

#	Question	Answer(s)
1	what if we inverse the vector b	In this class we only inverse a square matrix. So a vector cannot be inverted.
2	When we do gaussian elimination how do you know it is the inverse again?	When you eliminate the left side of the augmented matrix to an identity matrix, the right side of the augmented matrix will be the inverse
3	when doing gauss jordan, what goes on the right side of the augmented matrix?	It's similar to the normal Gaussian elimination, the same rules and operations, just on the right side we have multiple columns. Think about it like doing multiple Gaussian eliminations together.
4	When online calculators solve $Ax=b$ , would they typically actually perform the Gaussian Elimination on the system or calculate the inverse and multiply by b?	Computationally, they are pretty much the same thing, so it depends on the implementation.
5	Can you explain again the graph professor drew about if $Ax=b$ has infinite solutions, then A is not invertable?	If we have multiple solutions, we can map multiple x vectors to the same b, so we cannot undo this operation to find out which x we begin with.
6	is the null space the kernal of A?	yes
7	what is a kernal?	A kernal is the same as the null space for our discussion
8	we already have A right	Yes, A is given. We are trying to identify the null space of A
9	what does the null space say about a system	Hopefully, this is a little clear now in lecture, but we can use null space to look at certain transition problems.
10	What does it mean by how many solutions does it have in null space?	The null space is the set of all solutions to $Ax=0$ . The number of solutions determines the dimension of the null space. We will see more later in this lecture.
11	if either the columns or rows of a matrix are a linear combination of other rows or columns does that mean the matrix is linearly dependent?	Yes that means the columns (rows) are linearly dependent, and the matrix (if square) is not invertible.
12	for the first example, in GE the last row is $0 \ -1 \   \ 0 \ 1$ thought that was no solutions	The only solution we showed was the trivial solution, i.e. the zero vector. There are no nonzero solutions, which are generally the ones we care about. The zero vector is always in the null space, so we don't talk about.
13	do we need to write it in terms of t or can we leave it in terms of $x_2$ ?	Yeah you can leave it in terms of $x_2$ . Just mention $x_2$ is the free variable and can be any real number.
14	Why do we write the answers in terms of t, rather than like $x_1=-2x_2$	Yeah you can also write $x_1=-2x_2$ . Just mention $x_2$ is your free variable.
15	finding the null space is the same as finding the solution to any other equation right, except that we have the zero vector rather than another arbitrary vector	yep! so we can use the same tools we 've already been developing
16	could you please explain the last step again? im confused how you went from $t * \text{vector} \rightarrow \text{the span}$ . Thanks!	This is from the idea of linear combination. $t * \text{vector}$ is a linear combination, so we can rewrite that using span definition.
17	so do we have to get it to that final span part or if we stoped at $x = [-2t, t]$ would that be a fine answer for nullspace too?	No, we want to define the null space using the vector components. That's something we want in general as we start talking more about vector spaces today.

18	does the nullspace always include the 0 vector?	Yes!
19	what does the null space represent exactly? the set of vectors that have no effect?	Yes it represent the solutions to $Ax=0$ , so you can think that $x$ has no effect to the results under the operation of $A$
20	what is the span for example one??	The first sample only had the zero vector in the null space, so its just the zero vector.
21	So in this case, does it mean that all of $\text{Span}(-2,1)$ are mapped to 0?	yep!
22	for something to be invertible, there has to be trivial nullspce right?	Yes
23	do we say that this is the nullspace of matrix $A$ or vector $x$ ?	It's the null space of matrix $A$
24	Is the null space of a linearly independent matrix always going to be the zero vector?	yes. We'll define that fully now.
25	So is the nullspace for first example a single zero vector than? or is it a span of zero vectors? (Although it is redundant.. )	They are the same. The span of 0 vector is the same of the set that only has the 0 vector
26	do all of the five rules have to be true for $A$ to be invertible?	What we will find is that if one of the rules is true, the others will have to be true.
