EECS 16A Fall 2020

Designing Information Devices and Systems I

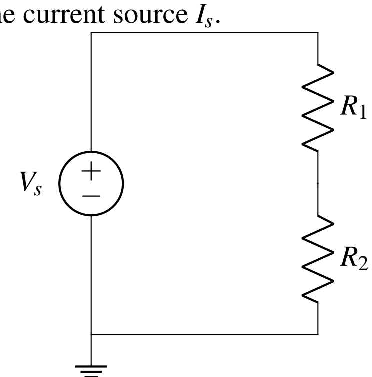
Discussion 7A

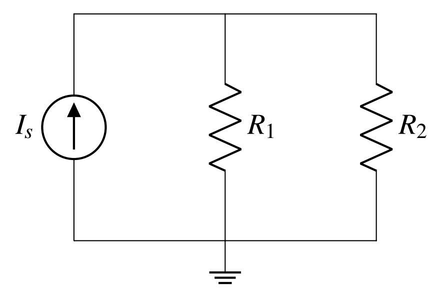
1. Current Divider

So far we've shown that for the circuit below to the left, the voltage across the resistor R_2 is expressed

$$V_{R_2} = \left(\frac{R_2}{R_1 + R_2}\right) V_s.$$

Let us derive a similar formula for the right circuit below, relating the current I_{R_2} through R_2 to the current through the current source I_s.





Answer:

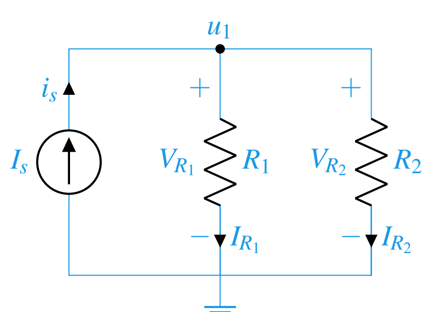
It would be an excellent idea to run through the circuit analysis procedure outlined in class:

Step 1 Selecting a ground node; this has already been done for us.

Step 2 Label all known voltages determined by voltage sources (there are none in this circuit).

Step 3 Label all remaining unknown nodes (denoted below u_1)/

Step 4 Label element voltages and currents (V_{R_1}, I_{R_1}, etc.)



Note: You can pick any direction you want for the currents, but once you have done so the voltages must abide by the passive sign convention!! Vice versa, you can pick the element voltage sign first but then the current has to abide by the passive sign convention.

Step 5 Write KCL equations for all nodes with unknown voltages (namely u_1)

$$I_s - I_{R_1} - I_{R_2} = 0$$

where we used the fact that $i_s = I_s$.

Step 6 Find expressions for all element currents in terms of element voltages and characteristics

$$V_{R_1} = I_{R_1} R_1 \qquad V_{R_2} = I_{R_2} R_2.$$

Step 7 Replace each element voltage expression using our node voltage terms (only u_1 for this circuit)

$$(u_1 - 0) = I_{R_1} R_1 \qquad (u_1 - 0) = I_{R_2} R_2.$$

Step 8 Plug our expressions found in step 7 into the KCL equations from step 5

$$I_s - \frac{u_1}{R_1} - \frac{u_1}{R_2} = 0 \qquad \longrightarrow \quad u_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = I_s \quad \rightarrow \quad u_1\left(\frac{R_2}{R_1\,R_2} + \frac{R_1}{R_2\,R_1}\right) = I_s \quad \rightarrow \quad u_1\left(\frac{R_2 + R_1}{R_1 R_2}\right) = I_s$$

Step 9 With the node voltage u_1 known explicitly now, replug back into our expressions from step 7 to finally identify the current passing through each resistor!

$$u_1 = \frac{R_1 R_2}{R_1 + R_2} I_s$$

From step 7 we plug in to find I_{R_1} and I_{R_1} from known circuit quantities

$$I_{R_1} = \frac{u_1}{R_1} \qquad \longrightarrow \qquad I_{R_1} = \frac{R_2}{R_1 + R_2} I_s$$

