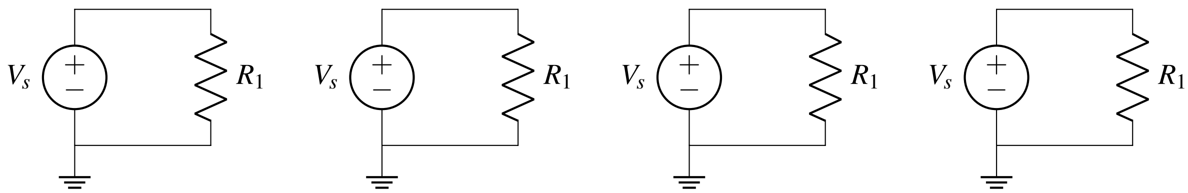
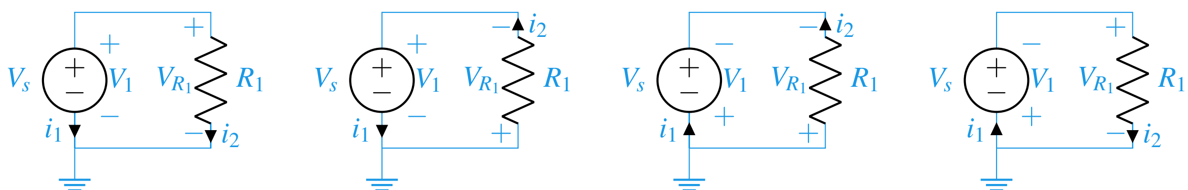
EECS 16A Designing Information Devices and Systems I
Fall 2020 Discussion 7B

1. Passive Sign Convention and Power

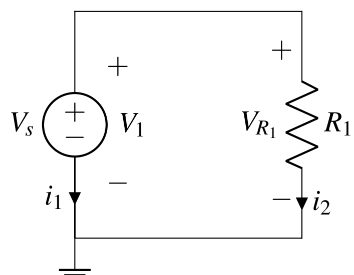
(a) We have made four copies of a circuit below. Following passive sign convention, there are four different possible labelings of current directions and voltage polarities for the circuit. For each copy, label each circuit's voltage source and resistor with current direction and voltage polarity labelings, keeping with passive sign convention.



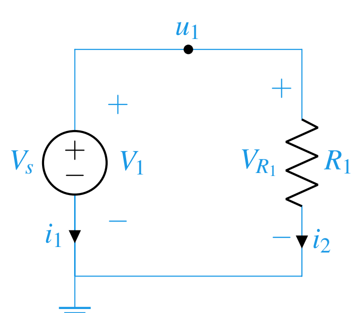
Answer:



(b) Suppose we consider one of the possible labelings you have found above. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5\,\text{V}$ and let $R_1 = 5\,\Omega$.



Answer: We'll start by solving the circuit for the unknown node potentials and currents.



The KCL equation for the one node in this circuit is:

$$i_1 + i_2 = 0$$

The element equations for the two elements in this circuit are:

$$u_1 - 0 = V_1 = V_s$$

$$u_1 - 0 = V_{R_1} = i_2 R_1$$

Solving the above equations with $V_s = 5\,\text{V}$ and $R_1 = 5\,\Omega$:

$$u_1 = 5\,\text{V}$$
$$i_1 = -1\,\text{A}$$
$$i_2 = 1\,\text{A}$$

From above, we can solve for the power dissipated across the resistor:

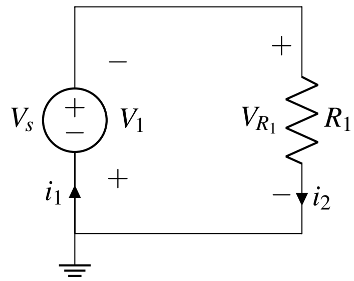
$$P_{R_1} = i_2 V_{R_1} = 1\,\text{A} \cdot 5\,\text{V} = 5\,\text{W}$$

Next we can solve for the power dissipated across the voltage source:

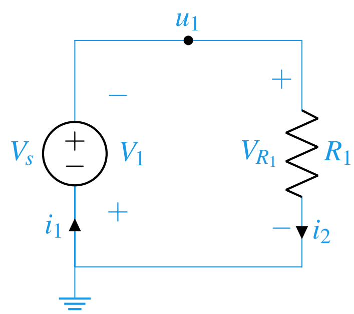
$$P_{V_s} = i_1 V_1 = i_1 V_s = -1\,\text{A} \cdot 5\,\text{V} = -5\,\text{W}$$

Notice we calculate a negative value for the power dissipated by the voltage source, implying the voltage source is adding power to the circuit.

