# EECS 16A Designing Information Devices and Systems I
# Spring 2022 Homework 2

## This homework is due February 4th, 2022, at 23:59.
## This homework is due February 7th, 2022, at 23:59.

**Submission Format**

Your homework submission should consist of a single PDF file that contains all of your answers (any handwritten answers should be scanned) as well as your IPython notebook saved as a PDF.

If you do not attach a PDF "printout" of your IPython notebook, you will not receive credit for problems that involve coding. Make sure that your results and your plots are visible. Assign the IPython printout to the correct problem(s) on Gradescope.

**1. Reading Assignment**

For this homework, please read Notes 2A, and 2B. They will provide an overview of Gaussian elimination, vectors, and matrices. You are always welcome and encouraged to read beyond this as well.
Please answer the following questions: How can Gaussian elimination help you determine if there are no solutions to a particular system of equations? How can Gaussian elimination help you determine if a particular system has a unique solution? How about an infinite number of solutions? Does a row of zeros always mean there are infinite solutions?

**2. Image Masks**

***Learning Objective:*** *Learn to setup imaging problems with matrices.*

For these word problems, you only need to setup the problem with Gaussian elimination or matrix-vector notation. Of course, you may solve for practice, but no additional credit is awarded.

After your first EECS16A lecture, you decide to try to build a single-pixel camera. You want to take a 2x2 image, i.e. 4 tiles, and based on the first lecture, you choose to take 4 measurements. Recall that each measurement is the sum of the illuminated tiles. For each measurement, you will use a different mask.

(a) Initially, you want to illuminate only one tile for each measurement. That is, you will first illuminate $x_1$, then you will illuminate $x_2$, etc. The outputs of your 4 measurements are $y_1$, $y_2$, $y_3$, and $y_4$ respectively. The 4 measurements you take are shown in Figure **??**. Explicitly setup the matrix problem for this in the $A\vec{x} = \vec{b}$ form.

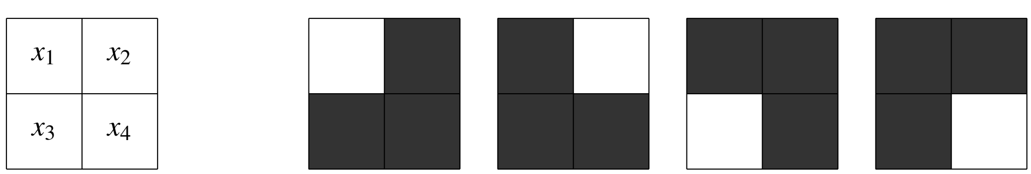


Image tile labels   Measurement 1   Measurement 2   Measurement 3   Measurement 4

Figure 1: Four image masks.

(b) While setting up your code to create the masks, you forget to turn off the illuminated tiles from the previous measurement. As a result, measurement one contains $x_1$, measurement two contains $x_1 + x_2$, etc. The outputs of your 4 measurements are $z_1$, $z_2$, $z_3$, and $z_4$ respectively. The 4 measurements you take are shown in Figure **??**. Explicitly setup the matrix problem for this in $A\vec{x} = \vec{b}$ form.

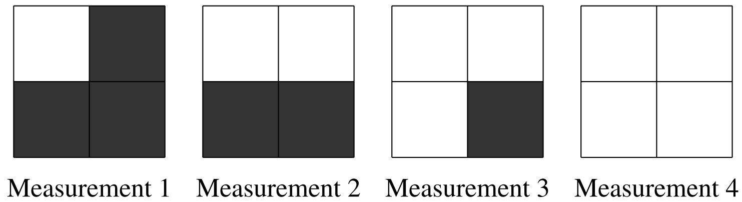


Figure 2: Four image masks.

(c) Your friend is also building their own single pixel camera. However, they make a different mistake in their code and during each measurement, instead of lighting up one tile, you light up the other 3 tiles instead. That is, instead of measuring $x_1$, they measure $x_2 + x_3 + x_4$. The output of the 4 measurements are $w_1$, $w_2$, $w_3$, and $w_4$. The 4 measurements from their setup are shown in Figure **??**. Explicitly setup the matrix problem for this in $A\vec{x} = \vec{b}$ form.

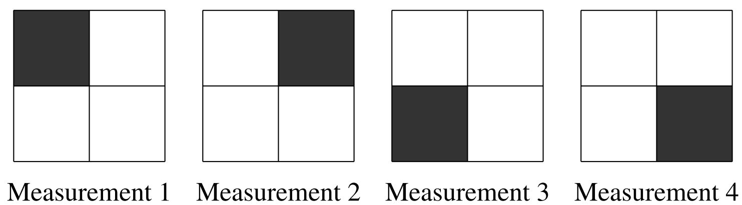


Figure 3: Four image masks.

