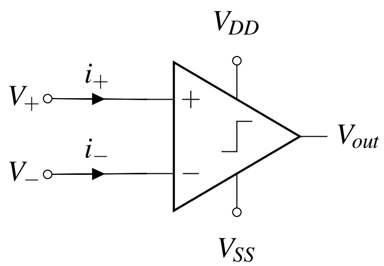
CSM 16A Designing Information Devices and Systems I
Fall 2020 Week 9

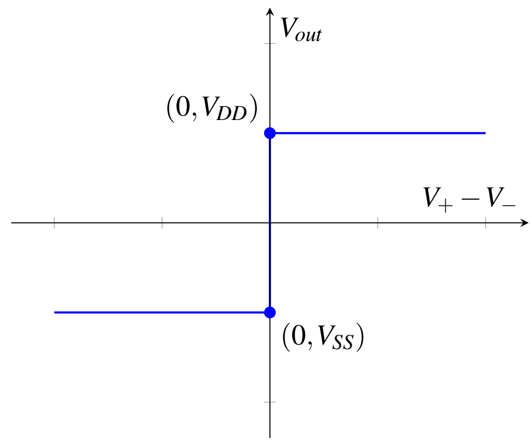
1. **Comparators**

   Learning Goal: This problem will help to understand comparator properties and design circuits with comparators.

   Relevant Notes: **Note 17C** goes over comparator properties.

   Comparators are typically drawn like the figure on the left, and their internal workings can be represented by the figure on the right.

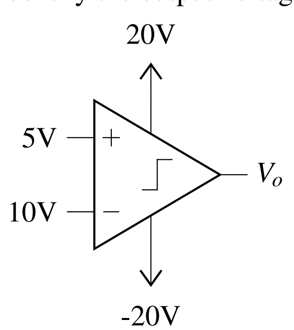




   Here, V_+ and V_- are input voltages, V_{DD} and V_{SS} are what we call the "supply rails", and V_{out} is the output voltage. From the diagram and knowing that V_{out} cannot exceed the supply rail voltages, we have a relationship between the outputs and the inputs:

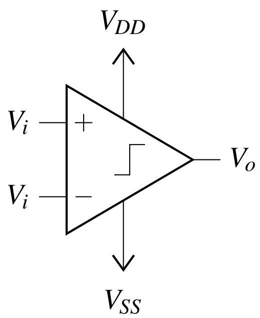
$$V_{out} = \begin{cases} V_{DD} & \text{, if } \quad V_+ > V_-) \\ V_{SS} & \text{, if } \quad V_+ < V_-) \\ \text{undefined} & \text{, if } \quad V_+ = V_-) \end{cases}$$

   (a) Identify the output voltage V_o for the following comparator:



   Answer: $V_- = 10\text{V}$ and $V_+ = 5\text{V}$, so $V_+ < V_-$, causing the comparator to rail to V_{SS}. So $V_o = -20\text{V}$.

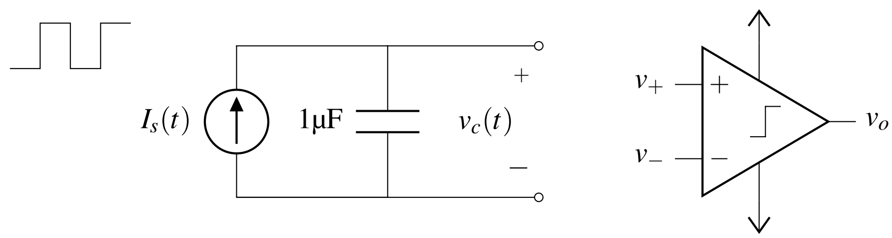
(b) Identify the output voltage V_o for the following comparator:



Answer: Since $V_- = V_+ = V_i$, the two input voltages into the comparator are equivalent. This results in undefined behavior, so $V_o =$undefined.

(c) Design a circuit by connecting the capacitive circuit and the comparator in the following figure such that $v_o = 2\text{V}$ if $v_c > 0.5\text{V}$ and $v_o = -1\text{V}$ if $v_c < 0.5\text{V}$. Draw your designed circuit.

You can use as many voltage sources you want. Label the values of the voltage sources you use.

