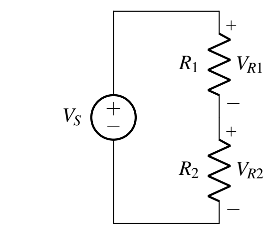
CSM 16A Designing Information Devices and Systems I
Fall 2020 Week 10

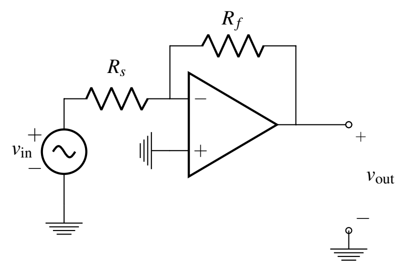
Reference: Op-Amp Example Circuits

Voltage Divider



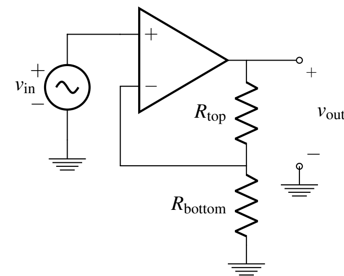
$$V_{R_2} = V_S\left(\frac{R_2}{R_1+R_2}\right)$$

Inverting Amplifier



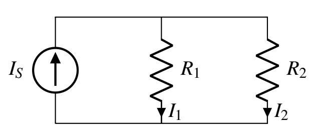
$$v_{out} = v_{in}\left(-\frac{R_f}{R_s}\right)$$

Noninverting Amplifier



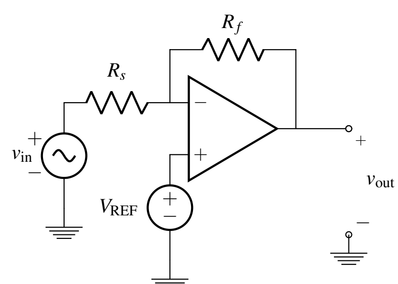
$$v_{out} = v_{in}\left(1+\frac{R_{top}}{R_{bottom}}\right)$$

Current Divider



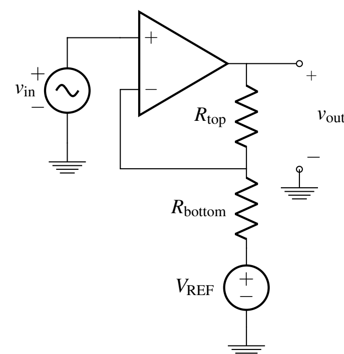
$$I_1 = I_S\left(\frac{R_2}{R_1+R_2}\right)$$

Inverting Amplifier with Reference



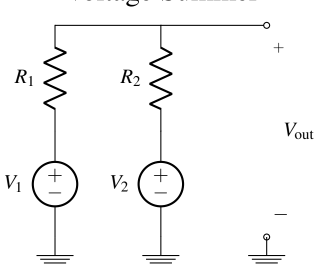
$$v_{out} = v_{in}\left(-\frac{R_f}{R_s}\right)+V_{REF}\left(\frac{R_f}{R_s}+1\right)$$

Noninverting Amplifier with Reference



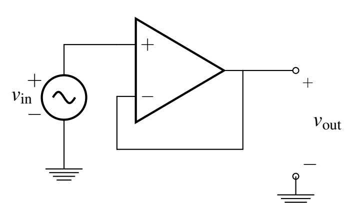
$$v_{out} = v_{in}\left(1+\frac{R_{top}}{R_{bottom}}\right)-V_{REF}\left(\frac{R_{top}}{R_{bottom}}\right)$$

Voltage Summer



$$V_{out} = V_1\left(\frac{R_2}{R_1+R_2}\right)+V_2\left(\frac{R_1}{R_1+R_2}\right)$$

