CSM 16A Designing Information Devices and Systems I
Fall 2020 Week 5

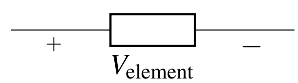
1. **Passive Sign Convention**

Learning Goal: This question practices labeling for passive sign convention.

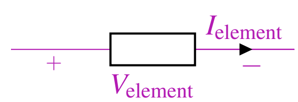
Relevant Notes: **Note 11A Section 11.3** introduces the standard circuit elements that are used in this question.

For the following components, label all the missing V_{element}, I_{element}, and +/- signs. *Hint: The value of the voltage and current sources shouldn't affect passive sign convention—remember that voltage and current can be negative!*

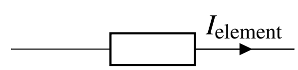
(a) .



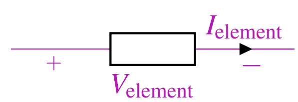
Answer:



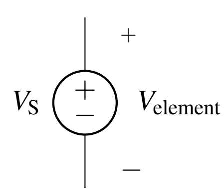
(b) .



Answer:

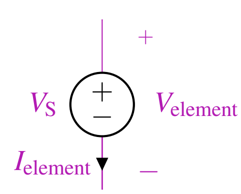


(c) .



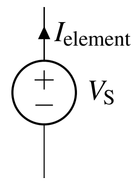
If $V_S = 1\text{V}$, what is V_{element}?
If $V_S = -1\text{V}$, what is V_{element}? Would this change the $+/-$ labels of V_{element}?

Answer:



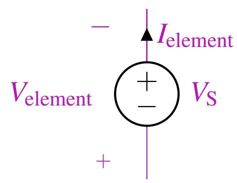
If $V_S = 1V$, $V_{element}$ would also be 1V due to our labeling of + and −.
If $V_S = -1V$, $V_{element} = -1V$ as well. This does NOT change the +/− labels of $V_{element}$, which do not depend on the voltage source's value. Once we have labeled the + and − a certain way in the diagram, we will follow passive sign convention with that specific labeling.

(d) .



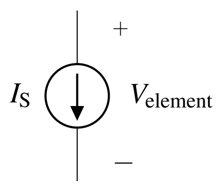
If $V_S = 3V$, what is $V_{element}$?
If $V_S = -3V$, what is $V_{element}$?

Answer:



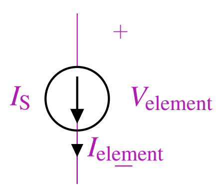
If $V_S = 3V$, $V_{element}$ is −3V because we have labeled our + and − opposing the direction of the voltage source. So $V_{element} = -V_S$.
Similarly, if $V_S = -3V$, then $V_{element} = 3V$.

(e) .



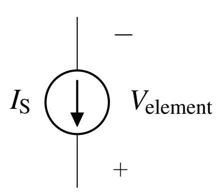
If $I_S = 5A$, what is $I_{element}$?
If $I_S = -5A$, what is $I_{element}$? Would this change the direction of the $I_{element}$ label?

Answer:



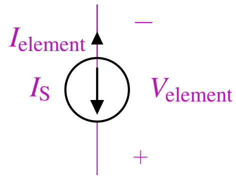
If $I_S = 5A$, then $I_{element} = 5A$, as it is in the same direction as I_S.
Using the property that $I_S = I_{element}$, if $I_S = -5A$, then $I_{element} = -5A$. This would not change the direction of the $I_{element}$ label.

(f) .



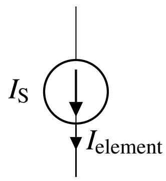
If $I_S = 1A$, what is $I_{element}$?
If $I_S = -1A$, what is $I_{element}$? Would this change the direction of the $I_{element}$ label?