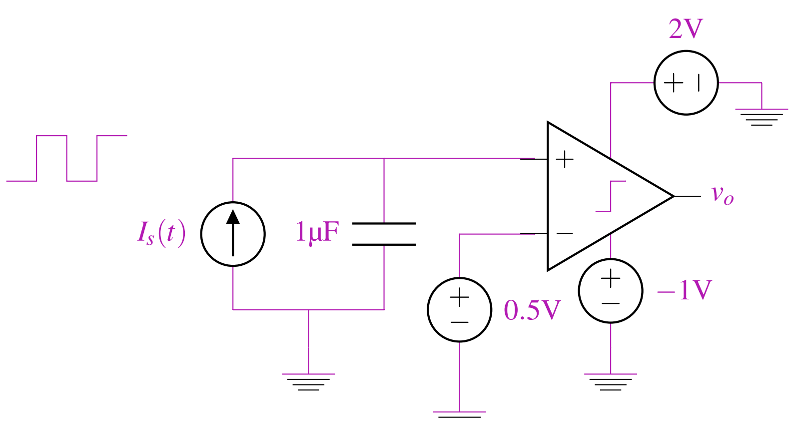


Answer: The problem is asking us to design a circuit that produces the following V_o:

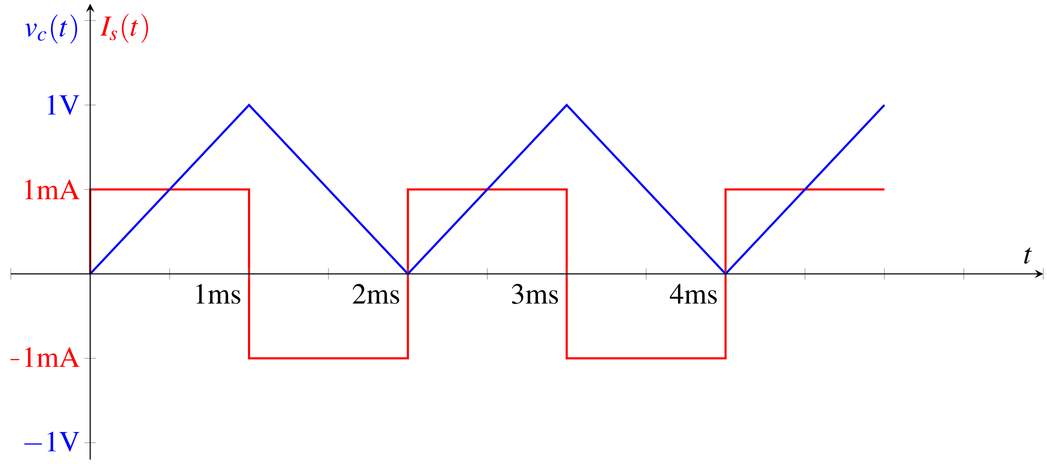
$$V_o = \begin{cases} 2\text{V} & \text{, if} \quad v_c > 0.5\text{V} \\ -1\text{V} & \text{, if} \quad v_c < 0.5\text{V} \\ ? & \text{, if} \quad v_c = 0.5\text{V} \end{cases}$$

We would like to use a comparator to achieve this. Notice that if we replace $V_{SS} = 2\text{V}$, $V_{DD} = -1\text{V}$, $V_- = 0.5\text{V}$, and $V_+ = v_c$, our comparator will output the desired values.

