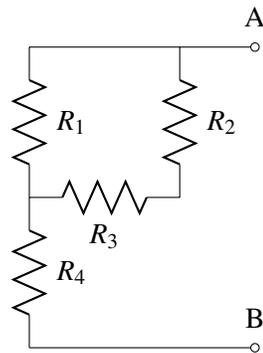




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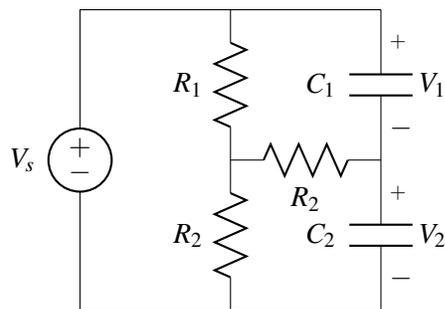
### 3. Simplifying Resistor Networks (6 Points)

Determine the equivalent resistance between points A and B in the circuit shown below. You may use the parallel operator  $\parallel$  in your final answer.



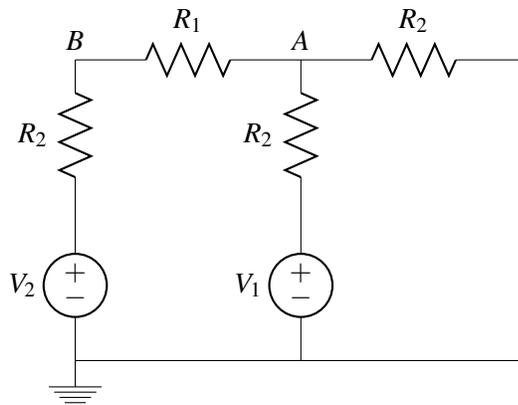
### 4. Capacitors (6 Points)

For the circuit given below, determine the voltage across each capacitor at steady state.  $R_2 = 2R_1$ ,  $C_2 = C_1$ , and  $V_s = 3\text{ V}$ .



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**5. Ladder Nodal (8 Points)**

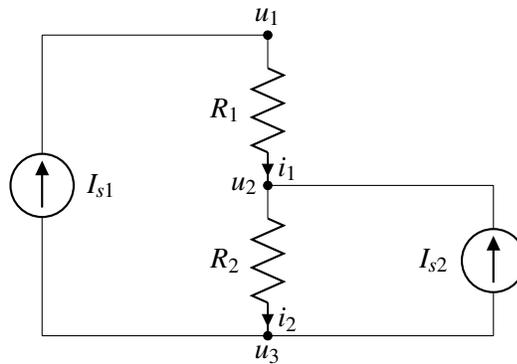


Use nodal analysis to write equations for each node  $A$  and  $B$ .

Let  $R_1 = 1\Omega$ ,  $R_2 = 2\Omega$ , and leave your expressions in terms of  $V_1$ ,  $V_2$ ,  $R_1$ ,  $R_2$ ,  $V_A$  and  $V_B$ . Do not simplify.

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**6. Flows Through Circuits (12 Points)**

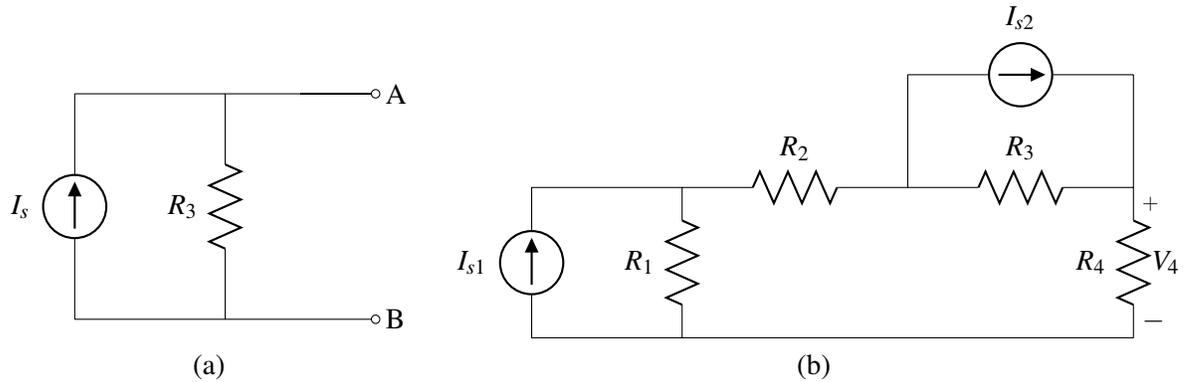


(a) (2 points) Redraw the circuit shown above with all current sources nulled.

(b) (5 points) Translate your circuit from part (a) into a graph and write its incidence matrix  $\mathbf{F}$ .

