## Reference Circuits

<table>
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<tr>
<th>Voltage Divider</th>
<th>Inverting Amplifier</th>
<th>Noninverting Amplifier</th>
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<td><img src="image" alt="Voltage Divider" /></td>
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<tr>
<td>$V_{R2} = V_S \left( \frac{R_2}{R_1 + R_2} \right)$</td>
<td>$v_{out} = v_{in} \left( -\frac{R_f}{R_s} \right)$</td>
<td>$v_{out} = v_{in} \left( 1 + \frac{R_{top}}{R_{bottom}} \right)$</td>
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<td>$I_1 = I_S \left( \frac{R_2}{R_1 + R_2} \right)$</td>
<td>$v_{out} = v_{in} \left( -\frac{R_f}{R_s} \right) + V_{REF} \left( \frac{R_f}{R_s} + 1 \right)$</td>
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1. Testing for Negative Feedback

While it is tempting to say "if the feedback voltage is connected to the negative op-amp terminal, then we have negative feedback," this is not always true. Here is a two-step procedure for determining if a circuit is in negative feedback:

- **Step 1: Zero out all independent sources**, replacing voltage sources with wires and current sources with opens as we did in superposition. You do not need to zero out the voltage sources that serve as the power supplies to the op-amp, since they are not treated as signals and almost considered part of the op-amp.

- **Step 2: Wiggle the output and check the loop.** The goal is to see how the feedback loop responds to a change. Assume that the output increases slightly. Check the direction of change of the feedback signal and the error signal from the circuit. Any change in the error signal will cause a new change in the output. This change is the feedback loop’s response to the initial change.
  - If the error signal decreases, then the output must also decrease. This is the opposite direction we initially assumed, i.e. the loop is trying to correct for the change. So the circuit is in negative feedback.
  - If the error signal instead increased, then the output would also increase. This is the same direction we initially assume, i.e. the initial increase lead to further increase. We call this positive feedback.

(a) Show that the voltage buffer circuit is in negative feedback.

![Diagram of voltage buffer circuit]

**Solution/Answer:** First, zero out all independent sources. For this problem, we just need to tie the input to ground.

![Diagram with input tied to ground]

Next, wiggle the output and check the loop. Below, we label the initial change in the output in red and label subsequent changes in orange:

![Diagram with initial change labeled in red and subsequent changes in orange]
We suppose that the $v_{out}$ goes up. $v_{out}$ is connected to $v_-$ so $v_-$ also goes up. Our op-amp equation is $v_{out} = A(v_+ - v_-)$, so $v_-$ going up will cause $v_{out}$ to go down. This is the opposite of what initially happened, so we are in negative feedback.

(b) Show that the inverting amplifier circuit is in negative feedback.

\[ v_{out} = A(v_+ - v_-) \]

We suppose that the $v_{out}$ goes up. $v_{out}$ is connected to $U_-$ so $U_-$ also goes up. Our op-amp equation is $v_{out} = A(v_+ - v_-)$, so $U_-$ going up will cause $v_{out}$ to go down. This is the opposite of what initially happened, so we are in negative feedback.
2. Multiple Inputs To One Op-Amp

(a) For the circuit above, find an expression for \( v_o \). \textit{(Hint: Use superposition.)}

\textbf{Answer:}

Let's call the potential at the positive input of the op-amp \( u_+ \). Using superposition, we first turn off \( v_{s2} \) and find \( u_+ \). The circuit then looks like:

We recognize the above circuit as a voltage divider. Thus,

\[ u_{+,s1} = \frac{R_2}{R_1 + R_2} v_{s1} \]

By symmetry, we expect \( v_{s2} \) to have a similar circuit and expression. The circuit for \( v_{s2} \) looks like:
The expression for $u_+$ with $v_{s2}$ is then:

$$u_{+,v_{s2}} = \frac{R_1}{R_1 + R_2} v_{s2}$$

From superposition, we know the output must be the sum of these.

$$u_+ = \frac{R_2}{R_1 + R_2} v_{s1} + \frac{R_1}{R_1 + R_2} v_{s2}$$

With $u_+$ determined, we can find the output voltage directly from the formula for a non-inverting amplifier. We can also derive it using the process below.

