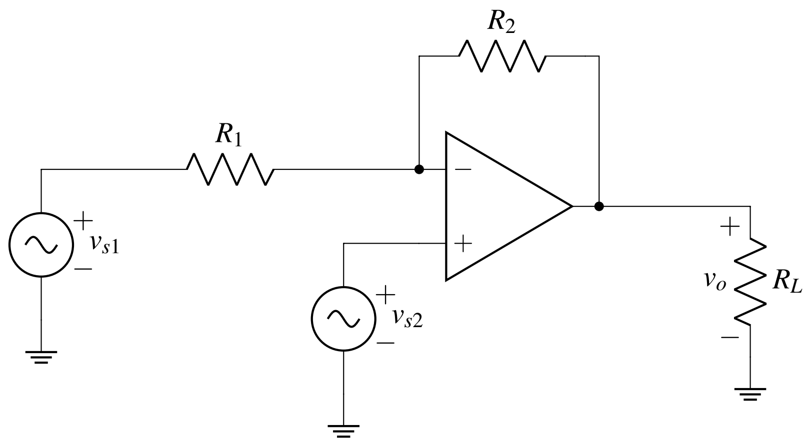
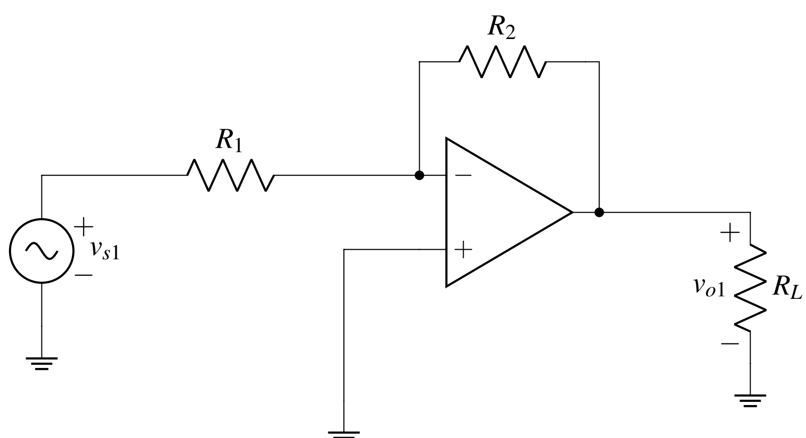
EECS 16A Designing Information Devices and Systems I
Fall 2020 Discussion 12A

1. Amplifier with Multiple Inputs

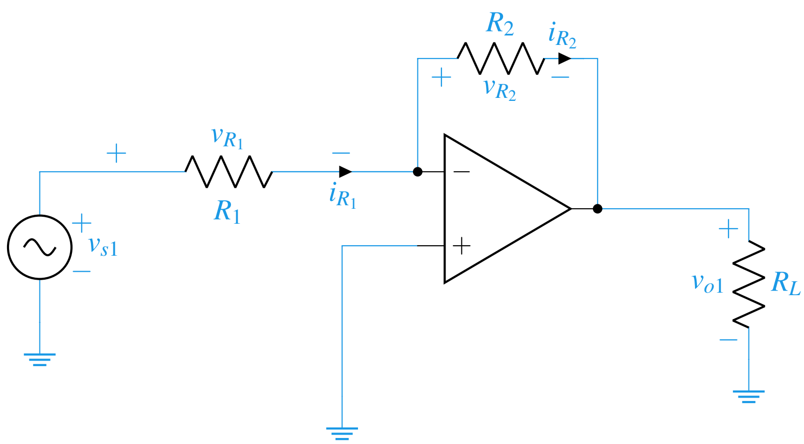
In this problem we will use superposition and the Golden Rules to find the output of the following op amp circuit with multiple inputs:



(a) First, let's turn off v_{s2}. Use the **Golden Rules** to find v_{o1} for the circuit below.