## 3. Vector-Vector Multiplication

***Learning Objective:*** *Practice evaluating vector-vector multiplication.*

For the following multiplications, state the dimensions of the result. If the product is not defined and thus has no solution, state this and justify your reasoning. For this problem $\vec{x} \in \mathbb{R}^N, \vec{y} \in \mathbb{R}^N, \vec{z} \in \mathbb{R}^M$, with $N \neq M$.

(a) i. $\vec{x}^T \cdot \vec{z}$

ii. $\vec{x} \cdot \vec{x}^T$

iii. $\vec{x} \cdot \vec{y}^T$

iv. $\vec{x} \cdot \vec{z}^T$

## 4. Multiply the Matrices

***Learning Objective:*** *Practice evaluating matrix-matrix multiplication.*

(a) We have two matrices $\mathbf{A}$ and $\mathbf{B}$, where $\mathbf{A}$ is a $3 \times 2$ matrix and $\mathbf{B}$ is a $2 \times 4$ matrix. Would the multiplication $\mathbf{AB}$ be a valid operation? If yes, what do you expect the dimensions of $\mathbf{AB}$ to be?

 

(b) Compute **AB** by hand, where **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 1 \end{bmatrix}, \text{ and } \qquad \mathbf{B} = \begin{bmatrix} 1 & 2 & -1 & 0 \\ -3 & 0 & 2 & -1 \end{bmatrix}$$

Compute **BA** too if the operation is valid. If it is invalid, explain why. Make sure you show the work for your calculations.

(c) Compute **AB** by hand, where **A** and **B** are given by

$$\mathbf{A} = \begin{bmatrix} 3 & 21 & 9 \\ -1 & 14 & 4 \\ 7 & -8 & 2 \end{bmatrix}, \text{ and } \qquad \mathbf{B} = \begin{bmatrix} -2 & 4 \\ -1 & 2 \\ 3 & -6 \end{bmatrix}$$

Compute **BA** too if the operation is valid. If it is invalid, explain why. Make sure you show the work for your calculations.

## 5. Filtering Out The Troll

**Learning Goal:** *The goal of this problem is to explore the problem of sound reconstruction by solving a system of linear equations.*

You were attending the 16A lecture the day before the first exam, and decided to record it using two directional microphones (one microphone receives sound from the x direction and the other from the y direction), because you don't trust the Zoom overlords. However, someone (we have *no* idea who) in the audience was trolling around loudly, adding interference to the recording! The troll's interference dominates both of your microphones' recordings, so you cannot hear the recorded speech. Fortunately, since your recording device contained two microphones, you can combine the two individual microphone recordings to remove the troll's interference.

The diagram shown in Figure **??** shows the locations of the speaker, the troll, and you and your two microphones (at the origin).

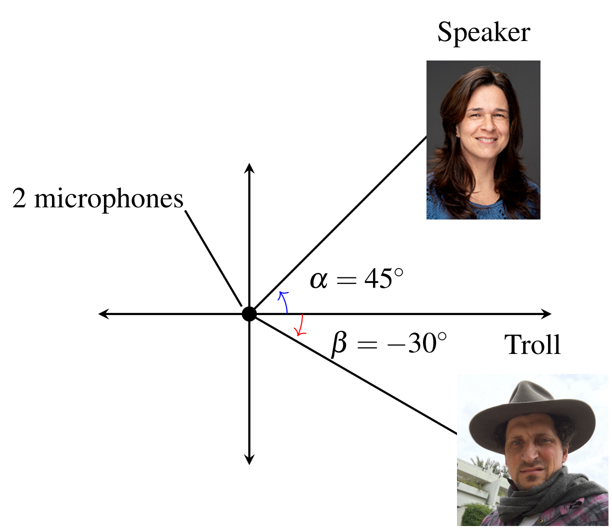


Figure 4: Locations of the speaker and the troll.

Since the microphones are directional, the strength of the recorded signal depends on the angle from which the sound arrives. Suppose that the sound arrives from an angle $\theta$ relative to the $x$-axis (in our case, these angles are 45°and $-30°$, labeled as $\alpha$ and $\beta$, respectively). The first microphone scales the signal by $\cos(\theta)$, while the second microphone scales the signal by $\sin(\theta)$. Each microphone records the weighted sum (or linear combination) of all received signals.

