

#	Question	Answer(s)
1	doesn't $Ax=b$ having a unique solution mean that the columns are linearly dependent, since A's columns can be written as a linear combination?	Linear dependence would mean that one of the columns of A can be written as a linear combination of the other columns of A. If b were a column of A, then we would have linear dependence.
2	are constructive proofs direct proofs?	Not exactly. Constructive proofs will literally construct an answer to prove existence. Direct proofs just generally proceed with logic. Sometimes constructive proofs are considered a subset of direct proofs.
3	In the "Summary: Equivalent Statements," doesn't it only hold that A is invertible if A is specifically a square matrix, not just any matrix with linearly independent columns?	Yes, A needs to be a square matrix to be invertible.
4	wouldn't you need the a_k multiplied by a scalar?	The scaling is handled in the linear combination.
5	where does the lowercase 'a' vector come from?	The a vectors are from the columns of the matrix A
6	wait.. shouldn't a_1, a_2, \dots, a_{k-1} not have arrows?	They are vectors (columns vectors of the matrix A)
7	Why is it necessary to put the $k-1$ and the $k+1$ when we write n anyway	We want to explicitly skip over a_k in the sum. Otherwise, its trivial to write a_k as a sum of vectors that includes itself.
8	why do we ignore the zero solutions case? why can't we try and prove $Ax = b$ has zero solutions?	We could do that, but that doesn't help us go toward the conclusion we are trying to prove. Linear dependence would mean we have multiple solutions.
9	Wouldn't it be faster to express a_k as half of the summation of $c_i a_i$ for all $1 \leq i \leq n$?	
10	How do you prove that there is no solution? Is there some type of matrix pattern to see with no solution?	We can always use Gaussian elimination to show we have no solutions.
11	what is x_{star} ?	we assume x_{star} is the unique solution (and we want to show this assumption is not correct)
12	is this a constructive proof or proof by contradiction?	This is a proof by contradiction.
13	why do you have to prove both?	"Doesn't have a unique solution" means (1) no solution; or (2) infinitely many solutions. So we need to prove both
14	what does x_{star} represent	we assume x_{star} is the unique solution (and we want to show this assumption is not correct)
15	Are we allowed to format our proofs like this? (A chart with two columns)	
16	what happened to $c_{sub k}$?	live answered
17	The unit vector is x^* ?	
18	If you prove one of the two statements, wouldn't it automatically disregard the other?	In the beginning we don't know whether the given $Ax=b$ has solution or not, so we need to discuss the two situations.
19	What does picking out A_k mean? Setting A_k to be 1?	
20	why don't we divide both sides of the equation by $c_{sub k}$	We have all the different c_k , not sure how that would help
21	Could you explain how the k -th element picks out the k -th column?	This is from the column view of matrix-vector multiplication
22	How does the 2nd matrix represent A_k ?	This is from the equation above where we said a_k is a linear combination of the other columns.

23	In a proof by contradiction, how can you tell whether it, is your assumption or your definitions that are wrong?	We will assume one thing, and all other definitions/derivations should be mathematically correct, so the only thing can be wrong is the assumption
24	Why -1?	
25	Did you just factor out a negative, is that why A is -A?	No, its because we have 0 on the RHS, so factoring out the -1 doesn't affect our conclusion.
26	You can factor out matrices?	
27	why isn't the A negative in the solution? shouldn't the items in the column be negative if the A is positive	Yes, she skipped a step of factoring out -1. The same conclusion applies.
28	how is the vector \vec{a} with 0, and 1, equal to the one with c_1 and c_2 ?	We defined the \vec{a}_k as a linear combination of the other vectors above. We just rewrote the two sides of the equations in matrix-vector form..
29	can you re-explain how that became zero vector again?	
30	are the C's supposed to be first or is $1 - 0 = -1$	Yes, she skipped a step of factoring out -1. The same conclusion applies.
31	where did the negative sign on the A go, I am confused	She skipped a step of factoring out -1. The same conclusion applies.
32	Where did the negative go? Did we divide both sides by it but it doesn't affect zero?	Yes, she skipped a step of factoring out -1. The same conclusion applies.
33	why is vector \vec{w} not $-c_1, -c_2, \dots, 1, \dots, -c_n$?	Yes, she skipped a step of factoring out -1. The same conclusion applies.
34	shouldn't it be positive one because the vector would be $1 - 0$	Yes, she skipped a step of factoring out -1. The same conclusion applies.
35	Why was the negative sign in front of $A[c_1, \dots, (-1), \dots, c_n] = 0$ removed?	We can multiply (-1) to both sides
36	Can you re-explain why the \vec{w} vector cannot be the zero vector?	The 2 vectors we added explicitly are nonzero and equal. So adding / subtracting them will give us a nonzero vector
37	how are the c values not negative? $0 - c_1 = -c_1$	we can multiply (-1) to both sides
38	Is it possible to set one of the C's equal to 1 to cancel out the -1 when it's multiplied out with A?	
39	Why can't all the c's add to 1? since they can be defined later? Then it becomes a 0 vector?	
40	how can you substitute $A\vec{w}$ for the 0 vector if $A\vec{w}$ doesn't equal 0	$A\vec{w} = 0$ here by our constructions
41	what about the case with no solutions?	live answered
42	What about the no sol case?	live answered
43	why is the orange highlight -1 instead of 1?	She skipped a step of factoring out -1. The same conclusion applies.
44	how did we get $A[0, \dots, 1, \dots, 0]$ equal to $A[c_1, \dots, 0, \dots, c_n]$?	They both equal to the k-th column of the matrix A
45	shouldn't we be able to just stop at the contradiction that the \vec{w} vector is not the zero vector but when you multiply it to A, you get the zero vector	We have to explicitly show the contradiction that there are actually more than 1 solutions.
46	What does she mean by proving there are multiple solutions	Recall that what we wanted to prove is linear dependence. Showing multiple solutions would prove that.
47	How come we don't have to prove the no solution possibility?	live answered

