CSM 16A Designing Information Devices and Systems I
Fall 2020 Week 6

1. Resistance is not Futile!

Learning Goal: Introduce students to the concept of resistivity in the perspective of circuit design and visualize the model when drawing wires and taking into considering cross-sectional area/length.

Please look into **Note 12 Section 12.3-12.4** to learn about resistivity and how different dimensions impact the resistance.

Resistivity is a **physical property** of the material that quantifies how much it opposes the flow of electric current.

Assume that in an ideal case, the cross-section and physical composition of the wire are uniform, We can find its resistance with the equation below:

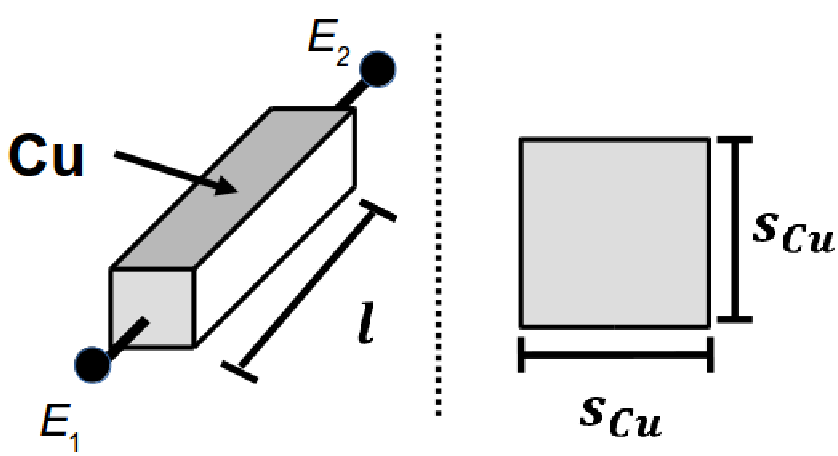
$$R = \rho \frac{L}{A}$$

Here, ρ stands for the resistivity of the wire, R stands for its resistance, A stands for the area of the cross section of the wire, and L stands for the length of the wire.

We will be frequently referencing some of the following variables:

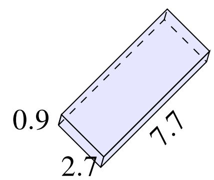
- A: the cross section area of a single wire.
- L: the length of a single wire.
- ρ_{cu}: resistivity for the material copper.
- ρ_{Al}: resistivity for the material aluminum.

(a) A copper (Cu) structure with a square cross-section is shown below. Given the material parameters, calculate the resistance R_{Cu} of the structure between E_1 and E_2.



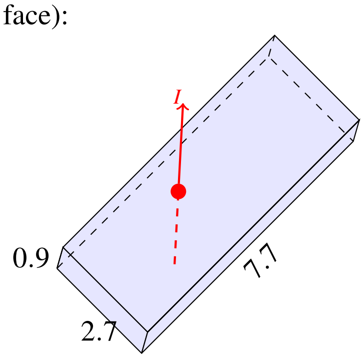
ρ_{Cu}	$1 \times 10^{-8}\,\Omega\,\text{m}$
s_{Cu}	$5\,\text{nm}$
l	$75\,\text{nm}$

(b) Now you are given a copper slab with dimensions 0.9cm, 2.7cm, and 7.7cm as denoted on the figure below. The dimensions and the resistivity of the slab remain the same throughout the rest of the problem.

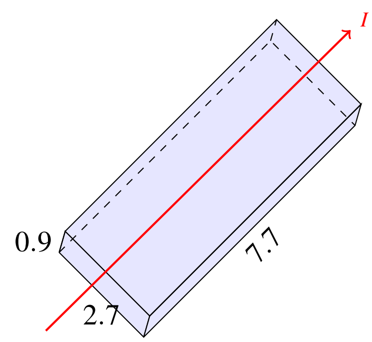


Suppose we connect opposite faces of the slab to a voltage source so that current can flow through it. Notice that there are **3 possible configurations** we can have for this slab, leading to 3 possible directions in which the current can flow (drawn in the answer choices below). Which direction will lead to the **highest** current flow *I* assuming we use the same voltage source all three times?
Assume the resistivity is the same throughout the slab and does not vary with respect to the direction of current flow.

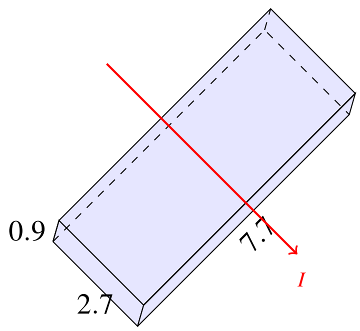
i. Parallel to 0.9 (current goes through 2.7, 7.7 face):



ii. Parallel to 7.7 (current goes through 0.9, 2.7 face):



iii. Parallel to 2.7 (current goes through 0.9, 7.7 face):



iv. Choices (i) and (iii) are tied for the maximum current flow.

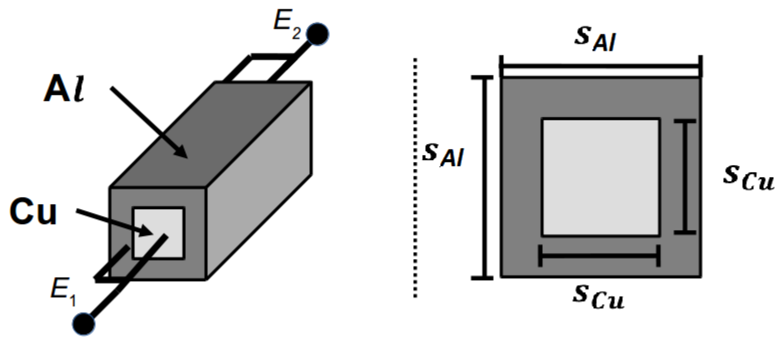
v. The direction doesn't matter; the resistor is physically the same, so the current will be the same.

(c) Suppose we have N wires similar to the one described in part (a). We align them side by side to form a bundle of wires. Find the overall resistance of this bundle. Are the wires connected in series or parallel?

(d) **(PRACTICE)** Again how can we connect these N identical wires so that the equivalent resistance is the highest?

(e) **(PRACTICE)** Consider part (c) again, but this time, instead of N copper wires, we split the number evenly between aluminum wires and copper wires. We arrange $N/2$ copper and $N/2$ aluminum wires side by side, and push them to form a new bundle of wires. What is the overall resistance of this wire? (In terms of ρ_{cu}, ρ_{Al}, L, and A)

(f) Now consider this *core-shell nanowire* structure, where the outside shell made of Al and the inside made of Cu. E_1 and E_2 terminals are both connected to the faces of the Cu and Al structure, as shown in the picture. Draw a circuit diagram of the nanowire as a set of resistors, using R_{Al} for the resistance of the Al layer and R_{Cu} for the resistance of the Cu layer.



(g) Based on your model from part (f) and the parameters given below, find the equivalent resistance R_{wire} between E_1 and E_2.

ρ_{Al}	$2 \times 10^{-8}\,\Omega\,\text{m}$
s_{Al}	$10\,\text{nm}$
ρ_{Cu}	$1 \times 10^{-8}\,\Omega\,\text{m}$
s_{Cu}	$5\,\text{nm}$
l	$75\,\text{nm}$

2. Equivalence in Resistive Networks

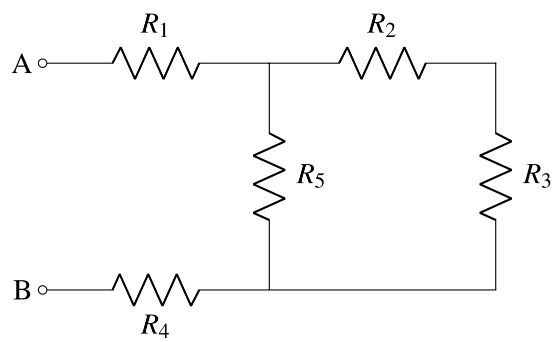
Learning Goal: Students will practice finding equivalences when given circuits with resistors in series and/or parallel.

Please look into **Note 15 Section 15.7.1 - 15.7.2** to see examples of finding circuit equivalences with resistors in series and resistors in parallel.

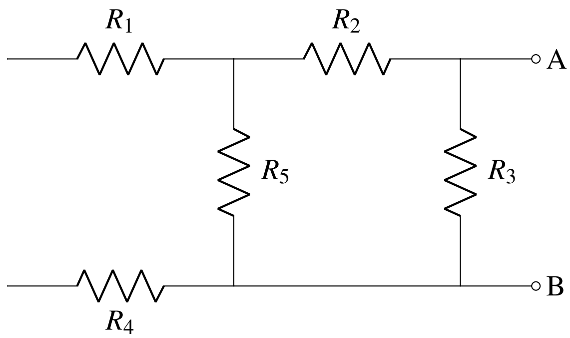
For all of the following networks find an expression or a numerical value for the equivalent resistance or capacitance between terminals A and B.

Hint: You can use the equivalence formulas for series and parallel combinations of resistors and capacitors for all of the subparts in this question.

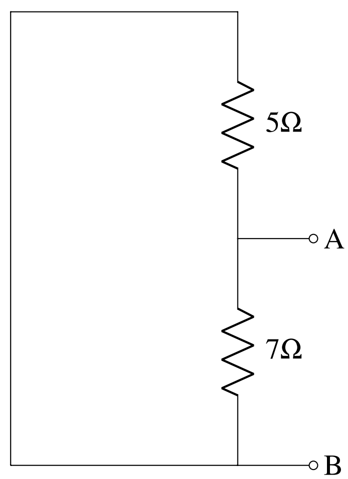
(a)



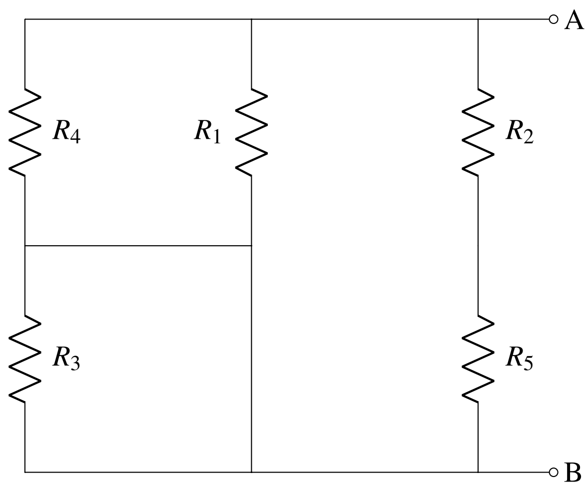
(b)



(c)



(d)



3. Midterms are a lot of Pressure

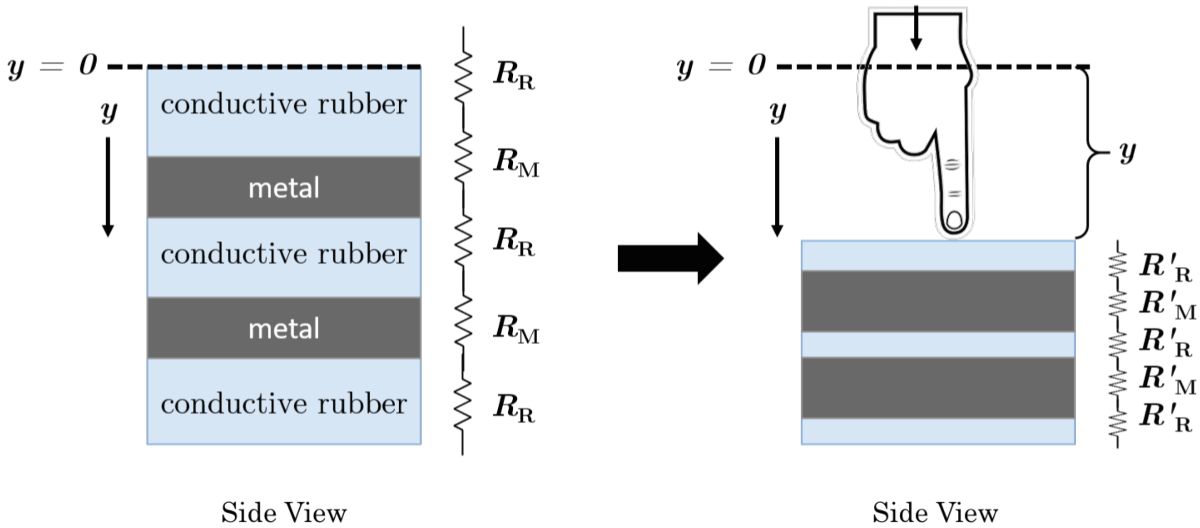
Learning Goal: Students will apply the concept of resistive sensors to a midterm style problem.

Please look into **Note 14 Section 14.4** to understand resistive sensors and convert physical models to circuits.

For this problem, we use something called "pressure sensitive rubber," which incorporates conductive rubber and metal into one system. As the rubber is pressed, the conductive rubber portions are compressed, which changes the resistance. The metal plates do not change dimensions.

The pressure sensitive rubber system is shown below, with a resistive model next to the diagram. The resistivity of rubber and metal are represented by ρ_R and ρ_M respectively. When the system is at rest (no touch), the resistances of the rubber and metal are represented by R_R and R_M. The area of the sensor, as seen from above, is A.

To use the material, a finger presses on top of the system, compressing the rubber regions, creating a change in resistance, also shown below. Please answer the following questions related to the system.



(a) Is the resistor model implementing resistors in series or parallel?

(b) If the values are $R_R = 1\,\text{k}\Omega$ and $R_M = 10\,\Omega$, what is the total resistance before pressing the system?

(c) During the press, the length of each rubber portion is reduced by a factor of 5. (Its length is now 1/5 of its original value.) The size of the metal plates does not change. What is the new total resistance during a press?

(d) The force required to compress the rubber is $F = ky$, where k is a constant and y is the distance compressed (from the origin). Derive an expression for the resistance as a function of the pressing force F.

Write your answer in terms of the initial resistances (R_R and R_M), the resistivities (ρ_R and ρ_M), the area of the sensor, A, and the constant, k. Assume all rubber layers compress the same amount and uniformly.