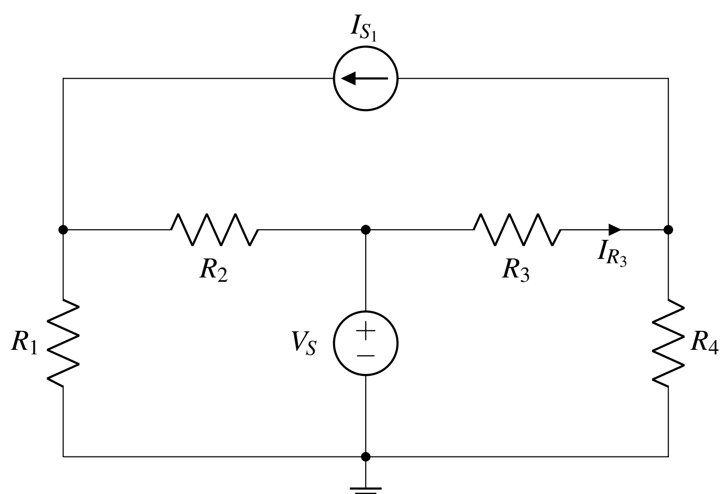
$$I_{R_2} = \frac{u_2}{R_2} \qquad \longrightarrow \qquad I_{R_2} = \frac{R_1}{R_1 + R_2} I_s \quad \square$$

2. Circuit Analysis

Provided the circuit below...

(a) ...use nodal analysis to solve for all node voltages.

(b) ...find the current I_{R_3} flowing through resistor R_3.



Answer:
Again it is a splendid idea to implement our circuit analysis procedure outlined in class!

Step 1 Selecting a ground node; this has already been done for us.

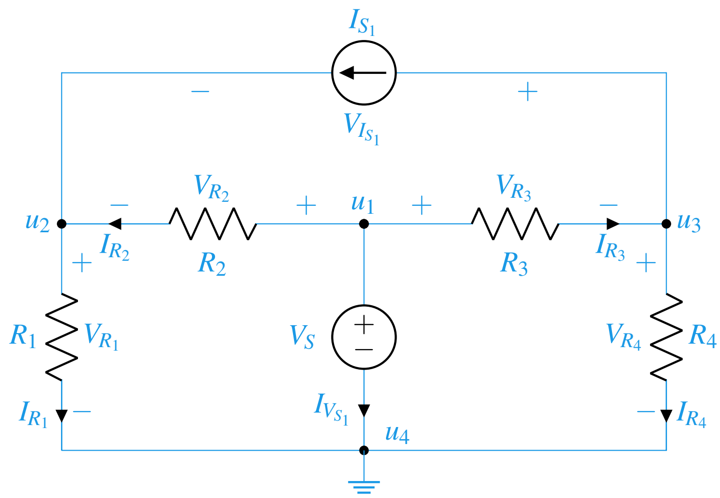
Step 2 Label all known voltages determined by voltage sources (denoted below as u_1, u_4).

We know $u_4 = 0$ since this node is directly linked to ground.

Also we know $u_1 = +V_s$ because the element linking node u_1 to ground is the voltage source V_s AND that source has the negative lead facing the ground side.

Step 3 Label all remaining unknown nodes (denoted below as u_2, u_3).

Step 4 Label element voltages and currents (V_{R_1}, I_{R_1}, etc.) .



Note: You can pick any direction you want for the currents, but once you have done so the voltages must abide by the passive sign convention!! Vice versa, you can pick the element voltage sign first but then the current has to abide by the passive sign convention.

Step 5 Write KCL equations for all nodes with unknown voltages (namely u_2, u_3):

$$I_{R_2} + I_{S_1} - I_{R_1} = 0$$
$$I_{R_3} - I_{S_1} - I_{R_4} = 0$$

Step 6 / 7 Find expressions for all element currents in terms of element voltages and characteristics.