Unity Gain Buffer



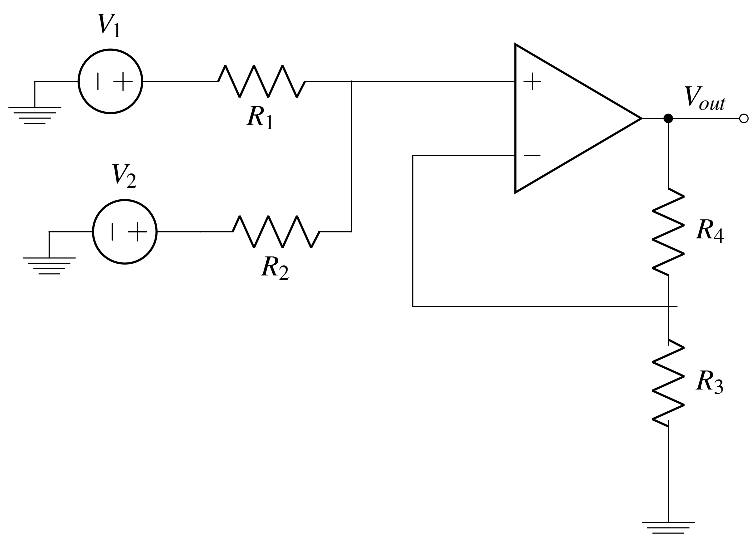
$$v_{out} = v_{in}$$

1. Voltage Summers

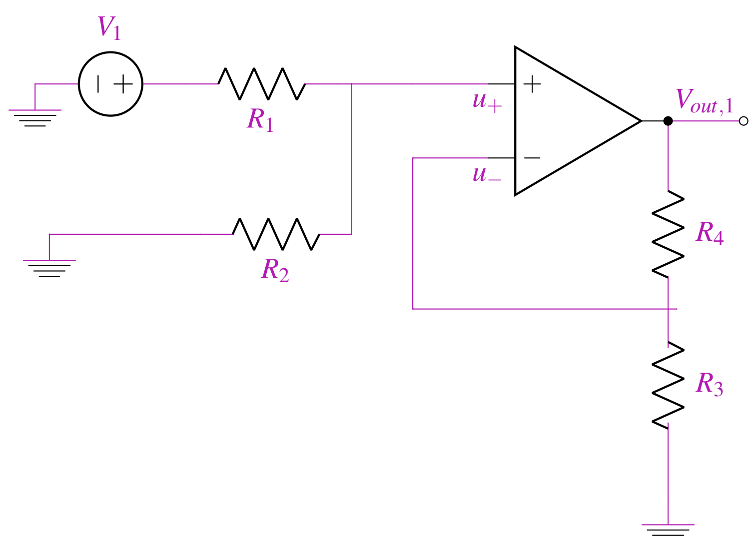
Learning Goal: This problem uses basic circuit analysis techniques to find the response of a summer circuit.

Relevant Notes: **Note 19** goes different op-amp circuit topology and corresponding derivations.

(a) Calculate V_{out} in terms of V_1 and V_2. Assume that $R_1 = R_2$. Use superposition.



Answer: Let's first consider the case when only V_1 is active. We deactivate voltage source V_2 by replacing it with a wire:



We see that u_+ is the middle node of a voltage divider with resistors R_1 and R_2. We also know that input current $I_+ = 0$. So we can find u_+ using:

$$u_+ = \frac{R_2}{R_1 + R_2} V_1.$$

Similarly u_- is the middle node of a voltage divider with resistors R_3 and R_4. We know that input current $I_- = 0$. We can find u_- as:

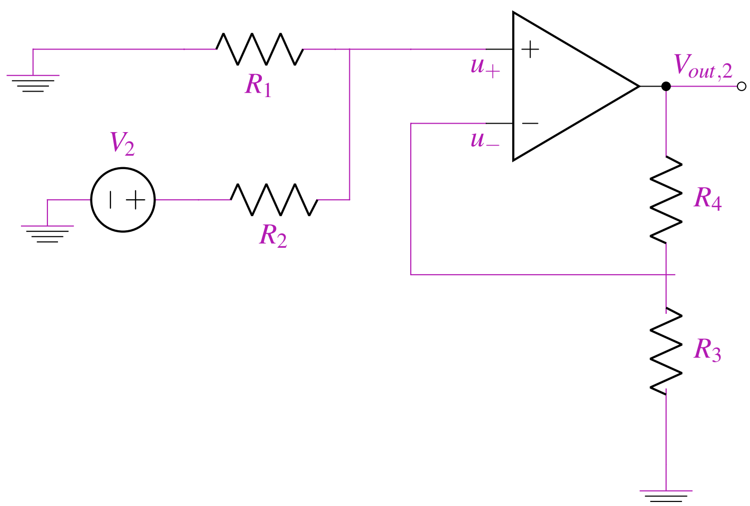
$$u_- = \frac{R_3}{R_3 + R_4} V_{out,1}.$$