From the negative feedback rule, $u_+ = u_-$. Using voltage dividers, we can express $u_-$ in terms of $v_o$:

$$u_- = \frac{R_4}{R_3 + R_4} v_o$$

$$v_o = \left(1 + \frac{R_3}{R_4}\right) u_- = \left(1 + \frac{R_3}{R_4}\right) u_+$$

Now, to find the final output, we can set $u_+$ to our earlier expression.

$$v_o = \left(1 + \frac{R_3}{R_4}\right) \left(\frac{R_2}{R_1 + R_2} v_{s1} + \frac{R_1}{R_1 + R_2} v_{s2}\right)$$

(b) How could you use this circuit to find the sum of different signals, i.e. $V_{s1} + V_{s2}$? What about taking the sum and adding a multiplying by 2, i.e. $2(V_{s1} + V_{s2})$?

**Answer:**

The circuit already finds the weighted sum of two inputs. By setting $R_1 = R_2$ and $R_3 = R_4$, we can take the exact sum of two inputs.

$$v_o = \left(1 + \frac{R_3}{R_4}\right) \left(\frac{R_2}{R_1 + R_2} v_{s1} + \frac{R_1}{R_1 + R_2} v_{s2}\right) = (1 + 1) \left(\frac{1}{2} v_{s1} + \frac{1}{2} v_{s2}\right) = v_{s1} + v_{s2}$$

Notice that the first half of this circuit ($R_1$ and $R_2$) form a voltage summer with coefficients less than one; the second half is just a non-inverting amplifier. Thus we can always use $R_1$ and $R_2$ to take an equally weighted sum of the inputs and then multiply greater than 1 using the non-inverting amplifier. If we set $R_1 = R_2$, we get $\left(\frac{1}{2} v_{s1} + \frac{1}{2} v_{s2}\right)$ into the op-amp. To get a total gain of 2, then the non-inverting op-amp needs a gain of 4, so we can pick $R_3 = 3R_4$. 
3. Modular Op-Amp Circuits

Let’s expand our toolbox of op-amp circuits that perform mathematical operations by designing blocks that implement the following operations

(a) Scale the input voltage so that: \( V_{\text{out}} = +5 V_{\text{in}} \)
(b) Scale and invert the input voltage so that: \( V_{\text{out}} = -2 V_{\text{in}} \)
(c) Sum two input voltages together so that: \( V_{\text{out}} = V_{\text{in}_1} + V_{\text{in}_2} \)
(d) (PRACTICE) Take the difference of two voltages so that: \( V_{\text{out}} = V_{\text{in}_1} - V_{\text{in}_2} \). (Hint: how do you invert the sign of a voltage?)

Use the reference above for help!
Would connecting any of these blocks together modify their intended functionality?

**Answer:**

(a) Since our scaling of +5 is positive, we employ a non-inverting amplifier without a \( V_{\text{REF}} \) supply:

![Non-inverting amplifier diagram]

The input to the circuit is an op-amp terminal, so there are no loading effects and it can be connected to any circuit without affecting functionality. The output is directly connected to the output of the op-amp, so it cannot be affected by loading effects and can be connected to any circuit without affecting functionality.

(b) To scale the input by \(-2\) we must use the inverting amplifier configuration:

![Inverting amplifier diagram]

The output is directly connected to the output of the op-amp, so it cannot be affected by loading effects and can be connected to any circuit without affecting functionality. The input to the circuit is now a resistor. This can cause loading effects, so we need to be careful connecting it to other circuits. For example, if the preceding circuit is a voltage divider, we need to include a unity-gain buffer to prevent loading.
(c) This one is tricky!

The voltage summing circuit (in the reference) can get us close, but the resulting voltage still must be scaled by $+2$ using a non-inverting amplifier. Further, we need an op-amp in the circuit anyway to serve as a buffer! The best mindset for this problem is to think:

*Can we use a voltage divider involving the two input voltages as an input for a non-inverting amplifier?*

Conceptually we can imagine a $1/2$ voltage divider where the usual ground node is replaced by a source, say $V_{in_2}$. Then our divider equation becomes $V_{out} = \frac{1}{2}(V_{in_1} + V_{in_2})$. Next we can feed that input into a non-inverting amplifier circuit:

\[
\begin{align*}
V_{in_1} & \quad R_1 \\
V_{in_2} & \quad R_1 \\
V_{out} & \quad V_{in_1} \\
& \quad R_2 \\
& \quad R_2
\end{align*}
\]

We check if our circuit modules can be combined together by examining how the circuit output voltages change with an attached load resistor. Since every circuit has at least 1 fixed input (a node that is isolated from $V_{out}$ entirely), and the golden rules ($u_+ = u_-$) apply for negative-feedback circuits, the circuits’ functionalities are unimpeded by the load.