Answer:

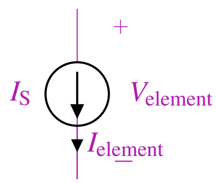


$I_{element}$ goes in the opposite direction of I_S, so if $I_S = 1A$, then $I_{element} = -1A$. If $I_S = -1A$, then $I_{element} = 1A$.

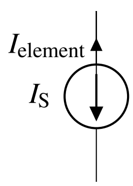
(g) **(PRACTICE)**



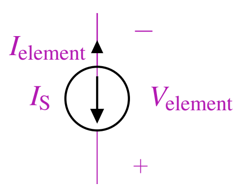
Answer:



(h) **(PRACTICE)**

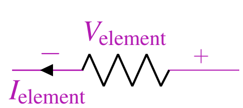


Answer:



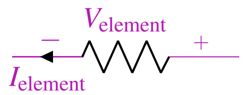
(i) **(PRACTICE)**

Answer:



(j) **(PRACTICE)**

Answer:

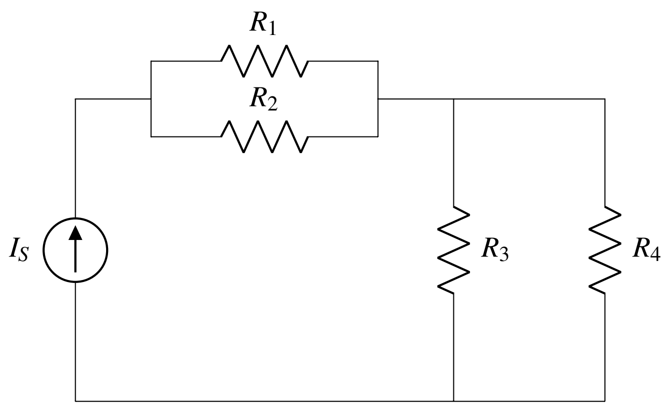


2. Nodes and Branches

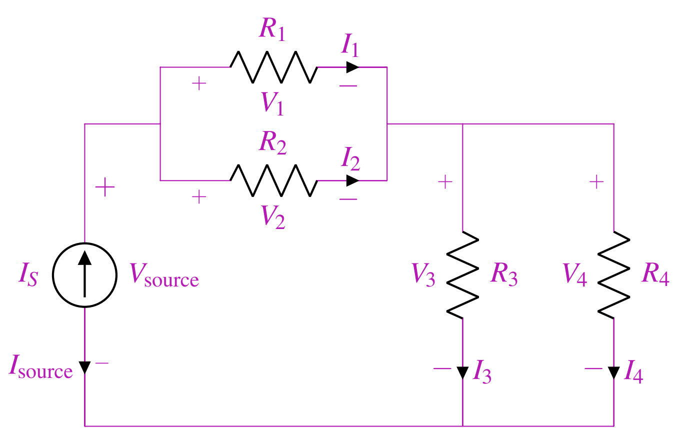
Learning Goal: The goal of this problem is to introduce nodal analysis, KVL, and KCL.

Relevant Notes: **Note 11A Section 11.4** introduces KVL and KCL.

(a) Label all the elements in the following circuit according to passive sign convention.

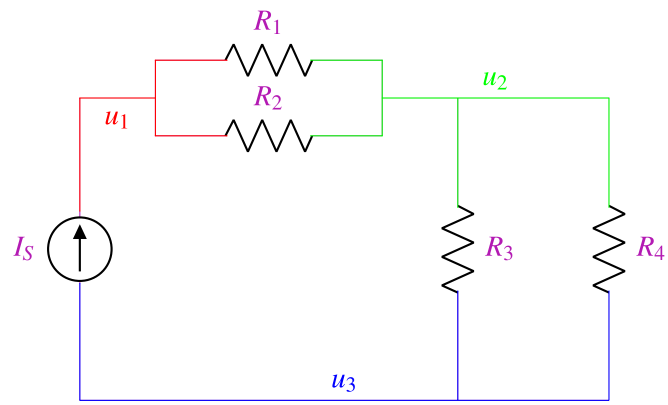


Answer: Note: we have increased the distance between R_1 and R_2 for greater clarity of the labelings. However, the circuit has not fundamentally changed.