(c) Suppose we choose a second labeling of the circuit as shown below. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5\,\text{V}$ and let $R_1 = 5\,\Omega$.



Answer: We'll solve the circuit the same way as last time.



The KCL equation for the one node in this circuit is:

$$-i_1 + i_2 = 0$$

The element equations for the two elements in this circuit are:

$$0 - u_1 = V_1 = -V_s$$

$$u_1 - 0 = V_{R_1} = i_2 R_1$$

Solving the above equations with $V_s = 5\,\text{V}$ and $R_1 = 5\,\Omega$:

$$u_1 = 5\,\text{V}$$
$$i_1 = 1\,\text{A}$$
$$i_2 = 1\,\text{A}$$

From above, we can solve for the power dissipated across the resistor:

$$P_{R_1} = i_2 V_{R_1} = 1\,\text{A} \cdot 5\,\text{V} = 5\,\text{W}$$

Next we can solve for the power dissipated across the voltage source:

$$P_{V_s} = i_1 V_1 = i_1(-V_s) = 1\,\text{A} \cdot -5\,\text{V} = -5\,\text{W}$$

Notice here that the circuit has the same power dissipated by all the elements. This is because with both labeling of currents, we followed the passive sign convention.

(d) Did the values of the element voltages and element currents change with the different labeling? Did the power for each circuit element change? Did the node voltages change? If a quantity didn't change with a difference in labeling, discuss what would have to change for quantity to change.

Answer: With a different labeling, element voltages and element currents will change. The quantities were $V_1 = 5\,\text{V}$ and $i_1 = -1\,\text{A}$ in (b) and in (c) $V_1 = -5\,\text{V}$ and $i_1 = 1\,\text{A}$. Flipping the direction of a labeled current or the polarity of a labeled voltage will lead to negation of the value.

The power dissipated by a circuit element will not change because we follow passive sign convention. Passive sign convention requires that if we flip the direction of an element current we also flip the polarity of the corresponding element voltage, so there is a double negation in the computation of power. The only way to get a different value of power would be to change the component values or the circuit diagram itself by removing or adding more circuit elements. A physical system will only have one behavior as governed by the laws of physics - how we compute our answer should not change how it behaves. Our labeled voltage polarities and current directions are more akin to measurement choices which can change what we see.

The node voltages too, did not change. The top node voltage in both labelings were $5\,\text{V}$ and the bottom node voltages were $0\,\text{V}$. What would have to change to alter these values is one of three things: either the location of ground, the circuit component values, or the circuit diagram itself.

2. Volt and ammeter

Consider the following circuit below. We have also included relevant NVA equations below it.

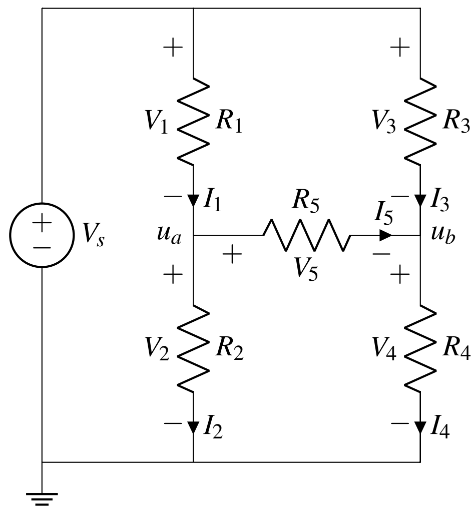


Figure 1: Circuit consisting of a voltage source V_s and five resistors R_1 to R_5.