27	so if $A$ is lin indep, is the nullspace always trivial meaning $x = 0$ vector	If the columns of $A$ are linearly independent, the nullspace is trivial (only 0 vector)
28	Can $Ax=0$ can only have unique solution $[0,0]$ ? There,Ãs no other unique solutions?	Yes. Recall our 2nd example. If the null space contains any other nontrivial vector, its whole span will also be in the nullspace.
29	did she write that when the nullspace is a vector unique solution then it is for sure the 0 vector aka the trivial case?	Yes. Recall our 2nd example. If the null space contains any other nontrivial vector, its whole span will also be in the nullspace.
30	what does $V$ not mean again? a vector with elements?	yes. Specifically, $v_0$ is in the nullspace of $A$
31	for $Atv$ would it not try to scale the matrix then multiply it by the vector or does it not even matter if it does that?	$Atv = tAv = t(Av)$ . So we do $A$ times $v$ first and get 0.
32	what does $X_1$ mean again?	The number of cars in Berkeley going to SF.
33	so if the nullspace of $A$ is non-trivial, $Ax=b$ has infinite solutions?	yep! because the nontrivial nullspace gives us some $Ax=0$
34	Does nontrivial mean that there is no solution or infinite solutions?	infinite solutions. you always have the solution $x=0$ for the equation $Ax=0$ , so it's not possible to have no solutions.
35	how did Prof get $X_1 + tv$	$tV$ is the null space, which is defined by $Ax=0$ . Our infinite solutions come from adding zero to both sides
36	We can find other solutions to $AX=B$ by adding the null space to $X$ . Wouldnt we get to the same vector each time cause adding the null space would just be the same as adding the 0 vector so we get the same vector $X$ that we started with?	If the null space is not trivial, we will have non-zero vectors in the null space.
37	can you repeat why $x_1-x_2 = 0$	live answered
38	what is the context of this traffic problem again? are we trying to find no traffic?	We are assuming no cars stay at each city. For example any car goes in SF ( $x_1$ ) will also go out ( $x_2$ ). And we are trying to find the number of cars ( $x_1, x_2, x_3$ ) going between the cities.

39	how did we do $R3 + 1$	It should be $R3 + R1$
40	in the previous question, what is $X1$ in the $X1 + tv$ solution	ohhh this $x1$ is one solution to $Ax=b$ (assume we know there is a solution $x1$ , and we want to find all other solutions)
41	How do we get this $A$ matrix in the traffic example?	The equations we got were looking at the traffic going in and out of each of the cities / nodes.
42	Is this assuming that the number of cars that goes into a certain city is the same number that leaves?	For this example, yes we used that assumption
43	how did she say that the number of free variable is the dimensions of the nullspace. We have 1 free var and a dimension of 3	The dimension of the null space is not the same as the dimension of vectors in the null space.
44	Hi, what was the conclusion we drew from the Berkeley SF Oakland example? We have a free variable & infinite solutions but I didn't get the null-space relation to it	Recall the null space is the solutions to $Ax=0$ . If we have infinite solutions, that means we must have some solution to $Ax=0$ , since we're always able to add zero. From GE, we associated the free variables with the null space.
45	how did you get 2 free variables?	2 free variables is coming from 2 independent graphs.
46	why is the nullspace 2?	live answered
47	Why does the number of free dimensions determine the dimensions of our nullspace?	live answered
48	how do we know the demension is 2? Is it just the number of free variables?	Yes
49	So dimensions of a nullspace just depends on the number of free variables?	Yes
50	sorry what does dlm mean	live answered
51	Why are the dimensions of the null space 2?	live answered
52	In the first car example, the dim of null space is 2?	In the first example the dimension will be 1 since we have 1 free variable
53	would the dimensions of the vectors be 1 dimension?	The dimension of the vectors depends on the number of elements in the vector. The dimensions of the space are defined by the number of vectors required to span the space.