Now we have to check if the circuit is in negative feedback. If we move the negative input of the op amp u_- upward, $V_{out,1} = A(u_+ - u_-)$ moves downward and as a result $u_- = \frac{R_3}{R_3+R_4}V_{out,1}$ moves downward. So the result of the initial stimulus goes in the opposite direction of the initial stimulus, which is the requirement for negative feedback.

Since the circuit is in negative feedback, we can apply the second golden rule:

$$u_+ = u_-$$
$$\implies \frac{R_2}{R_1+R_2}V_1 = \frac{R_3}{R_3+R_4}V_{out,1}$$
$$\implies V_{out,1} = \left(1+\frac{R_4}{R_3}\right)\frac{R_2}{R_1+R_2}V_1$$

Next we consider the case when only V_2 is active. We deactivate voltage source V_1 by replacing it with a wire:



Following a similar process we have:

$$u_+ = \frac{R_1}{R_1+R_2}V_2;$$
$$u_- = \frac{R_3}{R_3+R_4}V_{out,2}.$$

(Note that u_+ is now the voltage across R_1, while for the previous scenario u_+ was the voltage across R_2.)

Using the second golden rule we have:

$$u_+ = u_-$$
$$\implies \frac{R_1}{R_1+R_2}V_2 = \frac{R_3}{R_3+R_4}V_{out,2}$$
$$\implies V_{out,2} = \left(1+\frac{R_4}{R_3}\right)\frac{R_1}{R_1+R_2}V_2$$

Now using the superposition theorem, we can find V_{out}:

$$V_{out} = V_{out,1} + V_{out,2}$$
$$\implies V_{out} = \left(1+\frac{R_4}{R_3}\right)\frac{R_2}{R_1+R_2}V_1 + \left(1+\frac{R_4}{R_3}\right)\frac{R_1}{R_1+R_2}V_2$$

Note that a similar expression can be reached by using the equations for the voltage summer and the non-inverting amplifier. Pattern matching this circuit with the voltage summer and the non-inverting amplifier and then employing the given equations is a valid technique to solve this problem.

(b) What values should we select for R_1, R_2, R_3, and R_4 such that $V_{out} = V_1 + 2V_2$?

Answer: From the last part, we have

$$V_{out} = \left(1 + \frac{R_4}{R_3}\right)\frac{R_2}{R_1 + R_2}V_1 + \left(1 + \frac{R_4}{R_3}\right)\frac{R_1}{R_1 + R_2}V_2$$

Comparing with $V_{out} = V_1 + 2V_2$, we have:

$$\left(1 + \frac{R_4}{R_3}\right)\frac{R_2}{R_1 + R_2} = 1 \quad (1)$$

$$\left(1 + \frac{R_4}{R_3}\right)\frac{R_1}{R_1 + R_2} = 2 \quad (2)$$

Dividing equation (**??**) with equation (**??**) we get:

$$\frac{R_2}{R_1} = \frac{1}{2}R_1 \qquad = 2R_2$$

Now if we choose $R_2 = 100\Omega$ and $R_1 = 200\Omega$, we have

$$\frac{R_2}{R_1 + R_2} = \frac{100}{200 + 100} = \frac{1}{3} \quad (3)$$

$$\frac{R_1}{R_1 + R_2} = \frac{200}{200 + 100} = \frac{2}{3} \quad (4)$$

Plugging in the values of equation (**??**) in equation (**??**) (or equation (**??**) in equation (**??**)), we have:

$$\left(1 + \frac{R_4}{R_3}\right)\frac{1}{3} = 1$$

$$1 + \frac{R_4}{R_3} = 3$$

$$\frac{R_4}{R_3} = 2$$

$$R_4 = 2R_3$$

We can choose $R_3 = 100\Omega$ and $R_4 = 200\Omega$, so that the output voltage is:

$$V_{out} = \left(1 + \frac{200}{100}\right)\frac{100}{200 + 100}V_1 + \left(1 + \frac{200}{100}\right)\frac{200}{200 + 100}V_2 = (1+2)\frac{1}{3}V_1 + (1+2)\frac{2}{3}V_2 = V_1 + 2V_2.$$

2. Multi-stage Amplifier

Learning Goal: The objective of this problem is to understand how multiple stages of op-amp circuits can be used to achieve a specific circuit gain.

Relevant Notes: **Note 19 Section 19.5** goes over inverting and non-inverting amplifiers.

(a) What is the range of values that we can scale V_{in} by when using a non-inverting op amp? (What are possible values for the gain?)

Answer: Recall that when using a non-inverting op amp, the equation for V_{out} is given by

$$V_{out} = V_{in}(1 + \frac{R_2}{R_1})$$

where R_2 is the upper resistor and R_1 is the lower one (connected to ground). The circuit gain G is represented by

$$G = \frac{V_{out}}{V_{in}} = 1 + \frac{R_2}{R_1}$$

We can choose any values for resistance from [0, ∞); then, the minimum gain would be 1 if we chose $R_2 = 0$; the maximum gain approaches infinity as we choose some very large R_2 and/or a very small R_1. Hence, the range of gains is from 1 to infinity, and our range of values for V_{out} is $[V_{in}, \infty)$.

(b) What is the range of values that we can scale V_{in} by when using an inverting op amp? (What are the possible values for the gain?)

Answer: Recall that when using an inverting op amp, the equation for V_{out} is given by

$$V_{out} = -V_{in}\frac{R_2}{R_1}$$

. The circuit gain is represented by

$$G = -\frac{R_2}{R_1}$$

.

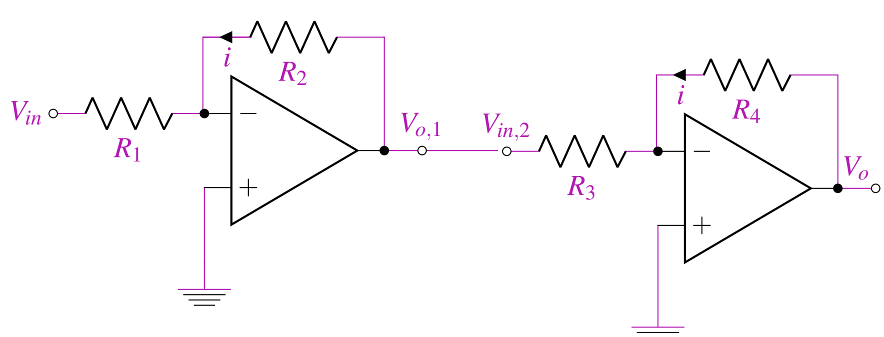
Again, we can choose any values for resistance from [0, ∞); then, the minimum absolute value scaling would be 0 if we chose $R_2 = 0$; the maximum absolute value scaling approaches infinity as we choose some very large R_2 and/or a very small R_1. Hence, the range of gains is from 0 to -∞, and our range of values for V_{out} is also $(-\infty, 0]$.

(c) Can you design a single inverting/ non-inverting amplifier with circuit gain $G = 0.5$? If not, what range of gain values is not reachable using a single inverting op amp or a single non-inverting op amp?

Answer: From part (a), we found that a non-inverting op amp can only reach gain values from [1, ∞). From part (b), we found that an inverting op amp can only reach gain values from (-∞, 0]. So using only a single op-amp, we cannot reach values between (0, 1), specifically $G = 0.5$.

(d) How would you construct a circuit using inverting/ non-inverting amplifiers so that the overall circuit gain is $G = 0.5$?