KCL	Ohm's law in terms of node voltages	Substitute Ohm's into KCL
$I_1 = I_2 + I_5$ $I_5 + I_3 = I_4$	$I_1 = \frac{V_1}{R_1} = \frac{V_s - u_a}{R_1}$ $I_2 = \frac{V_2}{R_2} = \frac{u_a - 0}{R_2}$ $I_3 = \frac{V_3}{R_3} = \frac{V_s - u_b}{R_3}$ $I_4 = \frac{V_4}{R_4} = \frac{u_b - 0}{R_4}$ $I_5 = \frac{V_5}{R_5} = \frac{u_a - u_b}{R_5}$	$\frac{V_s - u_a}{R_1} = \frac{u_a - 0}{R_2} + \frac{u_a - u_b}{R_5}$ $\frac{u_a - u_b}{R_5} + \frac{V_s - u_b}{R_3} = \frac{u_b - 0}{R_4}$

(a) The circuit diagram shown in Figure 1 has been redrawn in Figure 2 by adding a voltmeter (letter V in a circle and plus and minus signs indicating direction) to measure voltage $V_{ab} = u_a - u_b$. Assume that the voltmeter is ideal. Are the values of V_{ab} before adding the voltmeter and after adding the voltmeter different? If so, which of the given equations change when doing NVA?

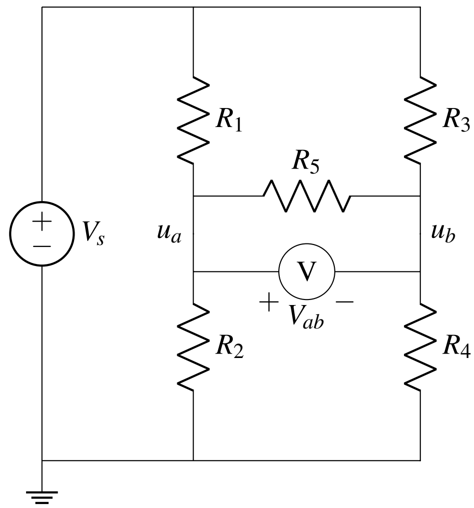
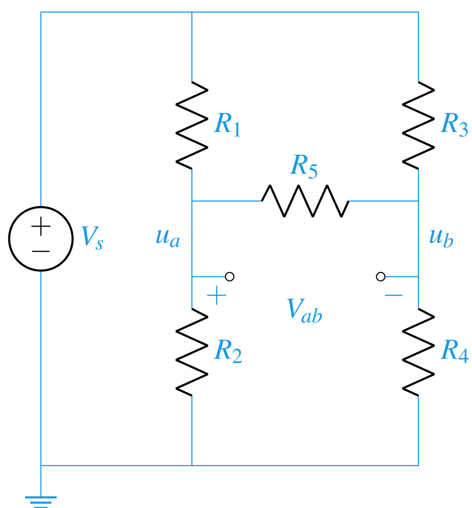


Figure 2: Circuit with voltmeter.

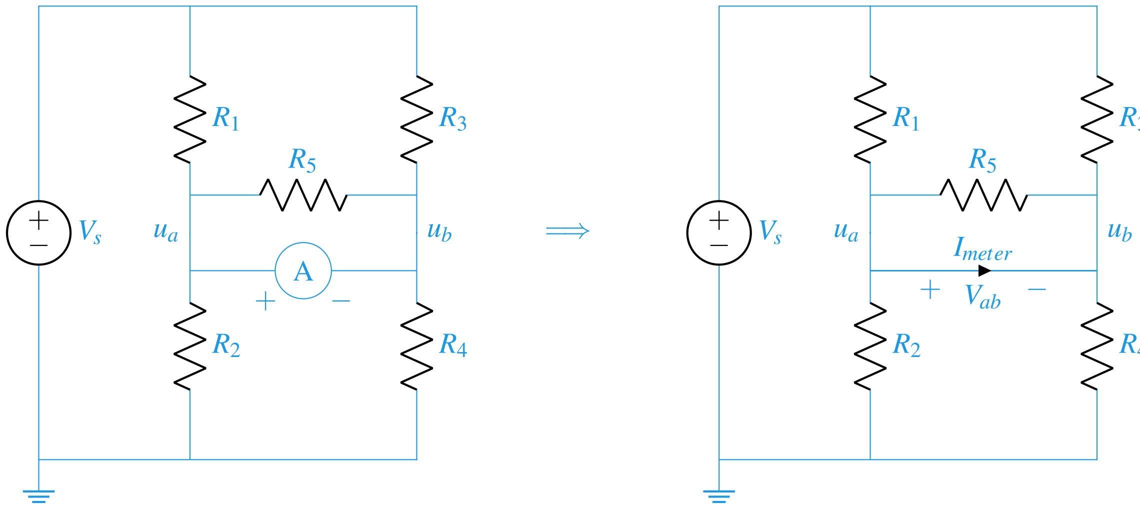
Answer: An ideal voltmeter behaves like an open circuit as depicted below:



Since no current can pass through the open circuit, the KCL equations will not have any extra current terms. The other equations do not change as well, so V_{ab} before the voltmeter was added and V_{ab} after the voltmeter was added are identical.