Answer:



Applying the Golden Rules, we know that the positive and negative terminals must be at the same voltage. Thus, the voltage at the negative terminal of the op-amp is 0. In addition, no current flows into the op-amp from the negative terminal due to its infinite input resistance (the negative terminal is connected to an "open" circuit).

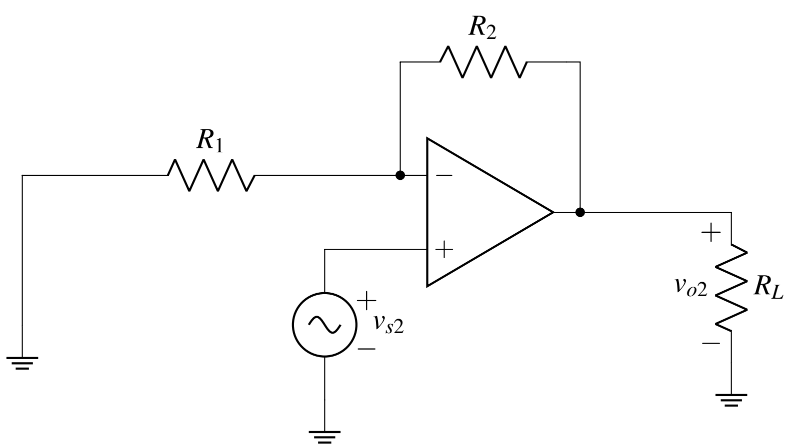
By KCL at the negative terminal of the op-amp, this means that the current going through R_1 and R_2 is $\frac{v_{s1}}{R_1}$. Taking the positive terminal of R_2 to be on the left (following PSC), the voltage drop across R_2 is $-v_{o1}$. By Ohm's law, we conclude:

$$\frac{-}{v_{o1}}R_2 = i_{R_2} = i_{R_1} = \frac{v_{s1}}{R_1}$$

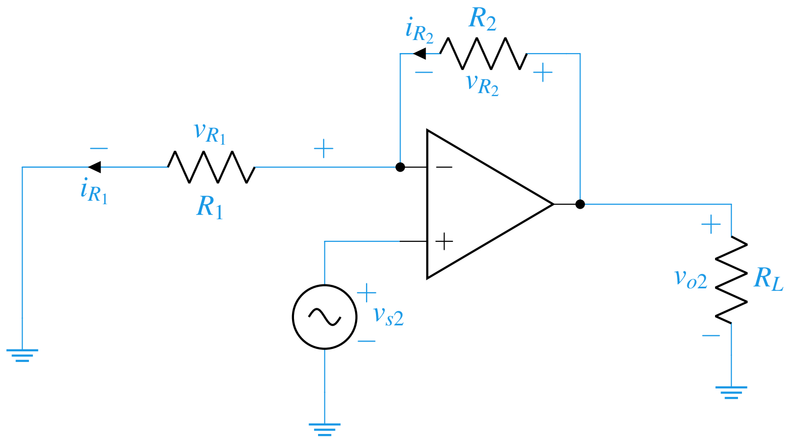
Rearranging we get:

$$v_{o1} = -v_{s1} \cdot \frac{R_2}{R_1}$$

(b) Now let's turn off v_{o1}. Use the **Golden Rules** to find v_{o2} for the circuit below.



Answer:



Applying the Golden Rules, we know that the positive and negative terminals must be at the same voltage. Thus, the voltage at the negative terminal of the op-amp is $V^- = v_{s2}$. In addition, since no current can enter into the negative terminal of the op-amp, R_1 and R_2 are in series. This means that the voltage at the negative terminal of the op-amp can be expressed in terms of v_{o2} using the voltage divider formula:

$$v^- = v_{o2}\left(\frac{R_1}{R_1+R_2}\right)$$

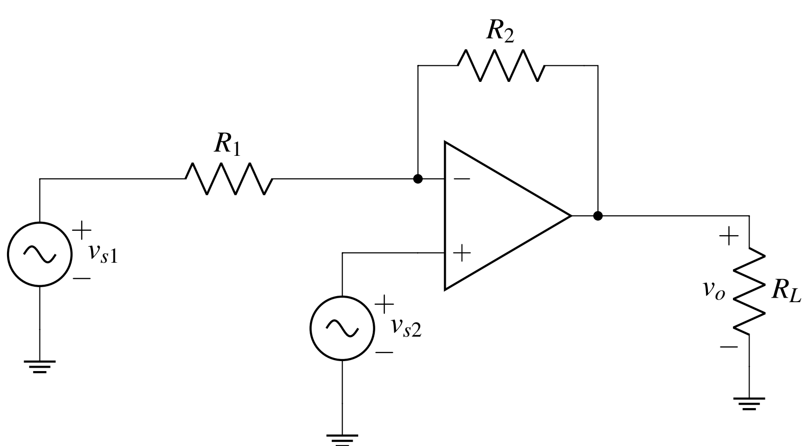
We also know that $v^- = v_{s2}$ and conclude:

$$v_{s2} = v_{o2}\left(\frac{R_1}{R_1+R_2}\right)$$

After rearranging, we have:

$$v_{o2} = v_{s2}\left(\frac{R_2}{R_1}+1\right)$$

(c) Use **superposition** to find the output voltage v_o for the circuit shown below.



Answer:

Using superposition we can now simply add the results from the two previous part to get the final answer:

$$v_o = v_{o1} + v_{o2} = -v_{s1}\cdot\frac{R_2}{R_1} + v_{s2}\left(\frac{R_2}{R_1}+1\right)$$

Reference: Inner products

Let $\vec{x}$, $\vec{y}$, and $\vec{z}$ be vectors in real vector space $\mathbb{V}$. A mapping $\langle \cdot, \cdot \rangle$ is said to be an inner product on $\mathbb{V}$ if it satisfies the following three properties:

(a) Symmetry: $\langle \vec{x}, \vec{y} \rangle = \langle \vec{y}, \vec{x} \rangle$

(b) Linearity: $\langle \vec{x}, \vec{y} + \vec{z} \rangle = \langle \vec{x}, \vec{y} \rangle + \langle \vec{x}, \vec{z} \rangle$ and $\langle c\vec{x}, \vec{y} \rangle = c\langle \vec{x}, \vec{y} \rangle$

(c) Non-negativeness: $\langle \vec{x}, \vec{x} \rangle \geq 0$, with equality if and only if $\vec{x} = \vec{0}$.

We define the norm of $\vec{x} = [x_1, x_2, \ldots, x_n]^T$ as $\|\vec{x}\| = \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2}$.

2. Mechanical Inner Products

For the following pairs of vectors, find the Euclidean inner product $\langle \vec{x}, \vec{y} \rangle = \vec{x}^T \vec{y}$.

(a)

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Answer: Recall that the inner product of two vectors $\vec{x}$ and $\vec{y}$ is $\vec{x}^T \vec{y}$, thus:

$$\begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = 1 + 3 = 4$$

(b)

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Answer: When working with real numbers, the inner product is commutative. Thus, using our work from the previous part, the inner product of these two vectors is 4.

(c)

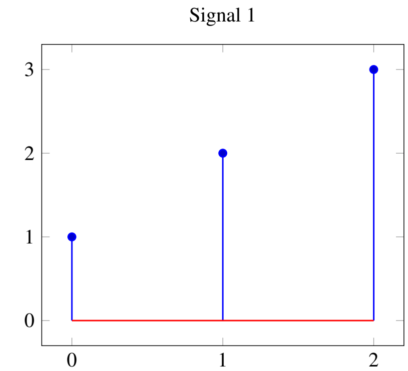
$$\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$$

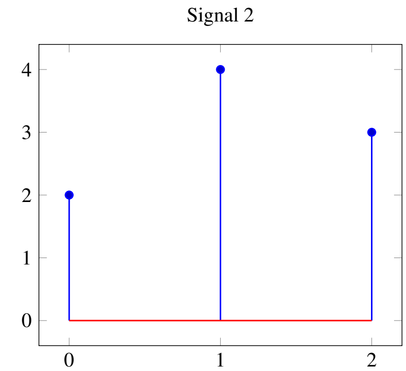
Answer:

$$\begin{bmatrix} 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix} = -3 + 3 = 0$$

3. Correlation

We are given the following two signals, $s_1[n]$ and $s_2[n]$ respectively.