Step 7 is where we substitute in the node voltage expressions for known quantities ($u_1 = +V_s, u_4 = 0$).

$$\begin{aligned}
I_{R_1}R_1 &= u_2 - u_4 &= u_2 \\
I_{R_2}R_2 &= u_1 - u_2 &= V_s - u_2 \\
I_{R_3}R_3 &= u_1 - u_3 &= V_s - u_3 \\
I_{R_4}R_4 &= u_3 - u_4 &= u_3
\end{aligned}$$

Step 8 Substitute expressions found in step 7 into the KCL equations from step 5.

$$\frac{V_s - u_2}{R_2} + I_{S_1} - \frac{u_2}{R_1} = 0$$
$$\frac{V_s - u_3}{R_3} - I_{S_1} - \frac{u_3}{R_4} = 0$$

Step 9 Notice that we can now directly solve for u_2 and u_3 from the results of step 8.

$$0 = \frac{V_s}{R_2} + I_s - u_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \qquad \longrightarrow u_2 = \frac{V_s R_1 + I_{S_1} R_1 R_2}{R_1 + R_2}$$
$$0 = \frac{V_s}{R_3} - I_s - u_3\left(\frac{1}{R_4} - \frac{1}{R_3}\right) \qquad \longrightarrow u_3 = \frac{V_s R_4 - I_{S_1} R_4 R_3}{R_4 + R_3} \qquad \square \textbf{ (a)}$$

Then we can substitute into the equations of Step 6 to find the currents through each resistor. This finally yields current I_{R_3}.

$$I_{R_3} = \frac{V_s - u_3}{R_3} = \frac{V_s - \frac{V_s R_4 - I_{S_1} R_4 R_3}{R_4 + R_3}}{R_3} = \frac{V_s + I_s R_4}{R_3 + R_4} \qquad \square \ (\mathbf{b})$$

3. Practice Problem: Bio-Molecule Detector

One application for electronics that has gained a lot of attention over the past several years is in so-called "bio-molecule" detection. The idea is to build a system that detects the presence of specific molecules and/or cells (e.g. specific viruses, proteins, etc.) in a biological sample; if this detection can be performed automatically and using relatively low-cost components, it can have a dramatic impact on a number of areas such as medical diagnosis, drug development, DNA sequencing, etc.

In this problem, we'll look at how some of the techniques we learned about in the touchscreen module can be applied to realize a hypothetical bio-molecule detector. (Real bio-molecule detection systems involve quite a bit more complexity than what we'll include here, but in many designs the same basic principles apply.)

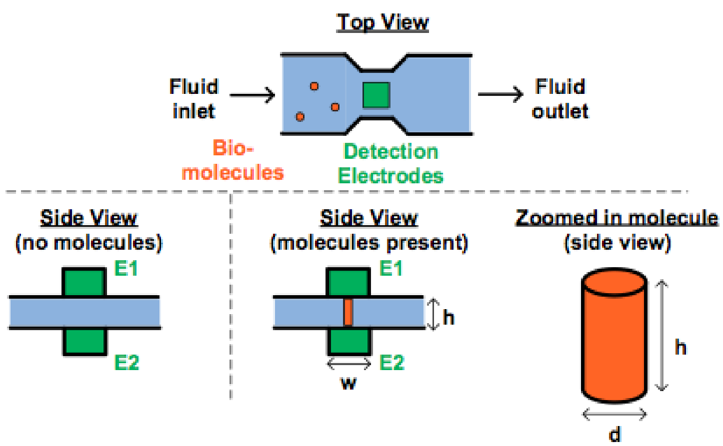


Figure 1: Bio-molecule detector.

