### Reference: Op-Amp Example Circuits

<table>
<thead>
<tr>
<th>Voltage Divider</th>
<th>Inverting Amplifier</th>
<th>Noninverting Amplifier</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
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<tr>
<td>$V_{R2} = V_S \left( \frac{R_2}{R_1 + R_2} \right)$</td>
<td>$v_{out} = v_{in} \left( -\frac{R_f}{R_S} \right)$</td>
<td>$v_{out} = v_{in} \left( 1 + \frac{R_{top}}{R_{bottom}} \right)$</td>
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<table>
<thead>
<tr>
<th>Current Divider</th>
<th>Inverting Amplifier with Reference</th>
<th>Noninverting Amplifier with Reference</th>
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<td><img src="image4" alt="Diagram" /></td>
<td><img src="image5" alt="Diagram" /></td>
<td><img src="image6" alt="Diagram" /></td>
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<tr>
<td>$I_1 = I_S \left( \frac{R_2}{R_1 + R_2} \right)$</td>
<td>$v_{out} = v_{in} \left( -\frac{R_f}{R_S} \right) + V_{REF} \left( \frac{R_f}{R_S} + 1 \right)$</td>
<td>$v_{out} = v_{in} \left( 1 + \frac{R_{top}}{R_{bottom}} \right) - V_{REF} \left( \frac{R_{top}}{R_{bottom}} \right)$</td>
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<tr>
<th>Voltage Summer</th>
<th>Unity Gain Buffer</th>
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<tr>
<td><img src="image7" alt="Diagram" /></td>
<td><img src="image8" alt="Diagram" /></td>
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<tr>
<td>$V_{out} = V_1 \left( \frac{R_2}{R_1 + R_2} \right) + V_2 \left( \frac{R_1}{R_1 + R_2} \right)$</td>
<td>$v_{out} = v_{in}$</td>
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1. Modular Circuit Buffer

Let’s try designing circuits that perform a set of mathematical operations using op-amps. While voltage dividers on their own cannot be combined without altering their behavior, op-amps can preserve their behavior when combined and thus are a perfect tool for modular circuit design. We would like to implement the block diagram shown below:

![Block Diagram](image)

In other words, create a circuit with two outputs $V_x$ and $V_y$, where $V_x = \frac{1}{2}V_{in}$ and $V_y = \frac{1}{3}V_x = \frac{1}{6}V_{in}$.

(a) Draw two voltage dividers, one for each operation (the $1/2$ and $1/3$ scalings). What relationships hold for the resistor values for the $1/2$ divider, and for the resistor values for the $1/3$ divider?

**Answer:** Recall our voltage divider consists of $V_{in}$ connected to two resistors ($R_1, R_2$) in series with the output voltage between ground and the central node. This yields the formula

$$V_{out} = \left( \frac{R_2}{R_1 + R_2} \right) V_{in}.$$  

For the $1/2$ operation ($V_x$ output) we recognize

$$\frac{1}{2} = \left( \frac{R_2}{R_1 + R_2} \right) \quad \rightarrow \quad R_1 + R_2 = 2R_2 \quad \rightarrow \quad R_1 = R_2 \equiv R_x.$$  

For the $1/3$ operation ($V_y$ output) we recognize

$$\frac{1}{3} = \left( \frac{R_2}{R_1 + R_2} \right) \quad \rightarrow \quad R_1 + R_2 = 3R_2 \quad \rightarrow \quad \frac{R_1}{2} = R_2 \equiv R_y.$$  

(b) If you combine the voltage dividers, made in part (a), as shown by the block diagram (output of the $1/2$ voltage divider becomes the source for the $1/3$ voltage divider circuit), do they behave as we hope (meaning $6V_{in} = 3V_x = V_y$)?
HINT: The following circuit and formula may be handy:

\[
V_{\text{out}} = \left( \frac{1}{2 + \frac{R_x}{2R_y}} \right) V_{\text{in}}
\]

**Answer:** Combining the voltage divider circuits yield

\[
V_{\text{out}} = \left( \frac{1}{2 + \frac{R_x}{3R_y}} \right) V_{\text{in}}
\]

To quickly access this combined system, we may identify \( V_z \) as the result of a new equivalent voltage divider (recognizing the \( R_y \) resistors in series and that series is in parallel with \( R_x \)). The load resistor becomes \( R_{eq} = \frac{3R_x R_y}{R_x + 3R_y} \). This yields

\[
V_z = \left( \frac{R_{eq}}{R_x + R_{eq}} \right) V_{\text{in}} = \left( \frac{1}{2 + \frac{R_x}{3R_y}} \right) V_{\text{in}}
\]

\[
V_y = \frac{1}{3} V_z = \left( \frac{1}{6 + \frac{R_x}{R_y}} \right) V_{\text{in}}
\]

From this stage it is evident that combining our dividers changes their behavior (although they preserve behavior in the limit \( R_y \gg R_x \)). The new values for \( V_z, V_y \) are dependent on values from both dividers, which means they can’t be treated independently! □.

(c) Perhaps we could use an op-amp (in negative-feedback) to achieve our desired behavior. Modify the implementation you tried in part (b) using a negative feedback op-amp in order to achieve the desired \( V_x, V_y \) relations \( V_x = (1/2)V_{\text{in}} \) and \( V_y = (1/3)V_x = (1/6)V_{\text{in}} \).

HINT: Place the op-amp in between the dividers such that the \( V_z \) node is an input into the op-amp, while the source of the 2nd divider is the output of the op-amp!

**Answer:** Use the op-amp as a voltage buffer. This means we short the op-amp’s negative input to its output, since the positive input must now match its output (by the golden rules).
Since no current flow into the positive op-amp input, we’ve successfully isolated the dividers so they can be used in a modular fashion!

NOTE: The $V_x, V_y$ outputs from this configuration would change with the addition of a load on either terminal. As a follow-up, think about ways to make each output agnostic to the loads attached!

2. Modular Op-Amp Circuits

Let’s expand our toolbox of op-amp circuits that perform mathematical operations by designing blocks that implement the following operations

(a) Scale the input voltage so that: $V_{out} = +5 \times V_{in}$
(b) Scale and invert the input voltage so that: $V_{out} = -2 \times V_{in}$
(c) Sum two input voltages together so that: $V_{out} = V_{in1} + V_{in2}$

Use the reference above for help!
Would connecting any of these blocks together modify their intended functionality?

**Answer:**

(a) Since our scaling of $+5$ is positive, we employ a non-inverting amplifier without a $V_{REF}$ supply:

(b) To scale the input by $-2$ we must use the inverting amplifier configuration:

(c) This one is tricky!
The voltage summing circuit (in the reference) can get us close, but the resulting voltage still must be scaled by $+2$ using a non-inverting amplifier. Further, we need an op-amp in the circuit anyway to serve as a buffer! The best mindset for this problem is to think: *can we use a voltage divider involving the two input voltages as an input for a non-inverting amplifier?*
Conceptually we can imagine a $1/2$ voltage divider where the usual ground node is replaced by a source, say $V_{in2}$. Then our divider equation becomes $V_{out} = \frac{1}{2}(V_{in1} + V_{in2})$. Next we can feed that input into a non-inverting amplifier circuit:

We check if our circuit modules can be combined together by examining how the circuit output voltages change with an attached load resistor. Since every circuit has at least 1 fixed input (a node that is isolated from $V_{out}$ entirely), and the golden rules ($u_+ = u_-$) apply for negative-feedback circuits, the circuits’ functionalities are unimpeded by the load.