Find the cross correlations, $\text{corr}_{s_1}(s_2)$ and $\text{corr}_{s_2}(s_1)$ for signals $s_[n]$ and $s_2[n]$. Recall

$$\text{corr}_x(y)[k] = \sum_{i=-\infty}^{\infty} x[i]y[i-k].$$

$\text{corr}_{\vec{s}_1}(\vec{s}_2)[k]$

$\vec{s}_1$	0	0	1	2	3	0	0
$\vec{s}_2[n+2]$							
$\langle \vec{s}_1, \vec{s}_2[n+2] \rangle$	+	+	+	+	+	+	=

$\vec{s}_1$	0	0	1	2	3	0	0
$\vec{s}_2[n+1]$							
$\langle \vec{s}_1, \vec{s}_2[n+1] \rangle$	+	+	+	+	+	+	=

$\vec{s}_1$	0	0	1	2	3	0	0
$\vec{s}_2[n]$							
$\langle \vec{s}_1, \vec{s}_2[n] \rangle$	+	+	+	+	+	+	=

$\vec{s}_1$	0	0	1	2	3	0	0
$\vec{s}_2[n-1]$							
$\langle \vec{s}_1, \vec{s}_2[n-1] \rangle$	+	+	+	+	+	+	=

$\vec{s}_1$	0	0	1	2	3	0	0
$\vec{s}_2[n-2]$							
$\langle \vec{s}_1, \vec{s}_2[n-2] \rangle$	+	+	+	+	+	+	=

$\text{corr}_{\vec{s}_2}(\vec{s}_1)[k]$

$\vec{s}_2$	0	0	2	4	3	0	0
$\vec{s}_1[n+2]$							
$\langle \vec{s}_2, \vec{s}_1[n+2] \rangle$	+	+	+	+	+	+	=

$\vec{s}_2$	0		0		2		4		3		0		0	
$\vec{s}_1[n+1]$														
$\langle \vec{s}_2, \vec{s}_1[n+1] \rangle$		+		+		+		+		+		+		=

$\vec{s}_2$	0		0		2		4		3		0		0	
$\vec{s}_1[n]$														
$\langle \vec{s}_2, \vec{s}_1[n] \rangle$		+		+		+		+		+		+		=

$\vec{s}_2$	0		0		2		4		3		0		0	
$\vec{s}_1[n-1]$														
$\langle \vec{s}_2, \vec{s}_1[n-1] \rangle$		+		+		+		+		+		+		=

$\vec{s}_2$	0		0		2		4		3		0		0	
$\vec{s}_1[n-2]$														
$\langle \vec{s}_2, \vec{s}_1[n-2] \rangle$		+		+		+		+		+		+		=

Answer: The linear cross-correlation is calculated by shifting the second signal both forward and backward until there is no overlap between the signals. When there is no overlap, the cross-correlation goes to zero. Both of these cross-correlations should have only zeros outside the range: $-2 \leq n \leq 2$.

$\text{corr}_{\vec{s}_1}(\vec{s}_2)[k]$

$\vec{s}_1$	0		0		1		2		3		0		0	
$\vec{s}_2[n+2]$	2		4		3		0		0		0		0	
$\langle \vec{s}_1, \vec{s}_2[n+2] \rangle$	0	+	0	+	3	+	0	+	0	+	0	+	0	= 3

$\vec{s}_1$	0		0		1		2		3		0		0	
$\vec{s}_2[n+1]$	0		2		4		3		0		0		0	
$\langle \vec{s}_1, \vec{s}_2[n+1] \rangle$	0	+	0	+	4	+	6	+	0	+	0	+	0	= 10

$\vec{s}_1$	0		0		1		2		3		0		0	
$\vec{s}_2[n]$	0		0		2		4		3		0		0	
$\langle \vec{s}_1, \vec{s}_2[n] \rangle$	0	+	0	+	2	+	8	+	9	+	0	+	0	= 19

$\vec{s}_1$	0		0		1		2		3		0		0	
$\vec{s}_2[n-1]$	0		0		0		2		4		3		0	
$\langle \vec{s}_1, \vec{s}_2[n-1] \rangle$	0	+	0	+	0	+	4	+	12	+	0	+	0	= 16

$\vec{s}_1$	0		0		1		2		3		0		0	
$\vec{s}_2[n-2]$	0		0		0		0		2		4		3	
$\langle \vec{s}_1, \vec{s}_2[n-2] \rangle$	0	+	0	+	0	+	0	+	6	+	0	+	0	= 6

$\text{corr}_{\vec{s}_2}(\vec{s}_1)[k]$

$\vec{s}_2[n]$	0		0		2		4		3		0		0	
$\vec{s}_1[n+2]$	1		2		3		0		0		0		0	
$\langle \vec{s}_2, \vec{s}_1[n+2] \rangle$	0	+	0	+	6	+	0	+	0	+	0	+	0	$=6$

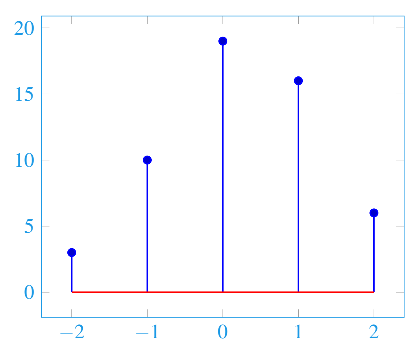
$\vec{s}_2[n]$	0		0		2		4		3		0		0	
$\vec{s}_1[n+1]$	0		1		2		3		0		0		0	
$\langle \vec{s}_2, \vec{s}_1[n+1] \rangle$	0	+	0	+	4	+	12	+	0 +		0	+	0	$=16$

$\vec{s}_2[n]$	0		0		2		4		3		0		0	
$\vec{s}_1[n]$	0		0		1		2		3		0		0	
$\langle \vec{s}_2, \vec{s}_1[n] \rangle$	0	+	0	+	2	+	8	+	9	+	0	+	0	$=19$

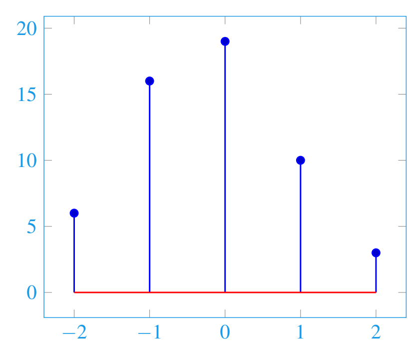
$\vec{s}_2[n]$	0		0		2		4		3		0		0	
$\vec{s}_2[n-1]$	0		0		0		1		2		3		0	
$\langle \vec{s}_2, \vec{s}_1[n-1] \rangle$	0	+	0	+	0	+	4	+	6	+	0	+	0	$=10$

$\vec{s}_2[n]$	0		0		2		4		3		0		0	
$\vec{s}_2[n-2]$	0		0		0		0		1		2		3	
$\langle \vec{s}_2, \vec{s}_1[n-2] \rangle$	0	+	0	+	0	+	0	+	3	+	0	+	0	$=3$

Cross-correlation of $\vec{s}_1$ and $\vec{s}_2$



Cross-correlation of $\vec{s}_2$ and $\vec{s}_1$



Notice that $\text{corr}_{\vec{s}_1}(\vec{s}_2)[k] = \text{corr}_{\vec{s}_2}(\vec{s}_1)[-k]$, i.e. changing the order of the signals reverses the cross-correlation sequence.