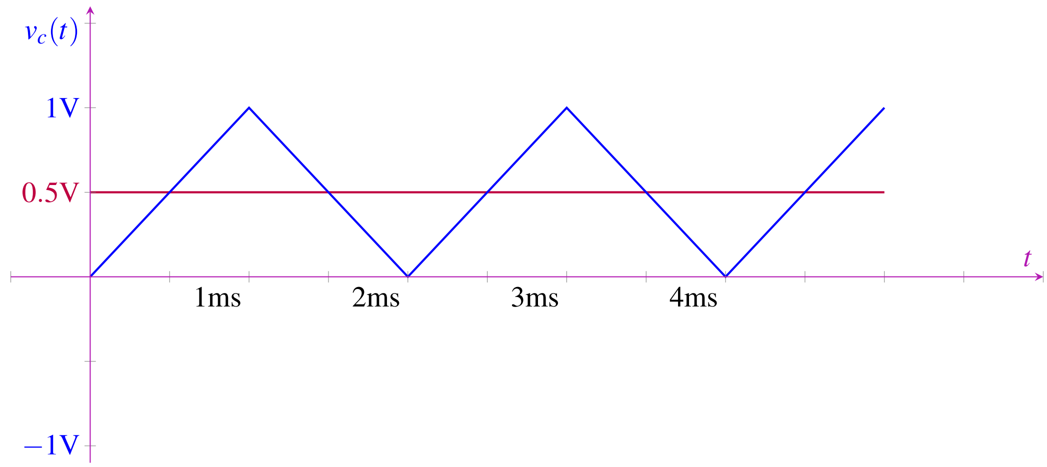
To construct this, we will connect V_{SS} and V_{DD} to two voltage sources of 2V and -1V, respectively. We will also connect V_- to a 0.5V voltage source. Finally, we want $V_+ = v_c$, which we can achieve by connecting the top terminal of v_c to V_+ and the bottom terminal to ground:



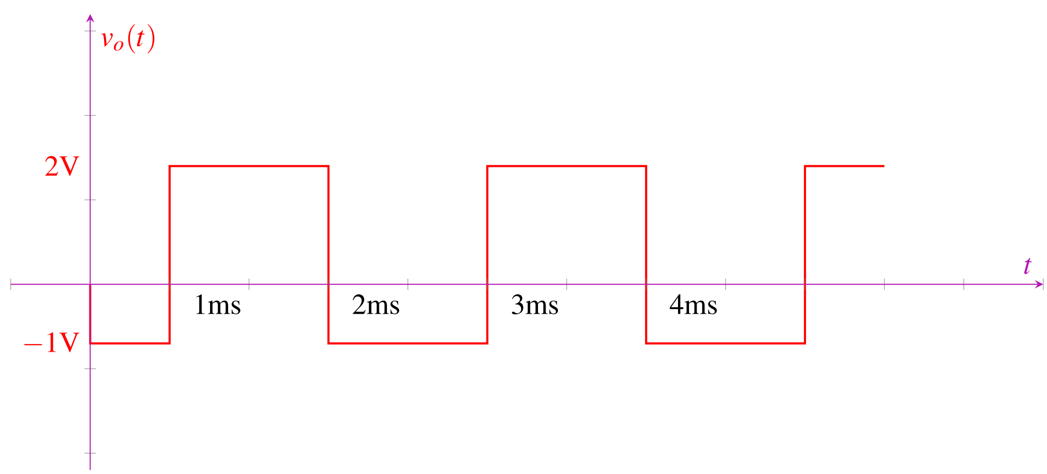
(d) Assume $I_s(t)$ and $v_c(t)$ are given in the following figure. Plot $v_o(t)$ vs. time for the circuit you designed in the last part.



Answer: Since our comparator is a function solely on $v_c(t)$, we look at when $v_c(t)$ is above or below 0.5V:



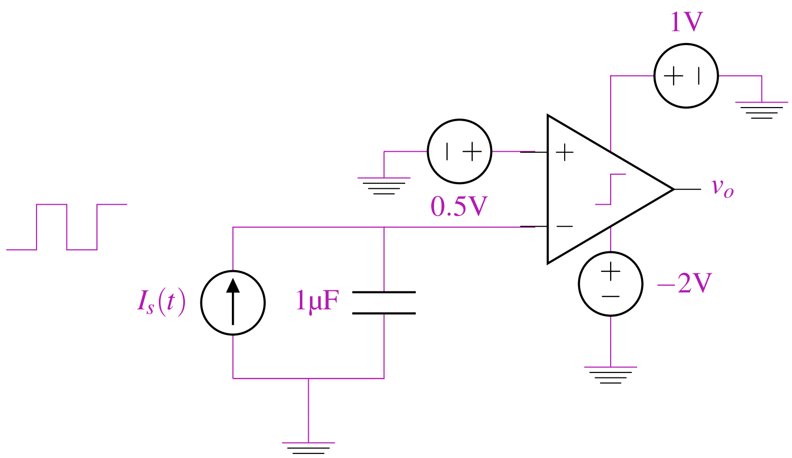
When the blue line is above the dark red one, the comparator rails to 2V. This happens periodically: between $t = 0.5$ms and $t = 1.5$ms, between $t = 2.5$ms and $t = 3.5$ms, etc. When it is below, it rails to -1V. This happens at all other times. Thus we have the following plot for $v_o(t)$:



(e) Redesign your circuit such that $v_o = -2\text{V}$ if $v_c > 0.5\text{V}$ and $v_o = 1\text{V}$ if $v_c < 0.5\text{V}$. Draw your redesigned circuit.

