

EECS 16A Designing Information Devices and Systems I Discussion 7A

1. Material Resistivity

- (a) Recall the 1D resistive touch screen model introduced in class. In this model, the top layer can be thought of as a resistor, while the bottom layer can be thought of as a wire. When the top layer is touched, it flexes at the touch point and makes contact with the bottom layer. This results in a voltage divider.

Material	Resistivity ρ ($\Omega \text{ cm}$)	Conductivity $1/\rho$ ($\Omega^{-1} \text{ cm}^{-1}$)
Silver	1.6×10^{-5}	6.3×10^4
Aluminum	2.7×10^{-5}	3.7×10^4
Carbon (graphite)	10×10^{-2}	10
Rubber	100×10^{12}	1×10^{-14}

Given the following list of materials and their resistivity/conductivity, which materials would be good to use as a top layer, and which would be good to use as a bottom layer? Why?

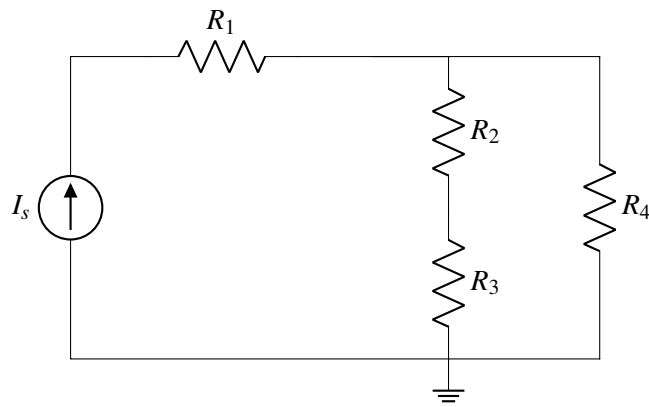
- (b) Let's say you want to make your own **10 cm** long resistor out of **graphite**. You need the resistance to be **1 Ω** . Recall the equation for resistance: $R = \rho \frac{L}{A} = \rho \frac{L}{W \cdot H}$
- What are some possible widths and heights of your resistor?
 - Can you think of advantages of having a wide and thin resistor? How about advantages of a narrow and thick resistor?

Answer:

- (a) We want the bottom layer to act like an ideal wire. Thus, it should be made out of highly conductive material. Silver or aluminum would be a good choice for the bottom layer. We want the top layer to act as a resistor. Thus, it should be made out of a material with a high resistivity. Carbon would be a good choice for the top layer. Although rubber has a higher resistivity, rubber is an insulator and will not conduct, which is why it wouldn't work for our purposes.
- (b) Any choice of W and H works for $W \times H = 1 \text{ cm}^2$ (such as $W=1 \text{ cm}$, $H=1 \text{ cm}$; $W=0.5 \text{ cm}$, $H=2 \text{ cm}$; etc.). A possible advantage for larger width is an increase in surface area, and thus better heat dissipation. Power generates heat which needs to be dissipated from the circuit. However, a smaller surface area as a result of a smaller resistor width can also be advantageous. A resistor with less surface area will take up less space on a circuit.

2. More Node Voltage Analysis!

Given $I_s = 1 \text{ A}$, $R_1 = 4 \Omega$, $R_2 = 2 \Omega$, $R_3 = 3 \Omega$, and $R_4 = 10 \Omega$, find the voltage across and the current through each of the resistors in the circuit below:



Answer: To solve this circuit, let's go through the steps of node voltage analysis that we've learned. We always begin with labeling our circuit diagram:

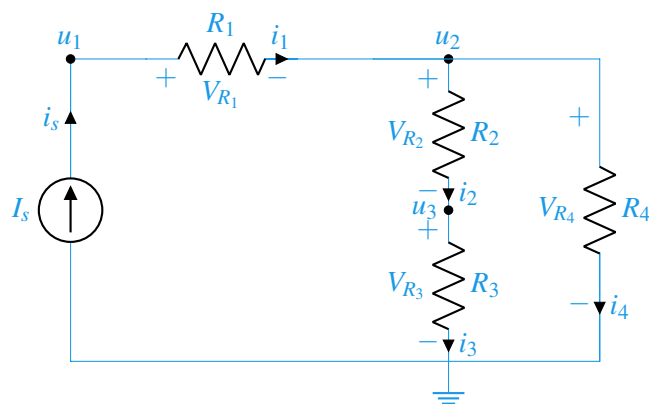
Step 1 Select a reference node. This has already been done for us, and we see that the node at the bottom of our circuit is our reference.

Step 2 Label the nodes. First, we label all of the known voltages determined by voltage sources, of which there are none in this circuit, and then label the remaining nodes, u_1 , u_2 , and u_3 .

Step 3 Label currents through non-wire elements. We have five elements in our circuit, so we draw the currents flowing through each of them as arrows, where the direction is arbitrary.

Step 4 Label element potentials based on passive sign convention. Remember, this means that our current should enter the positive terminal and exit the negative terminal of each of our elements.

At this point, we have labeled our diagram as in the one below:



Note that your diagram might not look exactly like this, but as long as you follow passive sign convention in your labelings and are consistent, your labeling is also valid! Now, let's start finding equations to solve for the unknown values!