(c) (5 points) Recall that KCL can be stated as  $\mathbf{F}^T \vec{i} = 0$  when all the sources are turned off. However, for circuits containing independent current sources, the equation is modified to be  $\mathbf{F}^T \vec{i} = \vec{b}$ . Find  $\vec{b}$  such that  $\mathbf{F}^T \vec{i} = \vec{b}$  represents the KCL constraint at every node in the above circuit.

**7. One Norton At A Time! (12 Points)**



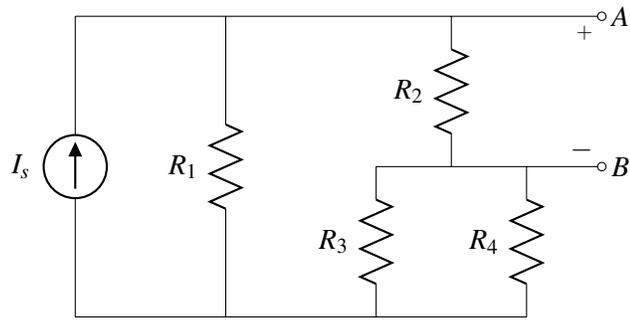
(a) (2 points) Given the circuit labeled (a), draw the Thevenin equivalent circuit for  $I_s = \frac{1}{5} \text{A}$  and  $R_3 = 20 \Omega$  and determine  $V_{th}$  and  $R_{th}$ .

(b) (5 points) Redraw the circuit labeled (b) with the following restrictions: you must use only voltage sources and the given four resistors. Determine the value of the voltage sources given  $R_1 = 30 \Omega$ ,  $R_2 = 10 \Omega$ ,  $R_3 = 20 \Omega$ ,  $R_4 = 10 \Omega$ ,  $I_{s1} = \frac{1}{3} \text{A}$ , and  $I_{s2} = \frac{1}{5} \text{A}$ . **Circuits drawn without  $R_1, R_2, R_3$ , and  $R_4$  will not be given full credit. Do not combine resistors.**

(c) (5 points) Calculate the voltage drop across  $R_4$ .

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**8. Equivalent Circuits (13 Points)**



Find  $V_{th}$  and  $R_{th}$  with respect to nodes  $A$  and  $B$  for the following circuit in terms of the variables  $I_s, R_1, R_2, R_3,$  and  $R_4$ . You may leave the parallel operator  $\parallel$  in your final answer.

**9. Resist the Touch (16 Points)**

In this question, we will be re-examining the 2-dimensional resistive touchscreen previously discussed in both lecture and lab. The general touch screen is shown in Figure 1 (a). The touchscreen has length  $L$  and width  $W$  and is composed of a rigid bottom layer and a flexible upper layer. The strips of a single layer are all connected by an ideal conducting plate on each side. The upper left corner is position  $(1, 1)$ .

The top layer has  $N$  vertical strips of denoted by  $y_1, y_2, \dots, y_N$ . These vertical strips have cross sectional area  $A$ , and resistivity  $\rho_y$ .

The bottom layer has  $N$  horizontal strips denoted by  $x_1, x_2, \dots, x_N$ . These horizontal strips all have cross sectional area  $A$  as well, and resistivity  $\rho_x$ .

Assume that all top layer resistive strips and bottom layer resistive strips are spaced apart equally. Also assume that all resistive strips are rectangular as shown by Figure 1 (b).

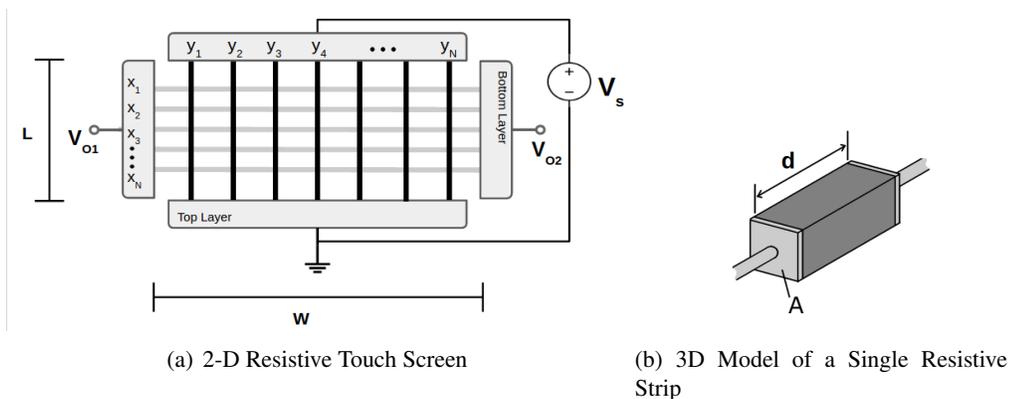


Figure 1:

- (a) (3 points) Figure 1(b) shows a model for a single resistive strip. Find the equivalent resistance  $R_y$  for the vertical strips and  $R_x$  for the horizontal strips, as a function of the screen dimensions  $W$  and  $L$ , the respective resistivities, and the cross-sectional area  $A$ .