You can use as many voltage sources you want. Label the values of the voltage sources you use.

Answer: This circuit exhibits similar comparative behavior to the one in part (c). The major difference is the value of the rails, where the lower voltage is now -2V and the upper one is 1V. Also, the condition on when to rail to the lower or upper voltage is flipped. We can achieve this by flipping the connected elements to v_+ and v_- relative to part (c), so $v_- = v_c$ (connected to the periodic capacitive circuit) and $v_+ = 0.5\text{V}$:

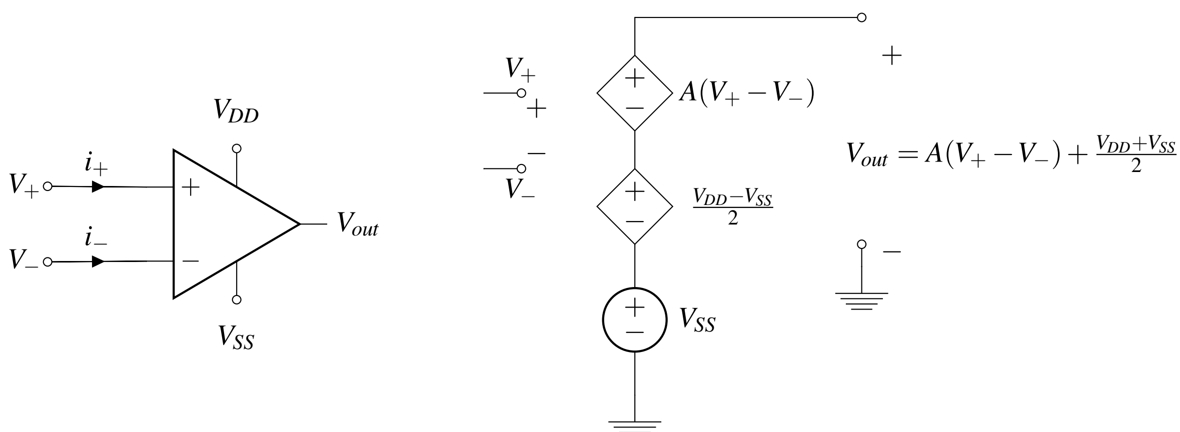


2. Operational Amplifiers

Learning Goal: This problem will help to understand op-amps in negative feedback and the operation of an inverting amplifier.

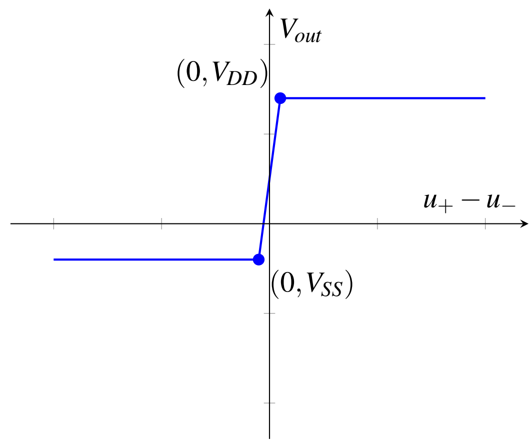
Relevant Notes: Note 18 goes over op-amp properties and derivation of circuit responses.

Operational amplifiers (op amps for short) are typically drawn like the figure on the left, and their internal workings can be represented by the figure on the right.