(b) Suppose you accidentally connect an ideal ammeter in part (a) to nodes u_a and u_b instead of an ideal voltmeter. Does the value of V_{ab} with the ammeter connected differ from the value of V_{ab} without the ammeter connected? If so, what equations change when doing NVA?

Answer: An ideal ammeter behaves like a wire or a short between two nodes as depicted below:



Placing it across the nodes u_a and u_b will unify them into one single node. So $u_a = u_b$. Thus V_{ab} changes and becomes $V_{ab} = u_a - u_b = 0V$.
Ohm's law for R_5 changes to $I_5 = \frac{0V}{R_5} = 0A$. The KCL equations change into:

$$I_1 = I_2 + I_{meter}$$
$$I_3 + I_{meter} = I_4$$

In fact, we can treat all the currents from R_1 and R_4 as entering and leaving the single node. We can reduce simply to the following one KCL equation for the node with node voltage $u_a = u_b = u$.

$$I_1 + I_3 = I_2 + I_4 \implies \frac{V_s - u}{R_1} + \frac{V_s - u}{R_3} = \frac{u - 0}{R_2} + \frac{u - 0}{R_4}$$

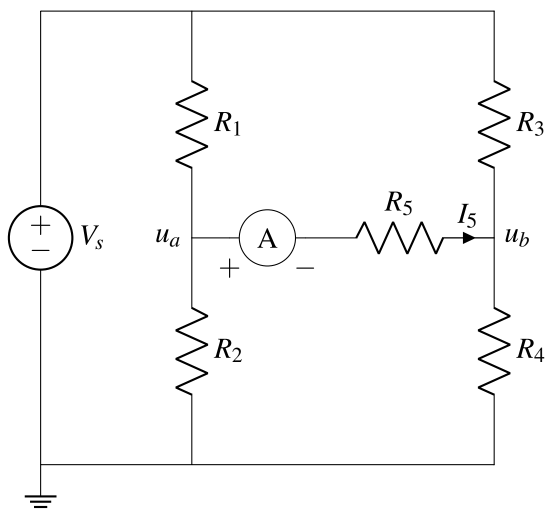
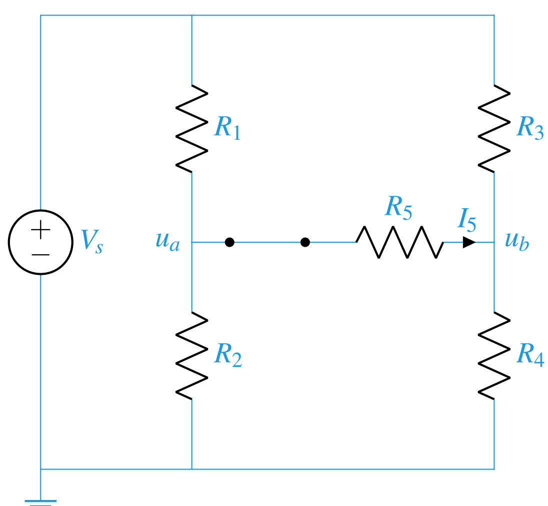


Figure 3: Circuit with ammeter.

(c) The circuit diagram shown in Figure 1 has been redrawn in Figure 3 by adding an ideal ammeter (letter A in a circle and plus and minus signs indicating direction) in series with resistor R_5. This will measure the current I_5 through R_5. Are the values of I_5 before adding the ammeter and after adding the ammeter different? If so, what equations change when doing NVA?

