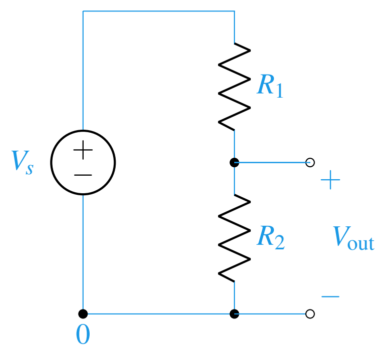
Step 6: Write down the IV relationships (Ohm's Law) of each of the non-wire elements.

Step 7: Use substitution to solve for $u_2 = V_{out}$.

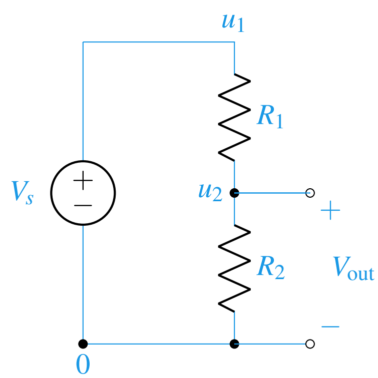
As an additional exercise, write out $\mathbf{A}\vec{x} = \vec{b}$ where $\vec{x}$ is a vector of your unknown currents and voltages. Fill in the rows of matrix $\mathbf{A}$ according to the equations you wrote.

Answer:

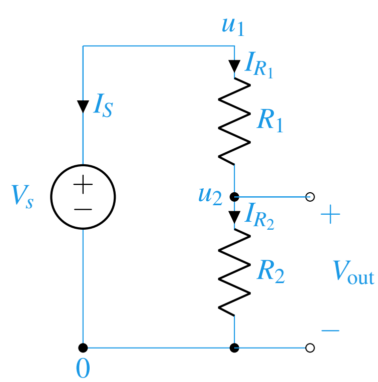
Step 1: Select a reference node,



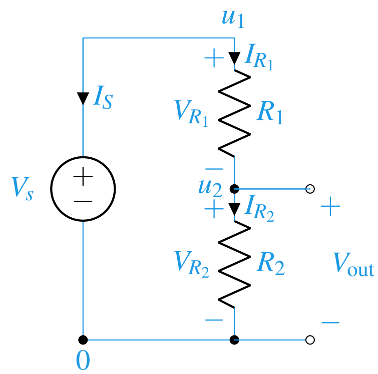
Step 2: Label all remaining nodes.



Step 3: Label the current through every non-wire element in the circuit.



Step 4: Label element voltages following **Passive Sign Convention**.



Step 5: Write KCL equations for all nodes with unknown voltages.

$$I_{R_1} = I_{R_2} \Rightarrow I_{R_1} - I_{R_2} = 0$$
$$I_S + I_{R_1} = 0$$

Step 6: Write down the IV relationships (Ohm's Law) of each of the non-wire elements.

$$V_s = u_1$$
$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{u_1 - u_2}{R_1} \Rightarrow I_{R_1}R_1 - u_1 + u_2 = 0$$
$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{u_2 - 0}{R_2} \Rightarrow I_{R_2}R_2 - u_2 = 0$$

Step 7: Use substitution to solve for $u_2 = V_{out}$.

$$\begin{aligned} I_{R_2} &= I_{R_1} \\ &\Rightarrow \frac{V_s - u_2}{R_1} = \frac{u_2 - 0}{R_2} \\ &\Rightarrow (V_s - u_2)R_2 = u_2 R_1 \\ &\Rightarrow u_2 = \frac{R_2}{R_1 + R_2} V_s \end{aligned}$$

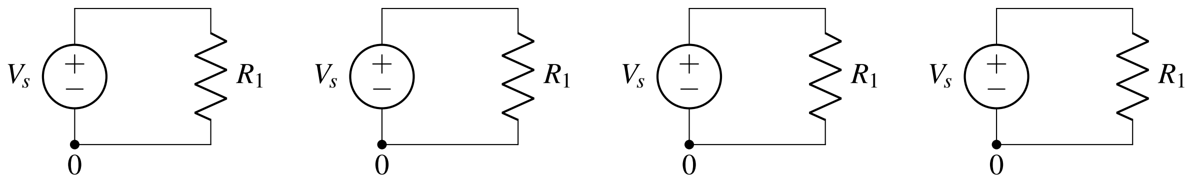
Next, lets write out $\mathbf{A}\vec{x} = \vec{b}$, fill the entries for $\mathbf{A}$ and $\vec{b}$ based on our equations:

$$\begin{bmatrix} ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? \\ ? & ? & ? & ? & ? \end{bmatrix} \begin{bmatrix} I_S \\ I_{R_1} \\ I_{R_2} \\ u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \\ ? \end{bmatrix}$$

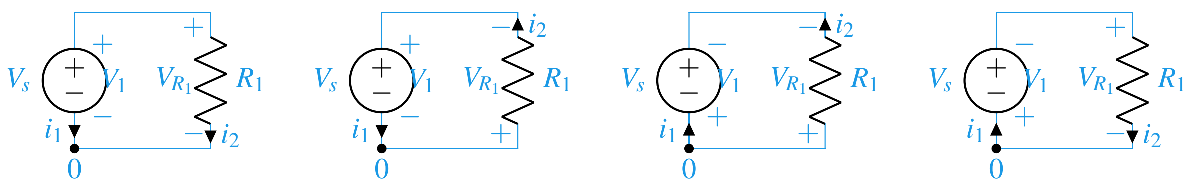
$$\begin{bmatrix} 0 & 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & R_1 & 0 & -1 & 1 \\ 0 & 0 & R_2 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_S \\ I_{R_1} \\ I_{R_2} \\ u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ V_s \\ 0 \\ 0 \end{bmatrix}$$

2. Passive Sign Convention and Power

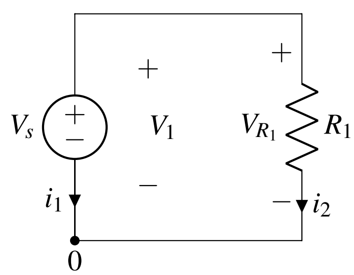
(a) Below are four copies of a the same single-resistor circuit. On each copy, provide a distinct choice of labels for each circuit's voltage polarities and current directions (there should be 4 possible choices in total!) while keeping with passive sign convention.