Here, V_+ and V_- are input voltages, V_{DD} and V_{SS} are what we call the "supply rails", and V_{out} is the output voltage. From the diagram and knowing that V_{out} cannot exceed the supply rail voltages, we have a relationship between the outputs and the inputs:

$$V_{out} = \begin{cases} V_{SS} & \text{, if} \quad A(V_+ - V_-) + \frac{V_{DD}+V_{SS}}{2} < V_{SS} \\ A(V_+ - V_-) + \frac{V_{DD}+V_{SS}}{2} & \text{, if} \quad V_{SS} \leq A(V_+ - V_-) + \frac{V_{DD}+V_{SS}}{2} \leq V_{DD} \\ V_{DD} & \text{, if} \quad V_{DD} < A(V_+ - V_-) + \frac{V_{DD}+V_{SS}}{2} \end{cases}$$



Typically gain A is quite large, meaning the the sloped region in the center is quite narrow.

(a) Much of EE16A will have our analysis be restricted to ideal op-amps. However, it is important to know non-ideal behaviors. What are some main differences between ideal and non-ideal op-amps?

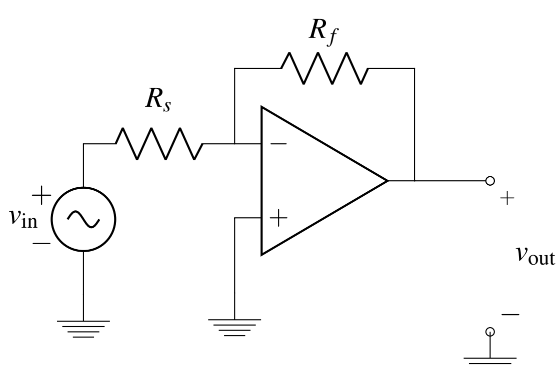
Answer: Ideal op-amps: Infinite gain $A = \frac{V_o}{V_i}$ (where $V_i = V_+ - V_-$). This means that the gain will be infinity if there was no limit on the rails, but it cannot reach infinity because it gets clipped at the positive or negative rails. infinite input resistance, zero input current. Non-ideal op amps: finite gain, finite input resistance, finite input current.

(b) If the gain of an operational amplifier/op amp is given by $A \rightarrow \infty$, so we can make some assumptions known as the "Golden Rules". What are the "Golden Rules" and when are they applicable?

Answer:

- Always true: $i_- = i_+ = 0\text{A}$
- Negative feedback only: $u_+ = u_-$

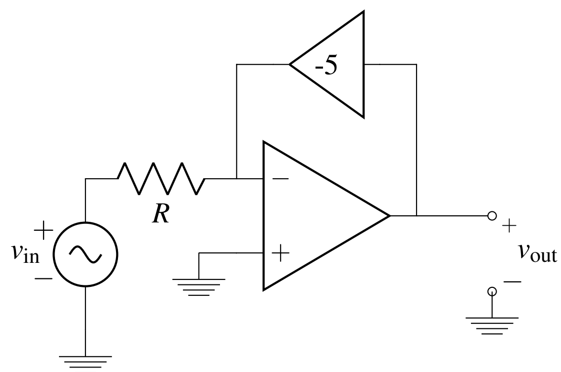
(c) Determine if the following system is in negative feedback.



Answer: This is in negative feedback, and in fact is a fairly well-known schematic called the inverting amplifier.

- For our initial stimulus, we'll kick v_{out} up
- How does u_- respond? Well, we can't assume $u_+ = u_-$ (because that requires the system to be in negative feedback), so we'll look at the resistors. Note that this is a voltage divider with u_- in the and, v_{out} at the top, so when v_{out} moves upward, u_- travels upward too
- If u_- goes up, that means v_{out} goes down because of the op amp—the opposite direction as the iniital stimulus!

(d) Determine if the following system is in negative feedback.

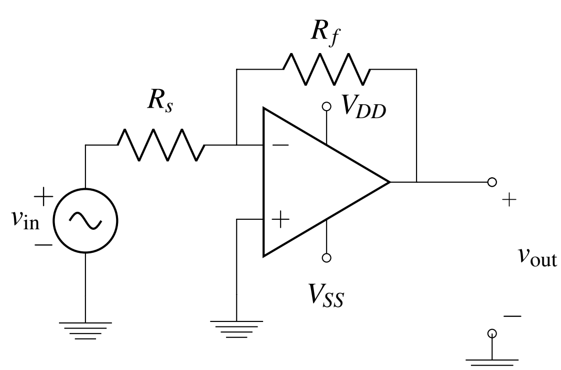


Note: The triangular block with the label "-5" in the figure above represents an amplifier with a factor of -5; In other words, it takes V_{out} as an input from the right side and outputs $-5V_{\text{out}}$ on the left side.