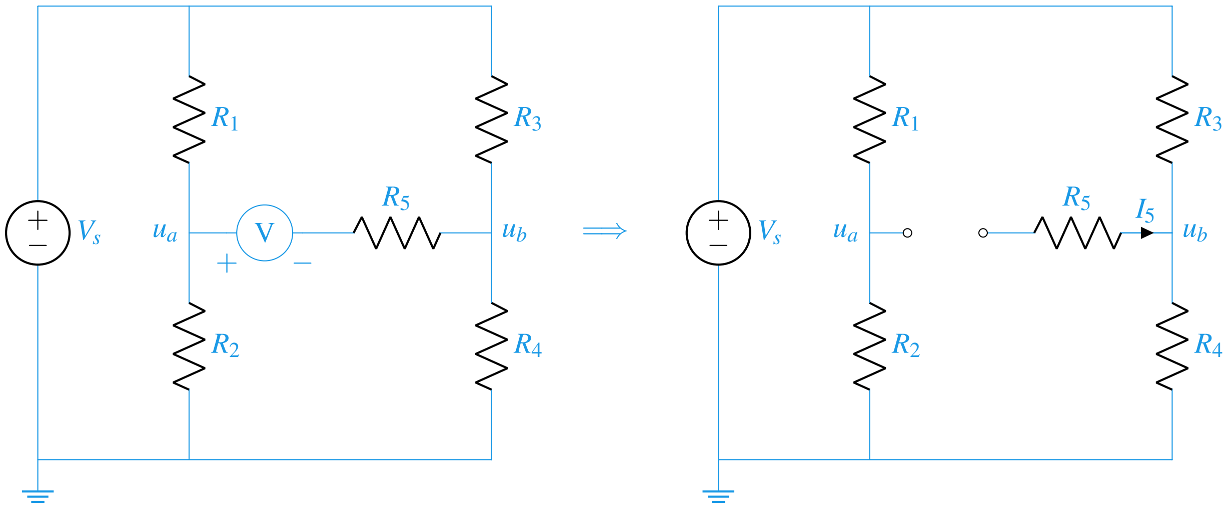
Answer:



Since the ideal ammeter behaves like a wire, the current I_5 when the ammeter has been placed in is no different from I_5 in the circuit given in Figure1. The NVA equations also do not change.

(d) Your friend accidentally connects a voltmeter in part (c) above, rather than an ammeter. Are the values of I_5 before adding the ammeter and after adding the ammeter different? If so, what equations change when doing NVA?

Answer: An ideal voltmeter behaves like an open circuit between the u_a node and the left of the R_5 resistor depicted below:



There will be no current flowing through R_5 as no current can flow through an open. The KCL equation on the new node of the left terminal the R_5 insures that $I_{open} = I_5 = 0A$. Ohm's law for R_5 changes.

$$I_5 = 0A = \frac{V_5}{R_5} \implies V_5 = 0V$$

The KCL equations change to:

$$I_1 = I_2 \implies \frac{V_s - u_a}{R_1} = \frac{u_a - 0}{R_2}$$
$$I_3 = I_4 \implies \frac{V_s - u_b}{R_3} = \frac{u_b - 0}{R_4}$$

3. Practice: Cell Phone Battery

As great as smartphones are, one of their drawbacks is that their batteries don't last a long time. For example, a Google Pixel phone, under typical usage conditions (internet, a few cat videos, etc.) uses 0.3W. We will model the battery as an ideal voltage source (which maintains a constant voltage across its terminals regardless of current) except that we assume that the voltage drops abruptly to zero when the battery is discharged (in reality the voltage drops gradually, but let's keep things simple).

Battery capacity is specified in mAh, which indicates how many mA of current the battery can supply for one hour before it needs to be recharged. The Pixel's battery has a battery capacity of 2770mAh at 3.8V. For example, this battery could provide 1000mA (or 3.8W) for 2.77 hours before the voltage abruptly drops from 3.8V to zero.

(a) How long will a Pixel's full battery last under typical usage conditions?

Answer:

300 mW of power at 3.8 V is about 79 mA of current. Our 2770 mA h battery can supply 79 mA for $\frac{2770\,\text{mA h}}{79\,\text{mA}} = 35\,\text{h}$, or about a day and a half.

(b) How many coulombs of charge does the battery contain? How many usable electrons worth of charge are contained in the battery when it is fully charged? (An electron has 1.602×10^{-19} C of charge.)

Answer:

One hour has 3600 seconds, so the battery's capacity can be written as $2770\,\text{mA h} \times 3600\,\text{s h}^{-1} = 9.972 \times 10^{6}\,\text{mA s} = 9972\text{As} = 9972\text{C}$.

An electron has a charge of approximately 1.602×10^{-19} C, so 9972 C is $\frac{9972\,\text{C}}{1.602 \times 10^{-19}\,\text{C}} \approx 6.225 \times 10^{22}$ electrons. That's a lot!

(c) Suppose the cell phone battery is completely discharged and you want to recharge it completely. How much energy (in J) is this? Recall that a J is equivalent to a W s.