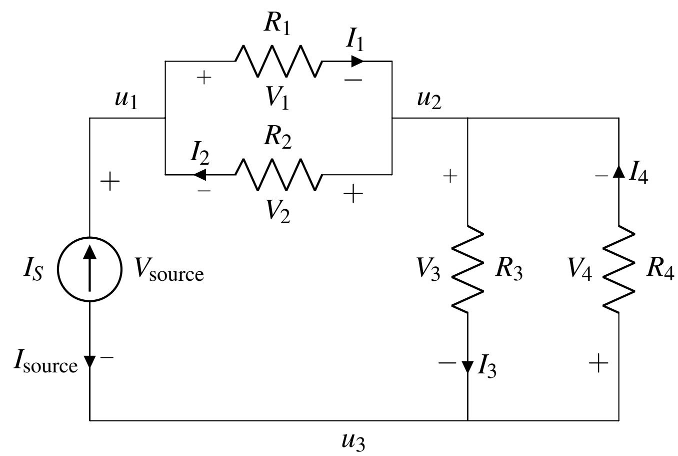


(b) Find and label all the distinct nodes in the circuit from part (a).

Answer: Each node consists of all the parts of the circuit that have the same voltage. In this circuit we have labeled three main nodes, u_1 (red), u_2 (green), and u_3 (blue):



(c) Some current and voltage labels are added to the circuit in part (a):



(i) If $I_S = 5\text{mA}$ and $I_1 = 2\text{mA}$, find I_{source} and I_2. Start by writing the KCL equation at node u_1.

(ii) If $I_3 = 4\text{mA}$ and $I_4 = -1\text{mA}$, find I_{source} and I_S. Start by writing the KCL equation at node u_3.

Answer:

(i) First, we know $I_{\text{source}} = -I_S = -5\text{mA}$, since it is in the same component and going in the opposite direction. Now, we want to find I_2 using KCL. KCL states that the sum of currents going into node u_1 equals the sum of currents going out of it. The current I_2 is going in, and currents I_1 and I_{source} are going out, so:

$$I_2 = I_1 + I_{\text{source}}$$

$$I_2 = I_1 + I_{\text{source}} = (2\text{mA}) + (-5\text{mA}) = -3\text{mA}$$

(ii) At node u_3, we have incoming currents I_3 and I_{source}, and outgoing current I_4. In addition, as shown in subpart (i), $I_{\text{source}} = -I_S$. Writing out the KCL equation at node u_3:

$$I_3 + I_{\text{source}} = I_4$$

$$I_{\text{source}} = -I_3 + I_4 = -(4\text{mA}) + (-1\text{mA}) = -5\text{mA}$$

$$I_S = -I_{\text{source}} = -(-5\text{mA}) = 5\text{mA}$$

(d) Consider the labeled circuit from part (c).

(i) If $V_{\text{source}} = 10\text{V}$ and $V_4 = -4\text{V}$, find V_1, V_2 and V_3. Start by writing the KVL equations for different loops.

(ii) Assume $R_1 = 3\text{k}\Omega$, $R_2 = 2\text{k}\Omega$, $R_3 = 1\text{k}\Omega$ and $R_4 = 4\text{k}\Omega$. Calculate I_1, I_2, I_3 and I_4 using the results from subpart (i).

