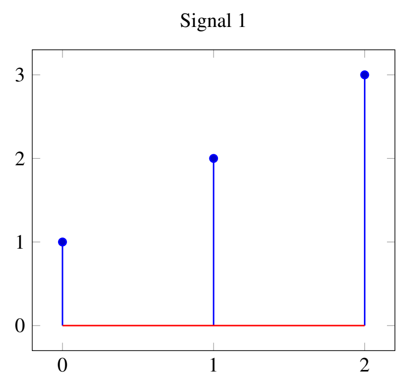
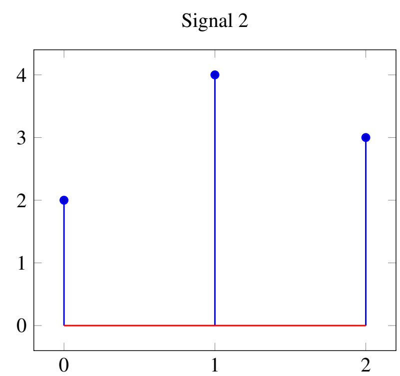
EECS 16A Designing Information Devices and Systems I
Fall 2020 Discussion 12B

1. Correlation

We are given the following two signals, $s_1[n]$ and $s_2[n]$ respectively.





Find the cross correlations, $\text{corr}_{s_1}(s_2)$ and $\text{corr}_{s_2}(s_1)$ for signals $s_{[}n]$ and $s_2[n]$. Recall

$$\text{corr}_x(y)[k] = \sum_{i=-\infty}^{\infty} x[i]y[i-k].$$

$$\text{corr}_{\vec{s}_1}(\vec{s}_2)[k]$$

$\vec{s}_1$	0	0	1	2	3	0	0	
$\vec{s}_2[n+2]$								
$\langle \vec{s}_1, \vec{s}_2[n+2] \rangle$	+	+	+	+	+	+	=	

$\vec{s}_1$	0	0	1	2	3	0	0	
$\vec{s}_2[n+1]$								
$\langle \vec{s}_1, \vec{s}_2[n+1] \rangle$	+	+	+	+	+	+	=	

$\vec{s}_1$	0	0	1	2	3	0	0	
$\vec{s}_2[n]$								
$\langle \vec{s}_1, \vec{s}_2[n] \rangle$	+	+	+	+	+	+	=	

$\vec{s}_1$	0	0	1	2	3	0	0	
$\vec{s}_2[n-1]$								
$\langle \vec{s}_1, \vec{s}_2[n-1] \rangle$	+	+	+	+	+	+	=	

$\vec{s}_1$	0		0		1		2		3		0		0	
$\vec{s}_2[n-2]$														
$\langle \vec{s}_1, \vec{s}_2[n-2] \rangle$		+		+		+		+		+		+		=

$\text{corr}_{\vec{s}_2}(\vec{s}_1)[k]$

$\vec{s}_2$	0		0		2		4		3		0		0	
$\vec{s}_1[n+2]$														
$\langle \vec{s}_2, \vec{s}_1[n+2] \rangle$		+		+		+		+		+		+		=

$\vec{s}_2$	0		0		2		4		3		0		0	
$\vec{s}_1[n+1]$														
$\langle \vec{s}_2, \vec{s}_1[n+1] \rangle$		+		+		+		+		+		+		=

$\vec{s}_2$	0		0		2		4		3		0		0	
$\vec{s}_1[n]$														
$\langle \vec{s}_2, \vec{s}_1[n] \rangle$		+		+		+		+		+		+		=

$\vec{s}_2$	0		0		2		4		3		0		0	
$\vec{s}_1[n-1]$														
$\langle \vec{s}_2, \vec{s}_1[n-1] \rangle$		+		+		+		+		+		+		=

$\vec{s}_2$	0		0		2		4		3		0		0	
$\vec{s}_1[n-2]$														
$\langle \vec{s}_2, \vec{s}_1[n-2] \rangle$		+		+		+		+		+		+		=

Answer: The linear cross-correlation is calculated by shifting the second signal both forward and backward until there is no overlap between the signals. When there is no overlap, the cross-correlation goes to zero. