1. Series and Parallel Capacitors

**Learning Goal:** This problem will help to understand how capacitors in series or parallel combination respond to a voltage source or a current source.

**Relevant Notes:** Note 16: Section 16.3 goes over the capacitance equivalence.

Find the voltage across and current through each capacitor for each of the following scenarios. Consider all the capacitors to be initially uncharged.

(a) $C_1$ and $C_2$ are in series:

![Series Capacitors Diagram]

(b) $C_1$ and $C_2$ are in parallel:

![Parallel Capacitors Diagram]
(c) \( C_1 \) and \( C_2 \) are in series:

\[
\begin{align*}
I_s & \quad + \quad C_1 \quad - \quad I_1 \\
V_1 & \quad + \quad C_1 \quad - \quad V_1 \\
I_1 & \quad + \quad C_2 \quad - \quad I_2 \\
V_2 & \quad + \quad C_2 \quad - \quad V_2
\end{align*}
\]

(d) \( C_1 \) and \( C_2 \) are in parallel:

\[
\begin{align*}
I_s & \quad + \quad C_1 \quad V_1 \quad + \quad C_2 \\
V_1 & \quad + \quad C_1 \quad - \quad V_1 \\
I_1 & \quad + \quad C_2 \quad - \quad I_2 \\
V_2 & \quad + \quad C_2 \quad - \quad V_2
\end{align*}
\]

2. Capacitive Touchscreen

**Learning Goal:** The goal of this problem is to model the capacitive touchscreen covered in lecture.

**Relevant Notes:** Note 17 Section 17.1 introduces the capacitive touchscreen and its circuit model. Note 16 Section 16.3 is helpful for creating the model, as it goes over capacitor equivalence.

Consider the following capacitive touchscreen configuration from lecture.

For the following parts, let \( \varepsilon = 10^{-11} \text{F/m} \), \( w_1 = 1 \text{cm} \), \( w_2 = 1 \text{cm} \), \( w_F = 3 \text{cm} \), \( l_1 = 5 \text{cm} \), \( l_2 = 5 \text{cm} \), \( l_F = 4 \text{cm} \), \( d = 5 \text{mm} \), \( h_1 = 5 \text{mm} \), and \( h_2 = 15 \text{mm} \).

(a) Draw a diagram representing the capacitance between \( E_1 \) and \( E_2 \) when there is no touch on the screen.
(b) Calculate the value of the capacitance between the two electrodes $E_1$ and $E_2$ when the screen is not being touched. Remember that $\varepsilon = 10^{-11}\text{F/m}$, $w_1 = 1\text{cm}$, $w_2 = 1\text{cm}$, and $d = 5\text{mm}$.

(c) Calculate (i) the capacitance between the finger and the top electrodes and (ii) the capacitance between the finger and the bottom electrodes, when the screen is being touched.
(d) Now consider what happens when we touch the screen. Let the blue line represent our finger, and assume there is a capacitance between your finger and each of the electrodes. The diagram looks like this:

Redraw the circuit diagram representing the capacitive touchscreen after being touched, so that the nodes representing $E_1$ and $E_2$ are on opposite ends of the diagram.

(e) Calculate the new capacitance between $E_1$ and $E_2$. Remember that $C_{F-E_1} = 6 \times 10^{-13} \text{F}$ and $C_{F-E_2} = 2 \times 10^{-13} \text{F}$. Has the effective capacitance changed from when there was no touch?
3. Charge Sharing and Conservation

In this question, we will explore how charges are conserved and shared when multiple charged (or uncharged) capacitors are connected together. Charge sharing and conservation is useful not only for dividing up the charges for different components (with different power demands) within a system, but also for storing and transferring the power in the case of limited access to the original voltage source.

Given the following circuit containing 2 switches $\phi_1, \phi_2$, the circuit repeatedly goes through a cycle of 2 phases (described below), continuously supplying voltage to the node $V_{out}$.

The two phases the circuit goes through are as follows:

(i) Close switch $\phi_1$ until $C_1$ (initial uncharged) is fully charged. Switch $\phi_2$ remains open.

(ii) Open switch $\phi_1$ and close switch $\phi_2$. Maintain this configuration until the charges on both capacitors stabilize.

(a) Draw out what the circuit would look like in Phase (i).

(b) Given that $C_1$ and $C_2$ are both initially uncharged, what would be the charges $Q_1$ and $Q_2$ on capacitors $C_1$ and $C_2$ respectively by the end of Phase i? What would $V_{out}$ be?
(c) Draw out what the circuit would look like in Phase (ii).

(d) Continuing from what the charges were on both capacitors at the end of Phase i, what would the charges on both capacitors $C_1$ and $C_2$ be by the end of Phase ii? What would $V_{out}$ be?