Answer:

(i) Consider the (counterclockwise) loop that contains V_{source}, V_1, and V_4. The KVL equation for this loop is

$$-V_{\text{source}} + V_1 - V_4 = 0$$

$$V_1 = V_{\text{source}} + V_4 = 10\text{V} + (-4\text{V}) = 6\text{V}$$

Since R_1 and R_2 connect the same nodes, u_1 and u_2, we can write the same KVL equation as above except replacing V_1 with $-V_2$:

$$-V_{\text{source}} - V_2 - V_4 = 0$$

$$V_2 = -V_{\text{source}} - V_4 = -10\text{V} - (-4\text{V}) = -6\text{V}$$

We can apply the same idea to conclude that $V_3 = -V_4 = -(-4\text{V}) = 4\text{V}$.

(ii) For a resistor, Ohm's Law relates the current, voltage, and resistance through the equation

$$V = IR$$

We can use Ohm's Law here because we know V_1, V_2, V_3, and V_4 from subpart (i) and we are given R_1, R_2, R_3, and R_4 in this subpart.

$$I_1 = \frac{V_1}{R_1} = \frac{6\text{V}}{3\text{k}\Omega} = 2\text{mA}$$

$$I_2 = \frac{V_2}{R_2} = \frac{6\text{V}}{2\text{k}\Omega} = -3\text{mA}$$

$$I_3 = \frac{V_3}{R_3} = \frac{4\text{V}}{1\text{k}\Omega} = 4\text{mA}$$

$$I_4 = -\frac{V_4}{R_4} = \frac{4\text{V}}{4\text{k}\Omega} = -1\text{mA}$$

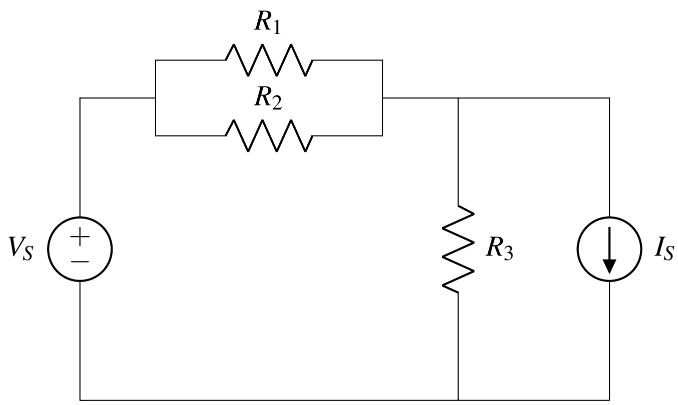
3. Intro to Circuit Solving

Learning Goal: The goal of this problem is to practice going through the main steps to solve circuits.

Relevant Notes: Note 11B introduces the circuit analysis procedure.

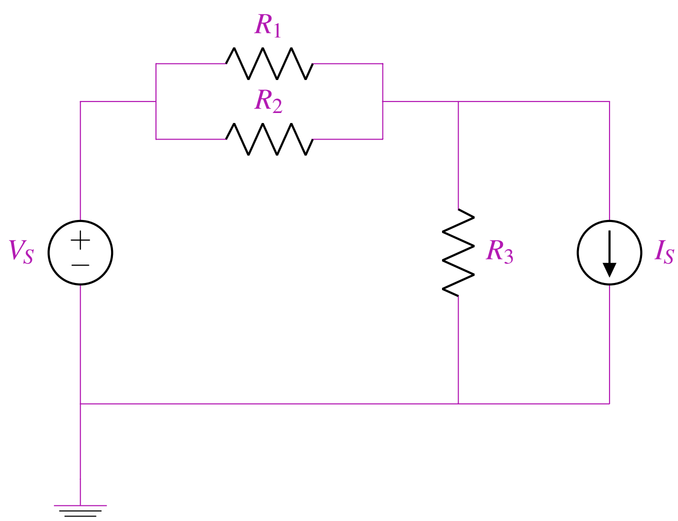
Solve the following circuit for all the branch currents and voltage differences across elements. Find the power dissipated in all elements. Follow the steps listed below to step through the problem. Use the following values for this problem: $V_S = 5\text{V}$, $R_1 = 2\text{k}\Omega$, $R_2 = 2\text{k}\Omega$, $R_3 = 4\text{k}\Omega$, and $I_S = 2\text{mA}$.