Step 5 Use KCL to write equations with our unknowns. Remember, this means that the current entering a node is the same as the current exiting a node.

$$\begin{aligned} i_s - i_1 &= 0 \text{ at node } u_1 \\ i_1 - i_2 - i_4 &= 0 \text{ at node } u_2 \\ i_2 - i_3 &= 0 \text{ at node } u_3 \end{aligned}$$

Step 6 Use Ohm's Law, $V = IR$, to write IV relationships of non-wire elements. Since $I = V/R$, we have the following equations for our elements:

$$\begin{aligned}i_1 &= \frac{V_{R_1}}{R_1} \\i_2 &= \frac{V_{R_2}}{R_2} \\i_3 &= \frac{V_{R_3}}{R_3} \\i_4 &= \frac{V_{R_4}}{R_4} \\i_s &= I_s\end{aligned}$$

We know that the difference in potentials across each element must be the voltage across that element, so we also have the following equations:

$$\begin{aligned}V_{R_1} &= u_1 - u_2 \\V_{R_2} &= u_2 - u_3 \\V_{R_3} &= u_3 - 0 = u_3 \\V_{R_4} &= u_2 - 0 = u_2\end{aligned}$$

Step 7 Solve the system of equations! Let's first rewrite the currents in terms of the node voltages:

$$\begin{aligned}i_1 &= \frac{u_1 - u_2}{R_1} \\i_2 &= \frac{u_2 - u_3}{R_2} \\i_3 &= \frac{u_3}{R_3} \\i_4 &= \frac{u_2}{R_4} \\i_s &= I_s\end{aligned}$$

Now, we can substitute these expressions for current into our KCL equations above:

$$\begin{aligned}I_s - \frac{u_1 - u_2}{R_1} &= 0 \\ \frac{u_1 - u_2}{R_1} - \frac{u_2 - u_3}{R_2} - \frac{u_2}{R_4} &= 0 \\ \frac{u_2 - u_3}{R_2} - \frac{u_3}{R_3} &= 0\end{aligned}$$

We see that $I_s = \frac{u_1 - u_2}{R_1}$ from the first KCL equation, so we can replace $\frac{u_1 - u_2}{R_1}$ in the second KCL equation with I_s , which is:

$$I_s - \frac{u_2 - u_3}{R_2} - \frac{u_2}{R_4} = 0$$

From the third KCL equation, we have that:

$$\frac{u_2 - u_3}{R_2} = \frac{u_3}{R_3}$$

We can write u_3 in terms of u_2 by rearranging:

$$\begin{aligned}u_2 - u_3 &= \frac{R_2 u_3}{R_3} \\ u_2 &= \frac{R_2 u_3}{R_3} + u_3 \\ u_2 &= \frac{R_2 u_3}{R_3} + \frac{R_3 u_3}{R_3} \\ u_2 &= \left(\frac{R_2 + R_3}{R_3}\right) u_3\end{aligned}$$

$$u_3 = \left(\frac{R_3}{R_2 + R_3}\right)u_2$$

This looks like our voltage divider formula, where u_3 is proportional to u_2 ! Substituting in for u_3 in our third KCL equation from above, we have:

$$I_s - \left(\frac{u_2 - \left(\frac{R_3}{R_2 + R_3}\right)u_2}{R_2}\right) - \frac{u_2}{R_4} = 0$$

We can rearrange this to find:

$$(R_2 R_4)I_s - R_4 u_2 + \left(\frac{R_4 R_3}{R_2 + R_3}\right)u_2 - R_2 u_2 = 0$$

$$(R_2 R_4)I_s = R_4 u_2 - \left(\frac{R_4 R_3}{R_2 + R_3}\right)u_2 + R_2 u_2$$

$$I_s = \frac{R_4 - \left(\frac{R_4 R_3}{R_2 + R_3}\right) + R_2}{R_2 R_4} u_2$$

$$u_2 = \frac{R_2 R_4}{R_4 - \left(\frac{R_4 R_3}{R_2 + R_3}\right) + R_2} I_s$$

Substituting the values given for I_s , R_2 , R_3 , and R_4 , we have:

$$u_2 = \frac{2\Omega * 10\Omega}{10\Omega - \left(\frac{10\Omega * 3\Omega}{2\Omega + 3\Omega}\right) + 2\Omega} * 1\text{ A}$$

$$u_2 = \frac{20}{6}\text{ V}$$

Then, u_3 is:

$$u_3 = \left(\frac{3\Omega}{2\Omega + 3\Omega}\right)\frac{20}{6}\text{ V}$$

$$u_3 = \left(\frac{3\Omega}{5\Omega}\right)\frac{20}{6}\text{ V}$$

$$u_3 = 2\text{ V}$$

Since we know $i_s = i_1$ from the first KCL equation, we can now find the voltage across each of the resistors:

$$V_{R_1} = u_1 - u_2 = i_1 R_1 = 1\text{ A} * 4\Omega = 4\text{ V}$$

$$V_{R_2} = u_2 - u_3 = \frac{20}{6}\text{ V} - 2\text{ V} = \frac{4}{3}\text{ V}$$

$$V_{R_3} = u_3 - 0 = u_3 = 2\text{ V}$$

$$V_{R_4} = u_2 - 0 = u_2 = \frac{20}{6}\text{ V} = \frac{10}{3}\text{ V}$$

Now, we can find the current through each of the resistors:

$$i_1 = i_s = 1\text{ A}$$

$$i_2 = \frac{V_{R_2}}{R_2} = \frac{\frac{4}{3}\text{ V}}{2\Omega} = \frac{4}{6}\text{ A} = \frac{2}{3}\text{ A}$$

$$i_3 = \frac{V_{R_3}}{R_3} = \frac{2\text{ V}}{3\Omega} = \frac{2}{3}\text{ A}$$

$$i_4 = \frac{V_{R_4}}{R_4} = \frac{\frac{10}{3}\text{ V}}{10\Omega} = \frac{1}{3}\text{ A}$$

Thus, we have solved for all of our unknown values! We could have also written these equations in matrix-vector form and solved for our unknown values using Gaussian Elimination.