The speech signal can be represented as a vector, $\vec{s}$, and the troll's interference as vector $\vec{r}$, with each entry representing an audio sample at a given time. The recordings of the two microphones are given by $\vec{m}_1$ and $\vec{m}_2$:

$$\vec{m}_1 = \cos(\alpha)\cdot\vec{s} + \cos(\beta)\cdot\vec{r} \tag{1}$$

$$\vec{m}_2 = \sin(\alpha)\cdot\vec{s} + \sin(\beta)\cdot\vec{r} \tag{2}$$

where $\alpha$ and $\beta$ are the angles at which the professor and the troll respectively are located with respect to the x-axis, and variables $\vec{s}$ and $\vec{r}$ are the audio signals produced by the professor and the troll respectively.

(a) Plug in $\alpha = 45° = \frac{\pi}{4}$ and $\beta = -30° = -\frac{\pi}{6}$ to Equations **??** and **??** to write the recordings of the two microphones $\vec{m}_1$ and $\vec{m}_2$ as a linear combination (i.e. a weighted sum) of $\vec{s}$ and $\vec{r}$.

(b) Solve the system using any convenient method you prefer from the earlier part to recover the important speech $\vec{s}$ as a weighted combination of $\vec{m}_1$ and $\vec{m}_2$. In other words, write $\vec{s} = c\cdot\vec{m}_1 + k\cdot\vec{m}_2$ (where $c$ and $k$ are scalars). What are the values of $c$ and $k$?

(c) Partial IPython code can be found in `prob2.ipynb`, which you can access through the Datahub link associated with this assignment on the course website. Complete the code to get the signal of the important speech. Write out what the speaker says. (Optional: Where is the speech taken from?)

*Note:* You may have noticed that the recordings of the two microphones sound remarkably similar. This means that you could recover the real speech from two "trolled" recordings that sound almost identical! Leave out the fact that the recordings are actually different, and have some fun with your friends who aren't lucky enough to be taking EECS16A.

**6. Linearity**

In this question, we will explore in further detail what exactly it means for a function to be linear. For each of the following, please specify the values of $a$ (a real number) for which the function is linear. Here, $x$ and $y$ are variables.

(a)

$$f(x,y) = (3-a)x + 2ay$$

(b)

$$f(x,y) = a^2x + 8y$$

(c)

$$f(x,y) = y + axy - 3x$$

(d)

$$f(x,y) = (x+ay)^2$$

## 7. Gaussian Elimination

**Learning Goal:** *Understand the relationship between Gaussian elimination and the graphical representation of linear equations, and explore different types of solutions to a system of equations. You will also practice determining the parametric solutions when there are infinitely many solutions.*

(a) In this problem we will investigate the relationship between Gaussian elimination and the geometric interpretation of linear equations. You are welcome to draw plots by hand or using software. Please be sure to label your equations with a legend on the plot.

i. Plot the following set of linear equations in the $x$-$y$ plane. If the lines intersect, write down the point or points of intersection.

$$x + 2y = 4 \tag{1}$$
$$2x - 4y = 4 \tag{2}$$
$$3x - 2y = 8 \tag{3}$$

ii. Write the above set of linear equations in augmented matrix form and do the first step of Gaussian elimination to eliminate the $x$ variable from equation 2. Now, the second row of the augmented matrix has changed. Plot the corresponding new equation created in this step on the same graph as above. What do you notice about the new line you draw?

iii. Complete all of the steps of Gaussian elimination including back substitution. Now plot the new equations represented by the rows of the augmented matrix in the last step (after completing back substitution) on the same graph as above. What do you notice about the new line you draw?

(b) Write the following set of linear equations in augmented matrix form and use Gaussian elimination to determine if there are no solutions, infinite solutions, or a unique solution. If any solutions exist, determine what they are. You may do this problem by hand or use a computer. We encourage you to try it by hand to ensure you understand Gaussian elimination. Remember that it is possible to end up with fractions during Gaussian elimination.

$$\begin{aligned} x + 2y + 5z &= 3 \\ x + 12y + 6z &= 1 \\ 2y + z &= 4 \\ 3x + 16y + 16z &= 7 \end{aligned}$$

(c) Consider the following system:

$$\begin{aligned} 4x + 4y + 4z + w + v &= 1 \\ x + y + 2z + 4w + v &= 2 \\ 5x + 5y + 5z + w + v &= 0 \end{aligned}$$

If you were to write the above equations in augmented matrix form and use Gaussian elimination to solve the system, you would get the following (for extra practice, you can try and do this yourself):

$$\left[\begin{array}{ccccc|c} 1 & 1 & 0 & 0 & 3 & 16 \\ 0 & 0 & 1 & 0 & -3 & -17 \\ 0 & 0 & 0 & 1 & 1 & 5 \end{array}\right]$$

How many variables are free variables? Determine the solutions to the set of equations.

**8. Homework Process and Study Group**

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.

 