There are other solutions to this problem. For example, what if instead, we used an inverting summer amplifier to sum the inputs:

\[
\begin{align*}
V_{in_1} & \quad R_1 \\
V_{in_2} & \quad R_1 \\
V_{out} & \quad V_{out} \\
& \quad R_2 \\
& \quad R_2
\end{align*}
\]

We have $V_{out} = -\frac{R_2}{R_1}V_{in_1} - \frac{R_2}{R_1}V_{in_2} = -\frac{R_3}{R_1}(V_{in_1} + V_{in_2})$. This is very close to what we want; we just have to invert it one more time. We can do this simply with another inverting amplifier. Then we just have to pick the right values for the resistors to get coefficients $= 1$. One easy selection for this is to make all the resistors equal:
We have \( V_{\text{out}} = -\frac{R}{R}(-\frac{R}{R}(V_{\text{in}1} + V_{\text{in}2})) = V_{\text{in}1} + V_{\text{in}2} \). One benefit over the first circuit over this second version is that it uses only 1 op-amp instead of 2. In practical applications, fewer components used means lower power and lower cost to produce.

In both circuit solutions, the final output is connected directly to the output of the op-amp, and inputs are connected to resistors. So like the previous part, the output will not be affected by loading and be connected anything. The input can load whatever precedes it, and so must be used with a little care, (e.g. the previous circuit may need to need to be buffered).

As an additional point, what if we wanted to build \( V_{\text{out}} = \frac{1}{2}(V_{\text{in}1} + V_{\text{in}2}) \) instead? We could just use 2 resistor and form a voltage summer circuit, no op-amp required. What if we wanted to use this modularly though, i.e. connect it to a subsequence circuit without consequence? Since voltage summer is just resistors, there is nothing to prevent loading. A quick fix is to put a voltage buffer at the output.

(d) We already solved a circuit that adds values together. One way to build a subtraction circuit is simply to invert the sign of one of those inputs using an inverting amplifier:

We could analyze this mechanically, but let’s try to solve intuitively. Let’s try with superposition. If we turn off \( V_{\text{in}2} \), then the output of the left side op-amp is 0. Then the two \( R_1 \) resistors are a voltage divider, the same as we analyzed in the previous part. If we turn off \( V_{\text{in}1} \), we again have a voltage divider. The output of the left side op-amp, facing the \( R_1 \) resistor, is a voltage source, so if we solve for that output, we can replace it with a voltage source. Once we replace it with a voltage source, we can analyze this circuit the same as the previous part. So using these op-amps circuits can give us some modularity in solving the circuits!

We have \( V_{\text{out}} = \frac{R_2 + R_3}{R_2} \left( \frac{R_1}{R_1 + R_1} V_{\text{in}1} + \frac{R_1}{R_1 + R_1} - \frac{R_1}{R_1} V_{\text{in}2} \right) = V_{\text{in}1} - V_{\text{in}2} \).
4. Practice: Dividers for Days

(a) Solve the following circuit for $v_x$.

Answer: Using superposition and employing the voltage divider equation twice, we get:

$$v_x = \frac{1}{2}V_1 + \frac{1}{2}V_2$$

(b) You have access to two voltage sources, $V_1$ and $V_2$. You can use two resistors (as long as $0 \leq R < \infty$). How would you design a circuit that produces a voltage $v_x = \frac{1}{3}V_1 + \frac{2}{3}V_2$?

Answer: Using superposition, we can find the output voltage for any two resistors $R_1$ and $R_2$ as:

$$v_x = \frac{R_2}{R_1 + R_2}V_1 + \frac{R_1}{R_1 + R_2}V_2$$

Thus, to create the $\frac{1}{3} - \frac{2}{3}$ ratio, we can use any nonzero resistances with value $R$ such that:

$$R_1 = 2R \quad R_2 = R$$

(c) You have two current sources $I_1$ and $I_2$. You also have a load resistor $R_L = 6k\Omega$. Similar to the first part, you can use whatever resistors you want (as long as they are finite integer multiples of $1k\Omega$). How would you design a circuit such that the current running through $R_L$ is $I_L = \frac{2}{5}(I_1 + I_2)$?

Answer: Use superposition, so think of the two currents as one summed current. Then, use KCL to determine how to divide the currents. Remember, the current divider formula is similar to that of the voltage divider, with the numerators flipped. This means that in the current divider, when calculating the current through one resistor we place the other resistor in the numerator, i.e.:
Using the above equations, we get that one possible resistor combination that creates $I_L = \frac{2}{3}(I_1 + I_2)$ is:

$R_L = 6 \, \Omega, R_1 = 4 \, \Omega$