Solve the following circuit for all the branch currents and voltage differences across elements. Find the power dissipated in all elements. Follow the steps listed below to step through the problem.



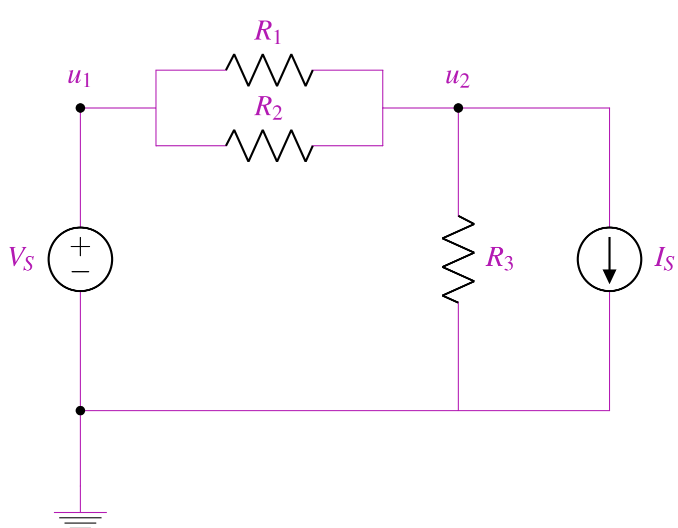
(a) Select a reference node.

Answer: You can select any node in the circuit as the ground node.



(b) Label nodes with voltage set by sources and the remaining nodes.

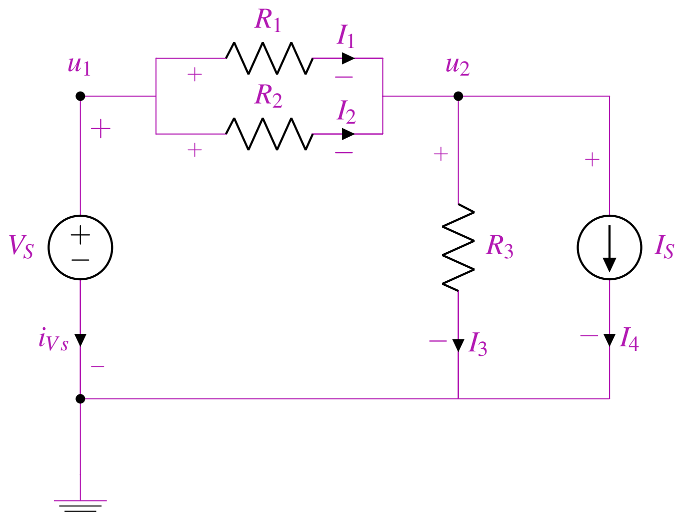
Answer: In this part, we label all the nodes except the reference.



(c) Label element voltages and currents.

Answer: We start with arbitrary current directions, you do not need to think about how charges are flowing at all. Then mark the element voltages according to Passive Sign Convention.

We do not need the element voltage for the current source and the element current for the voltage source. However, you are free to label these as well.



(d) Write the KCL equations for all the nodes without a voltage source.

Answer: If a node is connected to a voltage source, the corresponding node voltage is set by the voltage source. So we do not write KCL equations for nodes with voltage sources. So write KCL equation for only node u_2:

$$I_1 + I_2 = I_3 + I_4$$

(e) Write expressions for the element currents for resistors and current sources. Also write expressions for node voltages for the nodes connected to the voltage sources.

Answer: We write expressions for resistor currents (using Ohm's Law) and current sources:

$$I_1 = \frac{V_{R1}}{R_1} = \frac{u_1 - u_2}{R_1}$$

$$I_2 = \frac{V_{R2}}{R_2} = \frac{u_1 - u_2}{R_2}$$

$$I_3 = \frac{V_{R3}}{R_3} = \frac{u_2 - 0}{R_3} = \frac{u_2}{R_3}$$

$$I_4 = I_S$$

Then we write evaluate u_1, which is the node connected to the voltage source:

$$u_1 = V_S - 0 = V_S$$

(f) Substitute element current expressions into the KCL equations.