Answer: We can use two inverting op-amps to achieve overall gain $G = 0.5$. Say the first op-amp has gain G_1 and the second op-amp G_2, so $G = G_1 G_2$. From part (b), these gains can take on values from (-∞, 0], so multiplying them together helps reach $G = 0.5$.



For the first inverting op-amp, we have $G_1 = -\frac{R_2}{R_1}$, and for the second, $G_2 = -\frac{R_4}{R_3}$. Plugging these into the equation for G:

$$G = G_1 G_2 = \frac{R_2 R_4}{R_1 R_3} = 0.5$$

$$R_1 R_3 = 2 R_2 R_4$$

To reach out desired gain, we can pick any combination of R_1, R_2, R_3, and R_4 that satisfies this equation. One such solution is $R_1 = 200\Omega$ and $R_2 = R_3 = R_4 = 100\Omega$.

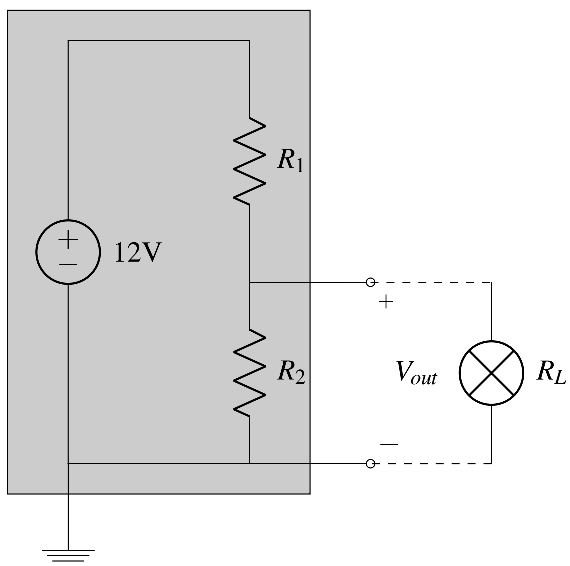
3. Op Amps as Buffers

Learning Goal: This problem helps understand the operating principle of an op-amp buffer and how it helps with loading.

Relevant Notes: **Note 19 Section 19.7** goes different op-amp circuit topology and corresponding derivations.

Now we will revisit a problem that you might have seen before, with our new knowledge of op-amps. We have access to a circuit inside a 'black box' as shown below, with two terminal coming out of it.

(a) We need a voltage of 6V power a light bulb with resistance R_L. Design R_1 and R_2 inside the black box so that the voltage across R_2 is exactly equal to this required voltage **when the bulb is not connected**; i.e. $V_{R_2} = V_{out} = 6\text{V}$.



Answer: The voltage across R_2 is given by

$$V_{R_2} = \frac{R_2}{R_1 + R_2} \times 12\text{V}$$

If we set $V_{R_2} = 6\text{V}$, we get

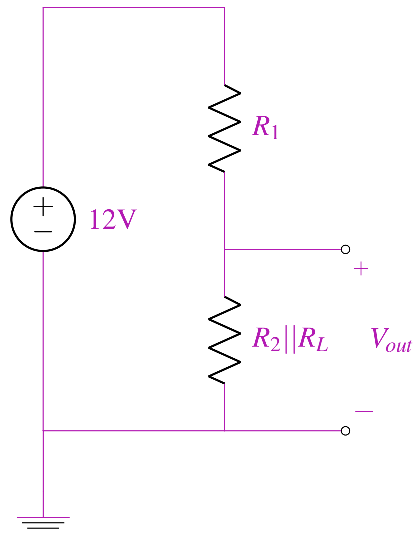
$$6\text{V} = \frac{R_2}{R_1 + R_2} \times 12\text{V}$$

$$\implies R_1 = R_2$$

For example, we can choose $R_1 = R_2 = 1\text{k}\Omega$.

(b) Now let us connect the bulb R_L across R_2. What is the voltage across R_1, R_2 and R_L when the bulb is connected when $R_L = R_2$? Use the values of R_1 and R_2 from the last part. Will the light bulb turn on? What happens if $R_L = 2R_2$?