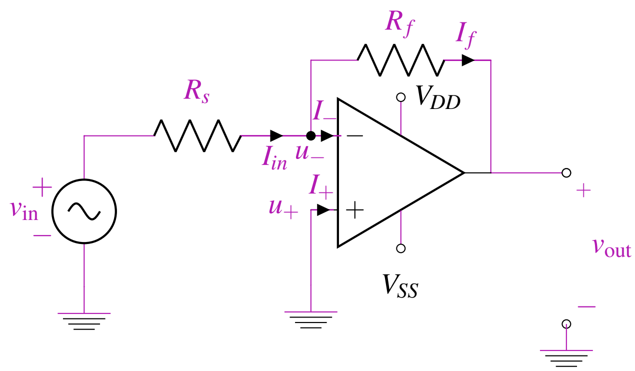
Answer: This is *not* in negative feedback! Going through the process of checking:

- For our initial stimulus, we'll kick v_{out} up
- With the $-5 \cdot v_{out}$, that means u_- goes down
- If u_- goes down, that means v_{out} goes up—the same direction as the initial stimulus!

(e) Find the expression of v_{out} for the following circuit:



Answer:



The first golden rule for op amps tell us:

$$I_+ = I_- = 0$$

Since we verified that this circuit is in negative feedback in part (c), the second golden rule also applies:

$$u_+ = u_-$$

Using KCL, we can write the following equations:

$$I_{in} = I_f + I_-$$

Since $I_- = 0$, we can write

$$I_{in} = I_f$$

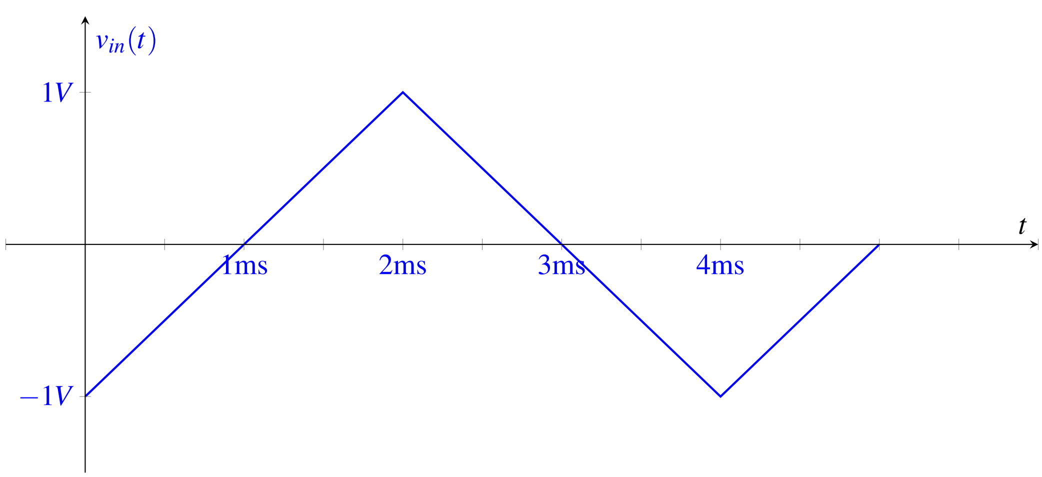
Since the positive input terminal is connected to ground, $u_+ = 0$, i.e. $u_- = u_+ = 0$.
Using the formula, V = IR:

$$I_{in} = \frac{v_{in} - u_-}{R_s} = \frac{v_{in}}{R_s}$$
$$I_f = \frac{u_- - v_{out}}{R_f} = \frac{-v_{out}}{R_f}$$

From the KCL equations, we know $I_{in} = I_f$. Substituting the equations from above,

$$\frac{v_{in}}{R_s} = \frac{-v_{out}}{R_f}$$
$$v_{out} = -\frac{v_{in} R_f}{R_s}$$

(f) Plot v_{out} vs. time for the following v_{in}. Assume $V_{DD} = 10\text{V}$, $V_{SS} = -10\text{V}$, $R_s = 100\Omega$, $R_f = 500\Omega$.



