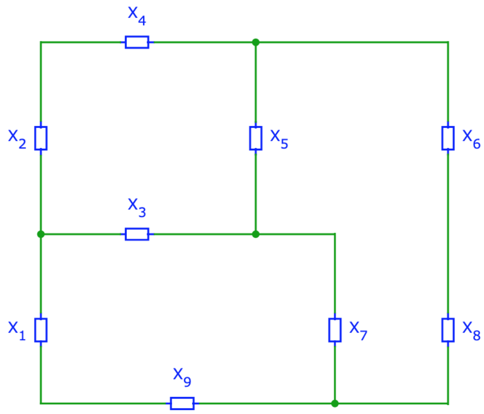
EECS 16A Designing Information Devices and Systems I
Fall 2020 Discussion 6B

1. Nodes and Branches

In the circuit shown below, label and count all nodes and branches.



Answer: There are seven nodes and nine branches.

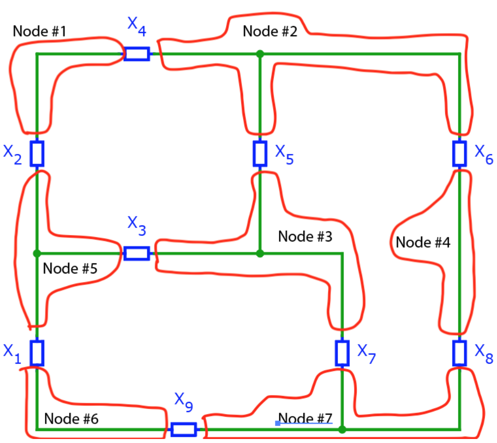


Figure 1: Labeled Nodes

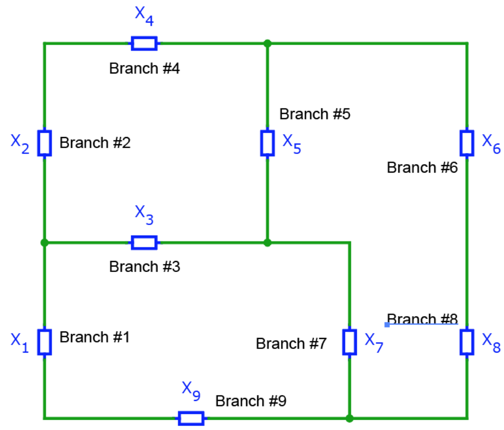
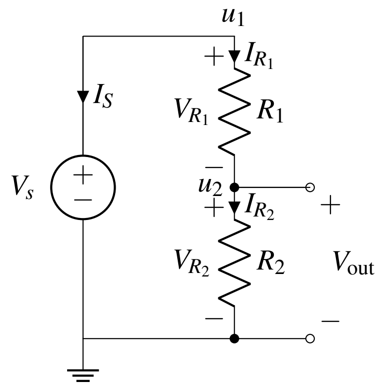


Figure 2: Labeled branches

2. Voltage Divider

For the circuit below, your goal will be to find the voltage V_{out} in terms of the resistances R_1, R_2, and V_s, using NVA (Node Voltage Analysis). The labeling steps (steps 1-4) have already been done for you.

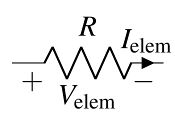


Here is a reminder of the labeling steps followed to get the circuit diagram above:

- **Step 1:** Select a reference (ground) node. Any node can be chosen for this purpose. We will measure all of the voltages in the rest of the circuit relative to this point.
- **Step 2:** Label all nodes with voltage set by voltage sources.
- **Step 3:** Label remaining nodes.
- **Step 4:** Label element voltages and currents, following **Passive Sign Convention** (discussed below).

Passive sign convention

The **passive sign convention** dictates that positive current should *enter* the positive terminal and *exit* the negative terminal of an element. Below is an example for a resistor:



As long as this convention is followed consistently, it does not matter which direction you arbitrarily assigned each element current to; the voltage referencing will work out to determine the correct final sign. When we discuss *power* later in the module, you will see why we call this convention "passive."

To achieve your goal of ***finding*** V_{out}, perform the rest of the NVA steps in the boxes below:

Step 5: Write KCL equations for all nodes with unknown voltages.

Step 6: Find expressions for all element currents in terms of element voltages and characteristics.

Step 7: Substitute all element voltages with node voltages found in your step 6 equations.

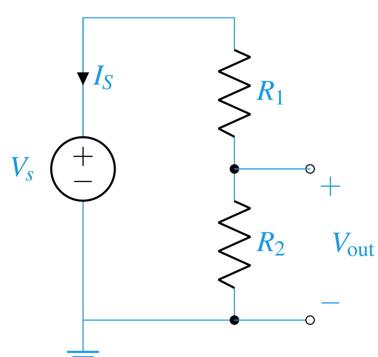
Step 8: Substitute expressions found in step 7 into the KCL equations from step 5.

Step 9: Solve for the node voltage values. At this point the analysis procedure is effectively complete - all that's left to do is solve the system of linear equations (by applying Gaussian Elimination, inverting $\mathbf{A}$, etc.) to find the values for the u's. Then we can go back to our Step 7 equations and calculate the I's. Note that in our circuit, $V_{out} = u_2 - 0 = u_2$.

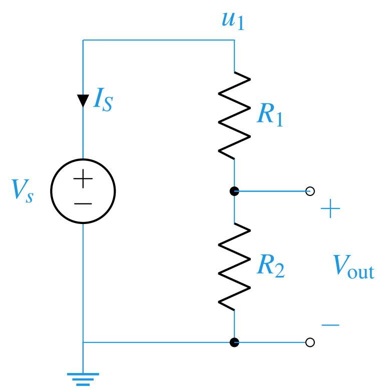
Answer:

Note: The solution will lead you though all of the steps. The result at the end of step 4 will be the circuit in the problem statement above.