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(b) (5 points) Consider a  $2 \times 2$  example for the touchscreen circuit.

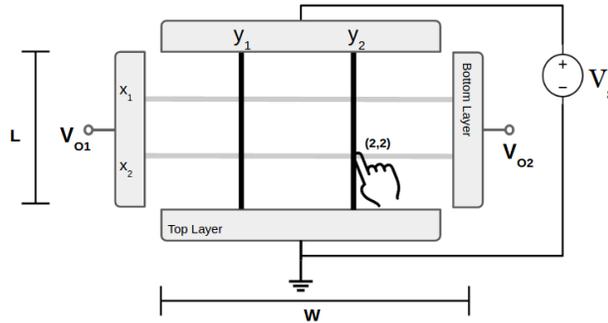
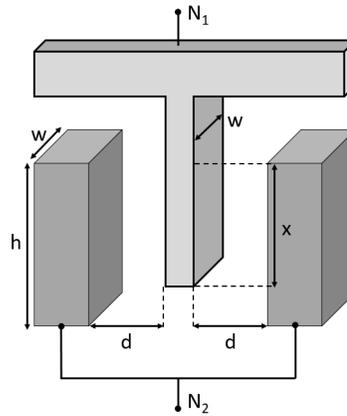


Figure 2:  $2 \times 2$  Case of the Resistive Touchscreen

Given that  $V_s = 3\text{ V}$ ,  $R_x = 2000\Omega$ , and  $R_y = 2000\Omega$ , draw the equivalent circuit for when the point  $(2,2)$  is pressed and solve for the voltage at terminal  $V_{O2}$  with respect to ground.

(c) (8 points) Suppose a touch occurs at coordinates  $(i, j)$  in Figure 1(a). Find an expression for  $V_{O2}$  as a function of  $V_s$ ,  $N$ ,  $i$ , and  $j$ .

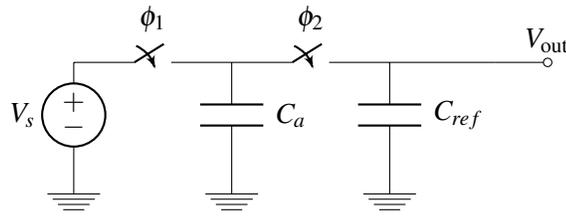
**10. MEMS Sensor (20 Points)**



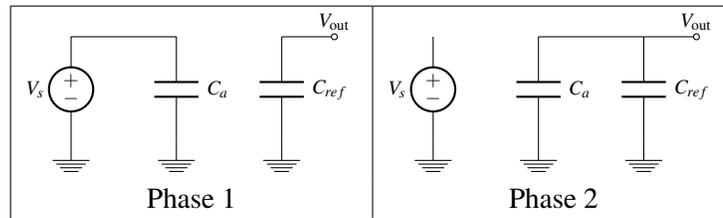
Above is the cross section of a MEMS accelerometer, which consists of a T-shaped mass between two sidewalls. When the device accelerates, the T structure, which is held up by springs, displaces up or down. Assume that all the structures are made of metal separated by air, and that the only relevant capacitances in this problem are parallel plate capacitances.

- (a) (4 points) Suppose both sidewalls are connected together with an ideal wire. Find the capacitance between the stem of the T mass and a sidewall as a function of  $x$ , the displacement of the T mass. Find the net capacitance between the two terminals  $N_1$  and  $N_2$  assuming the only parallel plate capacitance is that of the T mass stem and sidewall.

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The above circuit is designed to measure the displacement of the T structure. It cycles through two phases, which are depicted in the diagram below. For this problem, you may assume that switching is controlled through a microcontroller and that all capacitors reach steady state during each phase. Furthermore, assume that the T mass does not move during the measurement cycle and all capacitors are initially uncharged.



(b) (8 points) The circuit above is designed to measure the change in capacitance and to output a voltage. The phases of its operation are shown above. Find  $V_{out}$  in Phase 2 as a function of  $C_a$  and  $C_{ref}$ .

(c) (8 points) Let  $V_{out}[i]$  denote  $V_{out}$  at the  $i$ th measurement cycle. Suppose the circuit had been running for  $k$  cycles. What is  $V_{out}[k]$  in terms of  $V_{out}[k-1]$ ,  $C_a$  and  $C_{ref}$ ?