As shown in Figure 1, the detector works by flowing a liquid that may or may not contain the biomolecules through a region in the device that has electrodes on the top and bottom of the liquid channel. The electrodes (E1/E2 in Figure 1) are chemically "functionalized" (using e.g. some appropriately designed antibodies), so that if the specific bio-molecule of interest is present in the fluid sample, one or more of the molecules will get physically trapped between the two electrodes (bottom right of Figure 1). After all of the fluid has been cleared out of the device (i.e., so that if there are no bio-molecules present, there is only air in between the two electrodes E1/E2), we can then figure out whether or not one or more bio-molecules were trapped by measuring the resistance between the two electrodes.

(a) Let's first assume that we want to detect the presence of a bio-molecule by measuring resistance. If no bio-molecule is present, what should be the resistance between E1/E2? As shown in Figure 1, if each bio-molecule is a cylinder with diameter $d = 10\,\text{nm}$, height $h = 100\,\text{nm}$, and has a resistivity $\rho = 100\,\mu\Omega\,\text{m}$, what would be the resistance between E1 and E2 if only a single bio-molecule has been trapped? Note that you can assume that the trapped molecule is exactly vertically oriented when it is trapped – i.e., the top and bottom faces of the molecule are both aligned with surfaces of the electrodes.

If no bio-molecule is present between E1/E2, we are left with just air between the electrodes, so for all intents and purposes, the resistance between them should be *infinite*.

If there is a bio-molecule trapped between E1/E2, the resistance R should be:

$$R_{\text{molecule}} = \frac{\rho \cdot l}{A} = \frac{100 \times 10^{-6}\,\Omega\,\text{m} \cdot 100 \times 10^{-9}\,\text{m}}{\pi \cdot \left(\frac{1}{2} \cdot 10 \times 10^{-9}\,\text{m}\right)^2} = 127.3\,\text{k}\Omega$$

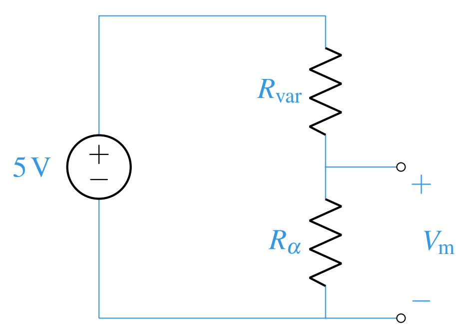
(b) Using the same numbers for d, h, and ρ as part (a), as a function of the number of trapped bio-molecules $N_{\text{molecules}}$, what is the resistance between E1 and E2? (Note that you can assume that $N_{\text{molecules}}$ is small enough that all of the molecules fit within the electrode area and that all of the molecules are still trapped in an exactly vertical orientation.)

By having many bio-molecules in the detector, the cross-sectional area becomes the sum of the areas of all of the molecules. Therefore, the net resistance between E1/E2 is:

$$R = \frac{127.3\,\text{k}\Omega}{N_{\text{molecules}}}$$

(c) Given your answers to parts (a) and (b), design a circuit that will output a voltage greater than 2.5 V if more than 5 molecules are trapped.

Design idea: use the relationship of the resistors in a voltage divider and the respective drops across them.



We use a voltage divider with a 5 V source and resistors R_α and R_{var} in series. R_{var} is the resistance across the molecules, and R_α is used to compare against R_{var}. We measure across R_α. We want the measured voltage $V_{\text{m}} > 2.5\,\text{V}$ when we have more than 5 molecules, so we want:

$$R_\alpha > R_{\text{var}}(6)$$

$$R_\alpha > \frac{127.3\,\text{k}\Omega}{6}$$

When we have 5 or less molecules, we want $V_{\text{m}} < 2.5\,\text{V}$ Therefore,

$$R_\alpha < R_{\text{var}}(5)$$

$$R_\alpha < \frac{127.3\,\text{k}\Omega}{5}$$

We can then set R_α to $\frac{127.3\,\text{k}\Omega}{5.5} = 23.15\,\text{k}\Omega$.