54	what is the dimension of nulls pace for the first example that has one free var?	1 free variable -> 1 dimensional space.
55	what was the key takeaway for the traffic example?	The dimension of null space is the number of free variables
56	Can you go over how to find 2 free vars again	It comes from having 2 independent traffic flow graphics. Conceptually, you can think of it as 2 separate traffic problems. each one has a solution independent of the other. So we have 2 free variables.
57	so for the first system, your nullspace has a dimension of 1? but you only need to measure 1 out of 3 vectors? i am confused	Yes the dimension is 1. We only have to measure 1 variable to know the other two variables.
58	i thought we had 2 euqations for example 2 since $x1 = x2$ so how do we end uup with 2 zero rows	For the example with 2 traffic flow graphics, each flow operations independent of the other, so we can have 2 independent solutions.
59	does that mean the number of dimensions of the nullspace for the berkeley, oakland, and sf example is only 1?	yup!

60	If we did $R_1 + R_2$ would we not only get one free variable?	No because the 2 equations' variables are not the same.
61	so is the nullspace $\mathbb{R}^2$ ?	No. It's a subspace of $\mathbb{R}^4$ in this question.
62	so the dimension of $e_{x1}$ is 1?	Yes
63	Since there is no traffic going from SF to SF, why would we have 1s in all the pivots?	This is not an example of the pump problem and transition matrix. The matrix represents the equations $x_1 = x_2$ , $x_2 = x_3$ , and $x_3 = x_1$
64	is the dimensions of the null space in the prior problem 1 or 2?	In the Berkeley SF Oakland problem it's 1. In the NYC Boston Berkeley SF example it's 2.
65	is the vector space just all the linear combinations of the vectors in $V_1$ ?	Yes!
66	Could you explain the line with plain english one more time?	"For all vectors $x, y, z$ in the vector space $V$ , and for all scalars $\alpha, \beta$ in the real numbers"
67	will we need to prove that a space is a vector space?	The knowledge of how to do that is expected, but the tools to do that are fairly fundamental and we've already seen a few times.
68	is $0$ is $0$ ?	Yes (5) means we need to have a $0$ vector in the set
69	when would a vector added to a zero vector not equal the original vector?	For our purposes, just about never
70	is $a_1x + a_2y$ also in the vector space?	Yes. You can combine the (1) and (2) properties to prove this
71	on 5, would it be different for $x + 0$ if zero vector wasn't an element of $V$ ?	If $0$ is not an element of $V$ , 5 will not be satisfied.
72	could you please remind me what axiom means again?	An axiom is just some definition or statement that is accepted. These axioms define the qualities of a vector space.
73	is number 6 means that we need at least one vector whose $-$ is also in the set?	6 needs to be satisfied for any $x$ vector. So for ANY $x$ vector there should EXIST $(-x)$ in the set
74	so is this currently the list of properties exhibited by any linear system. where the linear system is defined as anything that satisfies the given conditions?	Yup. Having these definitions allows us to generalize some operations across vector spaces that may not be using the column / list vector definition we use
75	are 7 and 9 the same	live answered
76	are 7 and 9 the same thing?	live answered
77	aren't 7 and 9 the same thing	live answered
78	what are 8 and 9 called?	8 and 9 are part of scaling
79	In number 5, how is the $0$ vector different from the zero vector?	It's almost always the zero vector. Unless you redefine the $+$ operation in some weird way (which we will not see in this course).
80	So what isn't a vector space?	For example, something that is just 2 vectors (as opposed to the span of 2 vectors).

81	What does 10 mean in words again?	There exists some scalar multiplier 1 that gives you the identity
82	why wouldn't just 2 vectors count as a vector space?	Just 2 vectors cannot satisfy the closure requirement of (1)
83	Doesnt it also not obey 1? We can have a negative scalar so it can escape the vector space	Yes you are correct
84	does the set of all scalars greater than 0 also violate property 5?	Yes
85	does the matrix work?	Yes it's a vector space
86	how can a set of scalars be a vector space?	Think of scalars as 1D vectors
87	wait so was $R^{2 \times 2}$ valid	Yes it's a vector space
88	Is $R$ a vector space?	Yes
89	are we expected to have all these memorized?	In terms of exams, you will be allowed some cheat sheet, but we generally care more about the interesting applications (span, null space, etc.)