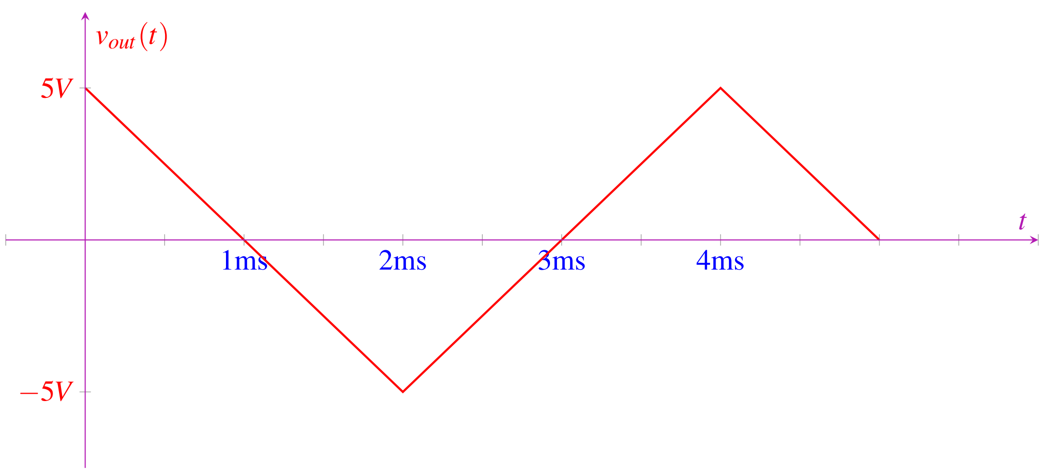
Answer:
Using the formula for v_{out} obtained in part (e) and the values for R_f and R_s, we can find the circuit gain:

$$G = \frac{v_{out}}{v_{in}} = -\frac{R_f}{R_s} = -\frac{500}{100} = -5$$

So the input will be multiplied by a factor of -5. We can substitute to find v_{out} along different times.

v_{in}	Time	v_{out} Calculation	v_{out}
-1	0	$-\frac{(-1)(500)}{100}$	5
0	1	$-\frac{(0)(500)}{100}$	0
1	2	$-\frac{(1)(500)}{100}$	-5
0	3	$-\frac{(0)(500)}{100}$	0
-1	4	$-\frac{(-1)(500)}{100}$	5
0	5	$-\frac{(0)(500)}{100}$	0

Using the v_{out} values we calculated above in the table, we can plot the v_{out} vs. time graph.



(g) What happens if R_f is changed to $R_f = 2000\Omega$. Plot v_{out} vs. time for the same v_{in}, where $V_{DD} = 10\text{V}$, $V_{SS} = -10\text{V}$, and $R_s = 100\Omega$.

Answer:
Similar to part (f), using the formula for v_{out} obtained in part (e) and the values for R_f and R_s, we can find the circuit gain:

$$G = \frac{v_{out}}{v_{in}} = -\frac{R_f}{R_s} = -\frac{2000}{100} = -20$$

So the input will be multiplied by a factor of -20. We can substitute to find v_{out} along different times. Using the v_{out} values we calculated above in the table, we can plot the v_{out} vs. time graph.

Here, we have to also consider the values given for V_{DD} and V_{SS}. The given bounds are $V_{DD} = 10\text{V}$, $V_{SS} = -10\text{V}$, but the values we obtained for V_{out} are between -20V and 20V. Therefore, we have to "clip" the graph at these points to not go past -10V and 10V, as shown in the graph below.

v_{in}	Time	v_{out} Calculation	v_{out}
-1	0	$-\frac{(-1)(2000)}{100}$	20
0	1	$-\frac{(0)(2000)}{100}$	0
1	2	$-\frac{(1)(2000)}{100}$	-20
0	3	$-\frac{(0)(2000)}{100}$	0
-1	4	$-\frac{(-1)(2000)}{100}$	20
0	5	$-\frac{(0)(2000)}{100}$	0