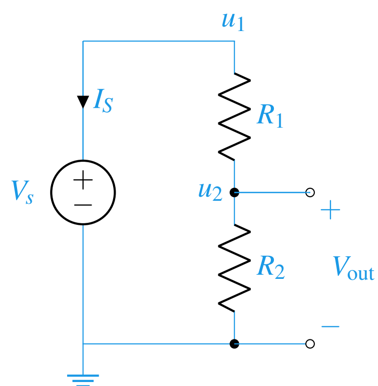
Step 1: Select a ground node,



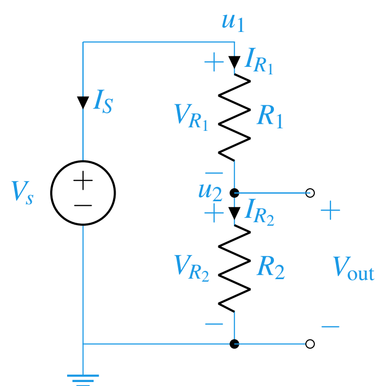
Step 2: Label all nodes with voltage set by voltage sources (denoted below as u_1),



Step 3: Label remaining nodes (denoted below as u_2),



Step 4: Label element voltages and currents following passive sign convention,



Step 5: Write KCL equations for all nodes with unknown voltages (namely u_2):

$$I_{R_2} = I_{R_1}$$

Step 6: Find expressions for all element currents in terms of element voltages and characteristics,

$$I_{R_1} = \frac{V_{R_1}}{R_1}$$
$$I_{R_2} = \frac{V_{R_2}}{R_2}$$

Step 7: Substitute all element voltages with node voltages found in your step 6 equations.

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{u_1 - u_2}{R_1} = \frac{V_s - u_2}{R_1}$$
$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{u_2 - 0}{R_2}$$

Where we used the fact that $u_1 = V_s$

Step 8: Substitute expressions found in 6 into the KCL equations from step 5,

$$I_{R_2} = I_{R_1}$$
$$\Rightarrow \frac{V_s - u_2}{R_1} = \frac{u_2 - 0}{R_2}$$
$$\Rightarrow (V_s - u_2)R_2 = u_2 R_1$$
$$\Rightarrow u_2 = \frac{R_2}{R_1 + R_2} V_s$$

Step 9: Since we only have one unknown there is no need to solve a system for equations, we just solve our equation from Step 8 for $u_2 = V_{\text{out}}$ to get:

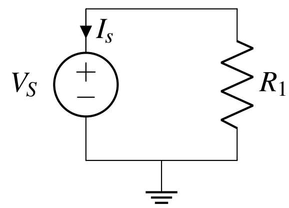
$$V_{\text{out}} = u_2 = \frac{R_2}{R_1 + R_2} V_s$$

Which is the voltage divider formula derived in lecture.

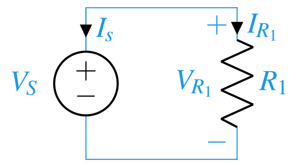
3. Practice: A Simple Circuit

Use KVL and/or KCL to solve the following circuits.

(a) For this problem assume $V_S = 1V$ and $R_1 = 1k\Omega$. Find the current, I_s flowing through the voltage source.



Answer: Notice that in this circuit we only have two nodes, and we know the node voltage of both of them.
Because of that, we can find the current flowing through I_s using KCL and KVL instead of precisely following the algorithm outlined in lecture. Labeling element voltages and currents we have:



Using KVL and Ohm's law we get:

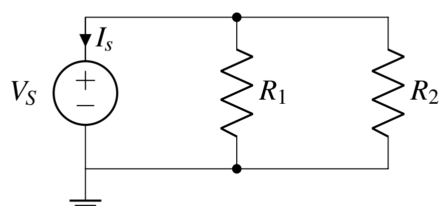
$$V_S = V_{R_1} \qquad \text{(KVL)} \tag{1}$$

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_S}{R_1} = 1mA \quad \text{(Ohm's law)} \tag{2}$$

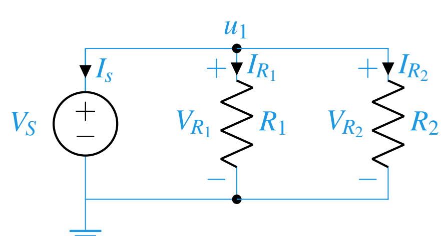
Finally through KCL on the top node we obtain:

$$I_s + I_{R_1} = 0 \Rightarrow I_s = -I_{R_1} = -\frac{V_S}{R_1} = -1mA \tag{3}$$

(b) For this problem assume $V_S = 1V$, $R_1 = 2k\Omega$, and $R_2 = 2k\Omega$. Find the current, I_s flowing through the voltage source.



Answer: Here we can follow the same procedure since we still have two terminals. Let's label again all element voltages and currents.



Using KVL and Ohm's law:

$$V_S = V_{R_1} = V_{R_2} \quad \text{(KVL)} \tag{4}$$

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_S}{R_1} \quad \text{(Ohm's law)} \tag{5}$$

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_S}{R_2} \quad \text{(Ohm's law)} \tag{6}$$

$$\tag{7}$$

Then writing out KCL and substituting from above we have:

$$I_{R_1} + I_{R_2} + I_S = 0 \tag{8}$$

$$\frac{V_s}{R_1} + \frac{V_s}{R_2} + I_S = 0 \Rightarrow I_S = -\left(\frac{V_s}{R_1} + \frac{V_s}{R_2}\right) \tag{9}$$

Plugging in,

$$I_S = -1mA \tag{10}$$

Notice that we did not make use of node u_1 or the ground node anywhere.