Answer:

For node u_2:

$$\frac{u_1 - u_2}{R_1} + \frac{u_1 - u_2}{R_2} = \frac{u_2}{R_3} + I_s$$
$$\frac{V_S - u_2}{R_1} + \frac{V_S - u_2}{R_2} = \frac{u_2}{R_3} + I_s$$

Let us write this equation to group the coefficients of the unknowns together. This equation has one unknown, u_2.

$$u_2\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = V_S\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - I_s$$

(g) Solve for the unknown node voltages (use Gaussian Elimination if necessary).

Answer: We have the element values: $V_S = 5\text{V}$, $R_1 = 2\text{k}\Omega$, $R_2 = 2\text{k}\Omega$, $R_3 = 4\text{k}\Omega$, and $I_S = 2\text{mA}$. Plugging in these values, we have:

$$u_2\left(\frac{1}{2000} + \frac{1}{2000} + \frac{1}{4000}\right) = 5\left(\frac{1}{2000} + \frac{1}{2000}\right) - 0.002$$

Now, we can solve this equation to find u_2:

$$u_2\left(\frac{5}{4000}\right) = 0.003$$
$$\implies u_2\left(\frac{5}{4000}\right) = 0.003$$
$$\implies u_2 = 2.4\text{V}$$

(h) Find each elemental current and voltage.

Answer: The current are the following:

$$I_1 = \frac{u_1 - u_2}{R_1} = \frac{5 - 2.4}{2000} = 1.3\text{mA}$$
$$I_2 = \frac{u_1 - u_2}{R_2} = \frac{5 - 2.4}{2000} = 1.3\text{mA}$$
$$I_3 = \frac{u_2}{R_3} = \frac{2.4}{4000} = 0.6\text{mA}$$
$$I_4 = I_S = 2\text{mA}$$
$$I_{V_s} = -I_1 - I_2 = -1.3\text{mA} - 1.3\text{mA} = -2.6\text{mA}$$

Then we calculate the element voltages:

$$V_{R1} = u_1 - u_2 = 5\text{V} - 2.4\text{V} = 2.6\text{V}$$
$$V_{R2} = u_1 - u_2 = 5\text{V} - 2.4\text{V} = 2.6\text{V}$$
$$V_{R3} = u_2 - 0 = 2.4\text{V}$$
$$V_{Vs} = u_1 - 0 = 5\text{V}$$
$$V_{Is} = u_2 - 0 = 2.4\text{V}$$

(i) Find the power dissipated by each element. Show that the power is conserved.

Answer: Dissipated power in each element can be calculated by multiplying the element current and element voltage. If the sign of dissipated power comes out negative, that means the corresponding element is delivering power.

$$
\begin{aligned}
P_{R1} &= V_{R1} \times I_1 = 2.6\text{V} \times 1.3\text{mA} = 3.38\text{watt} \\
P_{R2} &= V_{R2} \times I_2 = 2.6\text{V} \times 1.3\text{mA} = 3.38\text{watt} \\
P_{R3} &= V_{R3} \times I_3 = 2.4\text{V} \times 0.6\text{mA} = 1.44\text{watt} \\
P_{Vs} &= V_{Vs} \times I_{Vs} = 5\text{V} \times -2.6\text{mA} = -13\text{watt} \\
P_{Is} &= V_{Is} \times I_4 = 2.4\text{V} \times 2\text{mA} = 4.8\text{watt}
\end{aligned}
$$

Summing up all the power variables we get:

$$P_{R1} + P_{R2} + P_{R3} + P_{Is} + P_{Vs} = 0.$$

So power is conserved.