Answer:

The battery capacity is 2770 mA h at 3.8 V, which means the battery has a total stored energy of $2770\,\text{mA h} \cdot 3.8\,\text{V} = 10.5\,\text{W h} = 10.5\text{Wh} \cdot 3600\text{s} = 37.9\text{kJ}$.

(d) Suppose PG&E charges \$0.12 per kW h. Every day, you completely discharge the battery (meaning more than typical usage) and you recharge it every night. How much will recharging cost you for the month of October (31 days)?

Answer:

2770 mA h at 3.8 V is $2770\,\text{mA h} \cdot 3.8\,\text{V} = 10.5\,\text{W h}$, or 0.01 kW h. At \$0.12 per kW h, that is $\$0.12 \cdot 0.01$ per day, or $\$0.12 \cdot 0.01 \cdot 31 = \$0.037$, or about 4 cents a month. Compare that to your cell phone data bill! Whew!

(e) The battery has internal circuitry that prevents it from getting overcharged (and possibly exploding!). We will model the battery and its internal circuitry as a resistor R_{bat}. We now wish to charge the battery by plugging into a wall plug. The wall plug can be modeled as a 5 V voltage source and 200 mΩ resistor, as pictured in Figure 4. What is the power dissipated across R_{bat} for $R_{\text{bat}} = 1\,\text{m}\Omega$, $1\,\Omega$, and $10\,\text{k}\Omega$? (i.e. how much power is being supplied to the phone battery as it is charging?). How long will the battery take to charge for each of those values of R_{bat}?

Answer:

The energy stored in the battery is 2770 mA h at 3.8 V, which is $2.77\,\text{A h} \cdot 3.8\,\text{V} = 10.5\,\text{W h}$. We can find the time to charge by dividing this energy by power in W to get time in hours.

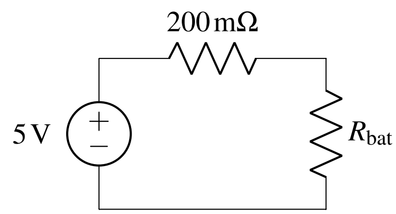


Figure 4: Model of wall plug, wire, and battery.

For $R_{\text{bat}} = 1\,\text{m}\Omega$:
The total resistance seen by the battery is $1\,\text{m}\Omega + 200\,\text{m}\Omega = 201\,\text{m}\Omega$ (because the wire and R_{bat} are in series), so by Ohm's law, the current is $\frac{5\,\text{V}}{0.201\,\Omega} = 24.88\,\text{A}$. The voltage drop across R_{bat} is (again by Ohm's law) $24.88\,\text{A} \cdot 0.001\,\Omega = 0.024\,88\,\text{V}$. Then power is $0.024\,88\,\text{V} \cdot 24.88\,\text{A} = 0.619\,\text{W}$, and the total time to charge the battery is $\frac{10.5\,\text{W}\,\text{h}}{0.619\,\text{W}} = 17\,\text{h}$.

For $R_{\text{bat}} = 1\,\Omega$:
The total resistance seen by the battery is $1\,\Omega + 0.2\,\Omega = 1.2\,\Omega$, the current through the battery is $\frac{5\,\text{V}}{1.2\,\Omega} = 4.167\,\text{A}$, and the voltage across the battery is by Ohm's law $4.167\,\text{A} \cdot 1\,\Omega = 4.167\,\text{V}$. Then the power is $4.167\,\text{A} \cdot 4.167\,\text{V} = 17.36\,\text{W}$, and the total time to charge the battery is $\frac{10.5\,\text{W}\,\text{h}}{17.36\,\text{W}} = 0.6\,\text{h}$, about $36\,\text{min}$.

For $R_{\text{bat}} = 10\,\text{k}\Omega$:
The total resistance seen by the battery is $10\,000\,\Omega + 0.2\,\Omega = 10\,000.2\,\Omega$, the current through the battery is $\frac{5\,\text{V}}{10\,000.2\,\Omega} \approx 0.5\,\text{mA}$, and the voltage across the battery is by Ohm's law $0.5\,\text{mA} \cdot 10\,\text{k}\Omega \approx 5\,\text{V}$ (up to 2 significant figures). Then the power is $5\,\text{V} \cdot 0.5\,\text{mA} = 2.5\,\text{mW}$, and the total time to charge the battery is $\frac{10.526\,\text{W}\,\text{h}}{0.0025\,\text{W}} = 4210\,\text{h}$.

