

(What is your) **Quest(?)**

Please read the questions carefully and answer appropriately.

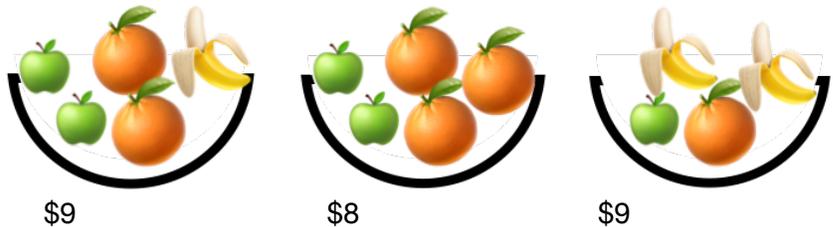
1. a. (2 points) The function $f(x, y) = 1 + 3x - 4y$ *$f(ax, ay) = 1 + 3ax - 4ay \neq a f(x, y)$*
- i. Exhibits homogeneity
 - ii. Is linear *No, because of i.*
 - iii. Both i and ii are true *No*
 - iv. None of the above are true

Briefly explain: *Not homogeneous, so not linear*

b. (4 points) For each of the following functions, circle **all the possible operations** that when put in the box \square , will result in a linear function. If none of the operations work, circle "none".

- | | | | | | |
|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|---------------------------------------|
| i. $f(x) = x \square 3$ | + | - | <input checked="" type="radio"/> | <input checked="" type="radio"/> | none |
| ii. $f(x) = \sin(y_0) \square x$ | + | - | <input type="radio"/> | <input checked="" type="radio"/> | none |
| iii. $f(x, y) = 3x \square y$ | <input checked="" type="radio"/> | <input checked="" type="radio"/> | <input type="radio"/> | <input type="radio"/> | none |
| iv. $f(x, y) = e^{2x} \square y$ | + | - | <input type="radio"/> | <input type="radio"/> | <input checked="" type="radio"/> none |

2. Ana and Miki go shopping. They see three gift baskets containing apples, oranges and bananas



(a) (5 points) Let $\vec{x} = \begin{bmatrix} x_a \\ x_o \\ x_b \end{bmatrix}$ be a vector, containing the prices of a single apple, single orange and single banana respectively. To find \vec{x} , we form the linear set of equations: $A\vec{x} = \vec{b}$. What are the entries of A and \vec{b} ?

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 2 & 3 & 0 \\ 1 & 1 & 2 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 9 \\ 8 \\ 9 \end{bmatrix}$$

(b) (3 points) Based on the system in part (b), find a solution for \vec{x} , via Gaussian Elimination (show your work):

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 2 & 2 & 1 & 9 \\ 2 & 3 & 0 & 8 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & 0 & 3 & 9 \\ 0 & -1 & 4 & 10 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 2 & 9 \\ 0 & -1 & 4 & 10 \\ 0 & 0 & 1 & 3 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 9 \\ 0 & -1 & 4 & 10 \\ 0 & 0 & 1 & 3 \end{array} \right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & -1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array} \right]$$

$x_a = 1$ $x_o = 2$ $x_b = 3$

3. (6 points) After performing Gaussian Elimination row reduction, you end up with a row-echelon form. For each of the following cases, circle whether the system has a unique solution, infinite solutions, or no solution. If any solutions exist, state the solution (if it is unique) or express the set of possible solutions.

(a)
$$\left[\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & 1 & 0 \end{array} \right]$$

(Unique, infinite solns., no solns.)

Solution(s): $0, 0$

(b)
$$\left[\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

(Unique, infinite solns., no solns.)

Solution(s): $0, 0$

(c)
$$\left[\begin{array}{cc|c} 1 & 3 & 0 \\ 0 & 1 & 1 \end{array} \right]$$

(Unique, infinite solns., no solns.)

Solution(s): $1, -3$

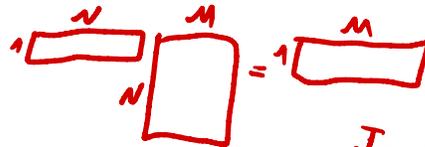
(d)
$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

(Unique, infinite solns., no solns.)

Solution(s): $x_3 = 1$
 $x_2 = \text{independent}$
 $x_1 = -3 - 2x_2$

4. (5 points) Let $A \in \mathbb{R}^{N \times M}$, $B \in \mathbb{R}^{N \times L}$, $\vec{a} \in \mathbb{R}^L$, $\vec{b} \in \mathbb{R}^M$, $\vec{c} \in \mathbb{R}^N$. For each of the following expressions, state the size of the resulting product if it exists, or circle "doesn't exist" if the product is not valid.

(a) $\vec{c}^T A \in \mathbb{R}^{1 \times M}$ or doesn't exist



(b) $\vec{b}^T A^T B \in \mathbb{R}^{1 \times L}$ or doesn't exist

$A^T B \in \mathbb{R}^{M \times L}$

(c) $(B\vec{c})^T A \in \mathbb{R}^{1 \times M}$ or doesn't exist

(d) $\vec{a} \vec{b}^T \vec{b} \vec{a}^T \vec{a} \vec{b}^T \in \mathbb{R}^{L \times M}$ or doesn't exist

(e) $\vec{b}^T \vec{b} \vec{a}^T \vec{a} \vec{b}^T \vec{b} \in \mathbb{R}^{1 \times 1}$ or doesn't exist

5. (0 points, but lots of credit) What is the airspeed velocity of an unladen swallow?

European or African?