48	how does the final line of the proof show that there must be multiple sols, if sols exist?	We showed that if there is a solution x_{star} , then we also have another solution $(x_{\text{star}} + w)$, so there are multiple solutions.
49	how do we prove that there is no solution?	live answered
50	Could we prove that there are an infinite amount of solutions	Yes, $(x_{\text{star}} + kw)$ where k is a scalar are all solutions
51	did we just used contradiction	Yes
52	How did we prove that there was no solution?	The original equation $Ax=b$ may have (1) no solution or (2) at least one solution, we don't know which one is true, so we need to discuss both situations. If (1) is true in the beginning then we automatically don't have one unique solution. If (2) is true in the beginning then we proved that there must be infinitely many solution. So either way the statement "no unique solution" is true.
53	How did we prove that there is no solution?	The original equation $Ax=b$ may have (1) no solution or (2) at least one solution, we don't know which one is true, so we need to discuss both situations. If (1) is true in the beginning then we automatically don't have one unique solution. If (2) is true in the beginning then we proved that there must be infinitely many solution. So either way the statement "no unique solution" is true.
54	What about no solution?	live answered
55	Why does $x_{\text{star}} + w$ show multiple solutions? Is it just that it, "is an additional solution just by being another equation?"	$(x_{\text{star}} + w)$ is also a solution to the equation $Ax=b$
56	I, "am still a little confused why we didnt prove no sol if we need both? could you please explain	The original equation $Ax=b$ may have (1) no solution or (2) at least one solution, we don't know which one is true, so we need to discuss both situations. If (1) is true in the beginning then we automatically don't have one unique solution. If (2) is true in the beginning then we proved that there must be infinitely many solution. So either way the statement "no unique solution" is true.
57	Can you reexplain why there is no solution for the previous problem?	The original equation $Ax=b$ may have (1) no solution or (2) at least one solution, we don't know which one is true, so we need to discuss both situations. If (1) is true in the beginning then we automatically don't have one unique solution. If (2) is true in the beginning then we proved that there must be infinitely many solution. So either way the statement "no unique solution" is true.
58	Is the top example wrong? Why did she rewrite it?	Since we are using a general θ , then the $[x - y]$ doesn't necessarily hold
59	If we had to prove a function is linear, as long as we prove those 2 statements, it, "is right?"	Yes. As long as you prove scalar multiplication and addition, then it is linear.
60	Would we have to specify $c_k \neq 0$?	Yes if we want to divide both sides by c_k .
61	does this mean that the matrix A acts as a mapping that takes in 1 vector and outputs another	yes
62	where is the 2 other 2 in $f(x) = 2x$	
63	Do these linear transformations work with affine linear equations?	Linear transforms are a subset of affine transforms
64	are trig functions linear? how come those can be used too?	They are not linear in terms of the angles. Once we select a particular angle then they are just scalars