Answer: When we connect the bulb across R_2, the equivalent circuit is the following:



The voltage V_{out} given by

$$V_{out} = \frac{R_2||R_L}{R_1 + R_2||R_L} \times 12\text{V}$$

When $R_L = R_2$:

$$R_2||R_L = R_2||R_2 = \frac{R_2}{2}.$$

Hence V_{out} is:

$$V_{out} = \frac{R_2||R_L}{R_1 + R_2||R_L} \times 12\text{V} = \frac{\frac{R_2}{2}}{R_1 + \frac{R_2}{2}} \times 12\text{V} = \frac{\frac{R_2}{2}}{R_2 + \frac{R_2}{2}} \times 12\text{V} = 4\text{V} < 6\text{V}$$

So the bulb will not turn on.

When $R_L = 2R_2$:

$$R_2||R_L = R_2||2R_2 = \frac{2R_2}{3}.$$

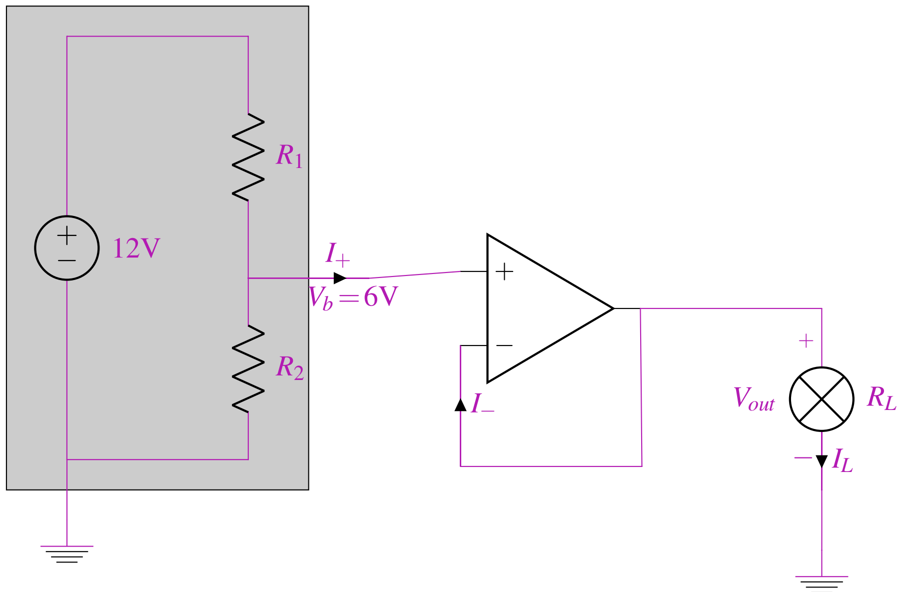
Hence V_{out} is:

$$V_{out} = \frac{R_2||R_L}{R_1 + R_2||R_L} \times 12\text{V} = \frac{\frac{2R_2}{3}}{R_1 + \frac{2R_2}{3}} \times 12\text{V} = \frac{\frac{2R_2}{3}}{R_2 + \frac{2R_2}{3}} \times 12\text{V} = 4.8\text{V} < 6\text{V}$$

Although, V_{out} is now closer to 6V, the bulb will still not turn on.

(c) Using your knowledge of op-amps, how could you resolve this issue of V_{out} changing based on the value of R_L? Think about how you might use an op-amp buffer.

Answer: We can introduce a 'buffer' op-amp as shown below:



A buffer op amp effectively 'decouples' its input and output, It does this by preventing the bulb from drawing current from the black box circuit. Remember that the input currents of an ideal op-amp:

$$I_+ = I_- = 0,$$

while the output current I_L can adjust itself to any extent, depending on the demand placed by V_{out}.
Let us use nodal analysis and the golden rules to formally solve this circuit. Firstly, we observe that the op-amp is in negative feedback configuration. Using the golden rules, we have that $V_+ = V_- = V_b = 6V$. Also, because the feedback connection is a short, $V_{out} = V_-$. Therefore, $V_{out} = V_b = 6V$. This is exactly what we want! The voltage across R_L equals $V_{out} = 6V$, which is above the required voltage.