90	when testing a matrix or scalar or vector for if it's a vector space you just substitute it in for the vector? idk how you would sub a scalar into $\mathbb{R}$ like that	Yes just substitute it in for the vector. If you have scalars, $\alpha(x + y) = \alpha x + \alpha y$ , where $x$ and $y$ are scalars
91	is the span of anything always a vector space? since the definition is all linear combos	yup!
92	Would $[3 \ 0 \ 4 \ 0]$ be a vector space?	Nope, check it by multiplying a scalar
93	how can a matrix be a vector space?	It satisfies all the properties. "Vector space" is kind of an abstract name. It is not necessarily a set of $n \times 1$ vectors
94	What is the difference between span and vector space?	The span of some vectors is a vector space. The terms mean different things though
95	what is .set again?	I believe that's actually "set"
96	wait so was $R^{(2 \times 2)}$ a vector space? Sorry I missed that	yes
97	What is that $F$ next to the $V$ ?	A set of scalars
98	haha thanks! (reL set/set)	live answered
99	Is 0 vector a subspace of 0 vector?	Any vector can always be a subspace / subset of itself.
100	how does the $\subseteq$ notation differ from the one that looks like it with the line under it?	With a line can mean that the spaces are equal. For a lot of our discussions, they are interchangeable, but we want to specifically define subspaces here.
101	are all null spaces subspaces of their respective vector space?	yep!
102	please clarification for second figure? what is the dashed line?	The dashed line is some line that does not include $(0, 0)$
103	what is an example of a subspace that is not just a line	live answered
104	why is the zero vector necessary?	Consider if you had the vector $v_1$ in the vector space. $v_1 - v_1$ must be a valid operation, which gives the zero vector. So zero must always be in the vector space.
105	is there a specific reason for the subspace to contain the zero vector?	Consider if you had the vector $v_1$ in the vector space. $v_1 - v_1$ must be a valid operation, which gives the zero vector. So zero must always be in the vector space.

106	so $R_2$ is subspace of $R_3$ ?	No. $R_2$ only has 2D vectors, and $R_3$ has 3D vectors. You can have a 2-dimensional space, i.e. a vector space defined by 2 vectors, in $R_3$ .
107	doesn't that mean axiom 6 is implied by axiom 5? why do we list it separately?	Often we will have definitions that are redundant and imply each other. But they are helpful to give us alternate ways / shortcuts to work through problems / proofs.
108	Could you scroll up to the top of the subspace page for a bit	live answered
109	in this example would $W_1$ and $V_1$ not be written with the double line?	either way is fine, as long as you keep the notation consistent
110	ok thanks	live answered
111	so is a matrix considered a vector?	yes in the general sense of a vector
112	How do we know if it contains the 0 vector	The set of upper triangular matrix contains $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
113	so if one of the columns in the matrix can be 0 0, then would it be considered to have the zero vector?	Nope, for the definition of vector space and subspace we need the entire matrix to be zero
114	wait what is the definition of Basis???	a basis of a vector space is the minimum set of vectors needed to represent all vectors in the vector space. For a more mathematical definition you can refer to the lecture note
115	How did she say that $R_2$ is a subset. Subset of what? It includes all $R_2$ !	A vector space can be a subset of itself.
116	does basis must be linear independent?	live answered
117	basis vectors must be linearly independent?	live answered
118	Can you please scroll up?	
119	the last example is a subset though yes?	yes
120	so the span is not a basis for $R_2$ ?	This span in particular, correct
121	when are the slides being posted?	