1. Op-Amp Rules and Negative Feedback Rules

Here is an equivalent circuit of an op-amp (where we are assuming that $V_{SS} = -V_{DD}$) for reference:

(a) What are the currents flowing into the positive and negative terminals of the op-amp (i.e., what are $I^+$ and $I^-$)? Based on this answer, what are some of the advantages of using an op-amp in your circuit designs?

The $u^+$ and $u^-$ terminals have no closed circuit connection between them, and therefore no current can flow into or out of them. This is very good because we can connect an op-amp to any other circuit, and the op-amp will not disturb that circuit in any way because it does not load the circuit (it is an open circuit).

(b) Suppose we add a resistor of value $R_L$ between $u_{out}$ and ground. What is the value of $v_{out}$? Does your answer depend on $R_L$? In other words, how does $R_L$ affect $A_{VC}$? What are the implications of this with respect to using op-amps in circuit design?

Notice that $u_{out}$ is connected directly to a controlled/dependent voltage source, and therefore $v_{out}$ will always have to be equal to $A_{VC}$ regardless of what $R_L$ is connected to the op-amp. This is very advantageous because it means that the output of the op-amp can be connected to any other circuit (except a voltage source), and we will always get the desired/expected voltage out of the op-amp.

For the rest of the problem, consider the following op-amp circuit in negative feedback:
(c) Assuming that this is an ideal op-amp, what is \( v_{\text{out}} \)?

Recall for an ideal op-amp in negative feedback, we know from the negative feedback rule that \( u^+ = u^- \). In this case, \( u^- = u_{\text{out}} = u^+ \).

(d) Draw the equivalent circuit for this op-amp and calculate \( v_{\text{out}} \) in terms of \( A, v_{\text{in}}, \) and \( R_L \) for the circuit in negative feedback. Does \( v_{\text{out}} \) depend on \( R_L \)? What is \( v_{\text{out}} \) in the limit as \( A \to \infty \)?

Notice that the op-amp can be modeled as a voltage-controlled voltage source. Thus, we have the following equation:

\[
\begin{align*}
    v_{\text{out}} &= A(v_{\text{in}} - v_{\text{out}}) \\
    v_{\text{out}} + Av_{\text{out}} &= Av_{\text{in}} \\
    v_{\text{out}} &= v_{\text{in}} \frac{A}{1+A}
\end{align*}
\]

Thus, as \( A \to \infty \), \( v_{\text{out}} \to v_{\text{in}} \). This is the same as what we get after applying the op-amp rule.

Notice that output voltage does not depend on \( R \). Thus, this circuit acts like a voltage source that provides the same voltage read at \( u^+ \) without drawing any current from the terminal at \( u^+ \). This is why the circuit is often referred to as a “unity gain buffer,” “voltage follower,” or just “buffer.”
2. Modular Circuit Buffer

Let’s try designing circuits that perform a set of mathematical operations using op-amps. While voltage dividers on their own cannot be combined without altering their behavior, op-amps can preserve their behavior when combined and thus are a perfect tool for modular circuit design. We would like to implement the block diagram shown below:

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In other words, create a circuit with two outputs $V_x$ and $V_y$, where $V_x = \frac{1}{2}V_{in}$ and $V_y = \frac{1}{3}V_x = \frac{1}{6}V_{in}$.
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(a) Draw two voltage dividers, one for each operation (the $1/2$ and $1/3$ scalings). What relationships hold for the resistor values for the $1/2$ divider, and for the resistor values for the $1/3$ divider?

**Answer:** Recall our voltage divider consists of $V_{in}$ connected to two resistors ($R_1, R_2$) in series with the output voltage between ground and the central node. This yields the formula

$$V_{out} = \left(\frac{R_2}{R_1 + R_2}\right)V_{in}.$$  

For the $1/2$ operation ($V_x$ output) we recognize

$$\frac{1}{2} = \left(\frac{R_2}{R_1 + R_2}\right) \quad \rightarrow \quad R_1 + R_2 = 2R_2 \quad \rightarrow \quad R_1 = R_2 \equiv R.$$  

For the $1/3$ operation ($V_y$ output) we recognize

$$\frac{1}{3} = \left(\frac{R_2}{R_1 + R_2}\right) \quad \rightarrow \quad R_1 + R_2 = 3R_2 \quad \rightarrow \quad \frac{R_1}{2} = R_2 \equiv R.$$  

(b) If you combine the voltage dividers, made in part (a), as shown by the block diagram (output of the $1/2$ voltage divider becomes the source for the $1/3$ voltage divider circuit), do they behave as we hope (meaning $V_y = \frac{1}{3}V_x = \frac{1}{6}V_{in}$)?
HINT: The following circuit and formula may be handy:

\[
V_{\text{out}} = \left(\frac{1}{2 + \frac{R_x}{R_y}}\right) V_{\text{in}}
\]

**Answer:** Combining the voltage divider circuits yield

\[
V_x = \left(\frac{R_{eq}}{R_x + R_{eq}}\right) V_{\text{in}} = \left(\frac{1}{2 + \frac{R_x}{3R_y}}\right) V_{\text{in}} \quad V_y = \frac{1}{3} V_x = \left(\frac{1}{6 + \frac{R_x}{R_y}}\right) V_{\text{in}}
\]

From this stage it is evident that combining our dividers changes their behavior (although they preserve behavior in the limit \(R_y \gg R_x\)).

The new values for \(V_x, V_y\) are dependent on values from both dividers, which means they can’t be treated independently! □

(c) Perhaps we could use an op-amp (in negative-feedback) to achieve our desired behavior. Modify the implementation you tried in part (b) using a negative feedback op-amp in order to achieve the desired \(V_x, V_y\) relations: \(V_x = (1/2)V_{\text{in}}\) and \(V_y = (1/3)V_x = (1/6)V_{\text{in}}\).

HINT: Place the op-amp in between the dividers such that the \(V_x\) node is an input into the op-amp, while the source of the 2nd divider is the output of the op-amp!

**Answer:** Use the op-amp as a voltage buffer.

This means we short the op-amp’s negative input to its output, since the positive input must now match its output (by the golden rules).
Since no current flow into the positive op-amp input, we’ve successfully isolated the dividers so they can be used in a modular fashion!

NOTE: The $V_x, V_y$ outputs from this configuration would change with the addition of a load on either terminal. As a follow-up, think about ways to make each output agnostic to the loads attached!

3. (Practice) An Inverting Amplifier

Calculate $v_{out}$ as a function of $V_s$ and $R_1$ and $R_2$.

Because the op-amp is in negative feedback, we know that $u^+ = u^- = 0\,\text{V}$. Therefore, $v_{out} = u^- - V_{R_2} = -I_{R_2}R_2$.

We also know that $I^+ = 0$, so $I_{R_1} = I_{R_2}$. Thus, $v_{out} = u^- - R_2 = -I_{R_1}R_2 = -I_{R_1}R_2 = -V_s \frac{R_2}{R_1}$. 