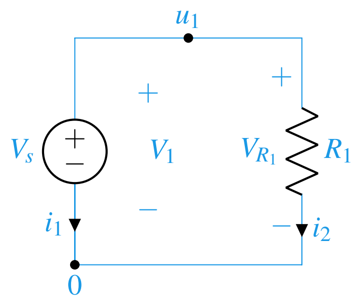
Answer:



(b) Suppose we consider one of the possible labelings you have found above. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5\,\text{V}$ and let $R_1 = 5\,\Omega$. *Recall that the power dissipated is the rate of electric energy converted into other forms and is given by the equation $P = IV$. When the power dissipated by an element is a negative value, it signifies that element is actually supplying electrical power to the circuit.*



Answer: We'll start by solving the circuit for the unknown node potentials and currents.



The KCL equation for the one node in this circuit is:

$$i_1 + i_2 = 0$$

The element equations for the two elements in this circuit are:

$$u_1 - 0 = V_1 = V_s$$

$$u_1 - 0 = V_{R_1} = i_2 R_1$$

Solving the above equations with $V_s = 5\,\text{V}$ and $R_1 = 5\,\Omega$:

$$u_1 = 5\,\text{V}$$
$$i_1 = -1\,\text{A}$$
$$i_2 = 1\,\text{A}$$

From above, we can solve for the power dissipated across the resistor:

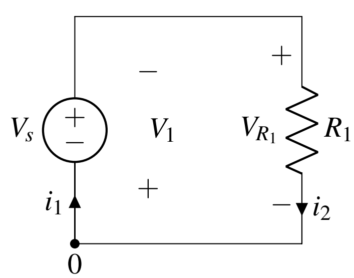
$$P_{R_1} = i_2 V_{R_1} = 1\,\text{A} \cdot 5\,\text{V} = 5\,\text{W}$$

Next we can solve for the power dissipated across the voltage source:

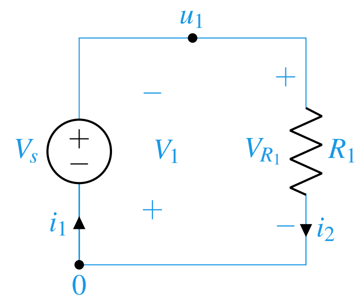
$$P_{V_s} = i_1 V_1 = i_1 V_s = -1\,\text{A} \cdot 5\,\text{V} = -5\,\text{W}$$

Notice we calculate a negative value for the power dissipated by the voltage source, implying the voltage source is adding power to the circuit.

(c) Suppose we choose a second labeling of the circuit as shown below. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5\,\text{V}$ and let $R_1 = 5\,\Omega$.



Answer: We'll solve the circuit the same way as last time.



The KCL equation for the one node in this circuit is:

$$-i_1 + i_2 = 0$$

The element equations for the two elements in this circuit are:

$$0 - u_1 = V_1 = -V_s$$

$$u_1 - 0 = V_{R_1} = i_2 R_1$$

Solving the above equations with $V_s = 5\,\text{V}$ and $R_1 = 5\,\Omega$:

$$u_1 = 5\,\text{V}$$
$$i_1 = 1\,\text{A}$$
$$i_2 = 1\,\text{A}$$

From above, we can solve for the power dissipated across the resistor:

$$P_{R_1} = i_2 V_{R_1} = 1\,\text{A} \cdot 5\,\text{V} = 5\,\text{W}$$

Next we can solve for the power dissipated across the voltage source:

$$P_{V_s} = i_1 V_1 = i_1(-V_s) = 1\,\text{A} \cdot -5\,\text{V} = -5\,\text{W}$$

Notice here that the circuit has the same power dissipated by all the elements. This is because with both labeling of currents, we followed the passive sign convention.

(d) Did the values of the element voltages and element currents change with the different labeling? Did the power for each circuit element change? Did the node voltages change? If a quantity didn't change with a difference in labeling, discuss what would have to change for quantity to change.

Answer: With a different labeling, element voltages and element currents will change. The quantities were $V_1 = 5\,\text{V}$ and $i_1 = -1\,\text{A}$ in (b) and in (c) $V_1 = -5\,\text{V}$ and $i_1 = 1\,\text{A}$. Flipping the direction of a labeled current or the polarity of a labeled voltage will lead to negation of the value.

The power dissipated by a circuit element will not change because we follow passive sign convention. Passive sign convention requires that if we flip the direction of an element current we also flip the polarity of the corresponding element voltage, so there is a double negation in the computation of power. The only way to get a different value of power would be to change the component values or the circuit diagram itself by removing or adding more circuit elements. A physical system will only have one behavior as governed by the laws of physics - how we compute our answer should not change how it behaves. Our labeled voltage polarities and current directions are more akin to measurement choices which can change what we see.

The node voltages too, did not change. The top node voltage in both labelings were $5\,\text{V}$ and the bottom node voltages were $0\,\text{V}$. What would have to change to alter these values is one of three things: either the location of the reference, the circuit component values, or the circuit diagram itself.