65	So would matrix matrix multiplication be mapping a list of vectors?	You can think of it as mapping a list of vectors, or as a mapping a whole matrix.
66	i dont understand how we prove that there is no solutions for the proof	We did not prove that. The original equation $Ax=b$ could have no solution in the beginning (say we have two equations $x+y = 1$ and $x+y = 2$). If this is true then we automatically don't have "one unique solution". The original equation $Ax=b$ could also have at least one solution (say we have $x + y = 1$ and $2x + 2y = 2$), and we proved that in this case there must be multiple solutions.
67	where and when are the slides posted?	Slides are posted to the course website, within a couple hours after lecture
68	long, lat	yup, that can be useful for defining global location
69	What does 1 represent in our state system?	With the arrows, it describe the percentage of each state that transitions. For example the arrow from B to C that is 1 says that 100% of B transitions to C.
70	is the systme the 3 reservoirs or the 3 reservoirs + 3 pumps	We will describe the state as the 3 reservoirs. The pumps describe how the state transitions.
71	Does the 1 represent that 100%, aka all of the water, move from the reservoirs or just 1%?	$1 = 100\%$
72	why does the water in resivoir a at time 1 return three elements? what do those mean?	$x(1)$ describe the state of all 3 reservoirs. $[1 \ 0 \ 1]$ would say that $x_a(1) = 1$, $x_b(1) = 0$, and $x_c(1) = 1$.
73	ohhhhh the x vector is the ssystem. sorry got it confused with X_a	live answered
74	does it mean 100% of water is always moving from $A \rightarrow A$, $B \rightarrow C$, and $C \rightarrow A$, how can $x(1)$ be $[1, 0, 1]$?	* $C \rightarrow B$. Its not 100% of the total water in the system moving from $A \rightarrow A$ but rather 100% of the water in A moving $A \rightarrow A$. Similarly 100% of the water in B moves to C, and 100% of the water from C moves to B.
75	Wouldnt the pumps betwn c and b need to transfer $X_c(t)$ and $X_b(t)$ quantities of water, not just one between b and c for that system to be true?	It would be $1 * X_c(t)$ and $1 * X_b(t)$.
76	both b and c are losing 1 and getting 1 so shouldn't it be same?	the "1" means 100%, so 100% of b moves to c, 100% of c moves to b, but they may have different amount in the beginning
77	If we had $X_a(t + 10)$, then the first row would have been 10, 0, 0. Right?	Because $x_a(t+1) = x_a(t)$, so we will have $x_a(t+10) = x_a(t)$
78	could you explain why r2 and r3 of Q arent swtiched to be the other way?	We derive this from the modelling equations we defined.
79	Can we divide a matrix by both sides for example: $Ax = Aw$ is also equal to $x = w$ with x and w being vectors	Generally no
80	Can you always square a matrix no matter what its size is?	No, the multiplication must be defined (the dimensions should match, so only square matrix can be squared)
81	to sqare a matric we do NOT square each element right? We apply multiplication normally yes?	yes we apply matrix multiplication normally
82	what would happen if we multiplied a 3x1 by itself? It fails the prior restriction we had so do we have transpose it first?	You are right, we cannot square a 3x1 matrix.

83	Shouldn't the first pump run have the output matrix be $[X_a(t+1), X_c(t+1), X_b(t+1)]$	That would be true if we made the matrix an identity matrix. But we want to keep the states in the same order, so we can do repeated operations on it over time, like we're doing here.
84	Is this a markov chain?	Yes, this is a form of Markov chain, but we will not formalize that or go into details of markov chains in this class.
85	Is any $n \times n$ matrix squared = identity matrix?	No. try this conterexample: $[2 \ 0; 0 \ 2]$ squared
86	What is Q?	Q is the transition matrix, describing how the system transitions at each time step.
87	how does $Q^*(x+1) = Q^*Q(x)$	$x(t+1) = Q^*x(t)$, so we can replace $Q^*x(t+1)$ with $Q^*Q^*x(t)$
88	why do we say that the state is not changing when Q^*Q is the identity matrix?	When we multiply Q^*Q (identity matrix) to a state vector, the vector does not change
89	shouldn't $x(t)$ and $x(t+1)$ switched..?	They should be switched.
90	why is $x(t+1)$ the vector multiplied?	They should be switched.
91	why do we multiply by the $x(t+1)$ matrix instead of $x(t)$?	They should be switched.
92	\wedge vector	They should be switched.
93	why isn't the first vector $x(t)$ instead of $x(t+1)$	They should be switched.
94	should $x(t+1)$ and $x(t)$ be switched in the vector columns	They should be switched.
95	shouldn't the $x(t+1)$ and $x(t)$ be in different vectors?	They should be switched.
96	So would it basically be nonconservative if it's not the identity matrix?	not necessarily. The first example where B gives 100% to C and C gives 100% to B is also conservative.
97	what is a conservative system?	
98	What's the difference between a conservative and non-conservative system	
99	does non conservative depend on the total water or the water in each reservoir? example if A pumps double the water back to itself. But B and C pump 1/2 to each other.	(Non)-Conservative depends on the total water in the system
100	Why is $x_c(t+2)$ instead of $(t+1)$	should be $(t+1)$
101	I think you're missing an arrow from c to b with a value of 1/6	Nope there is no c to b arrow
102	why is it $X_c(t+2)$ not $X_c(t+1)$	should be $(t+1)$
103	when determining how much each node is contributing, do we look the matrix it in a row view?	live answered
104	Is this system conservative>?	yes
105	Why is it $[x_a(t_1), x_b(t_1) x_c(t_2)]$? and not $x_c(t_1)$?	should be $x_c(t+1)$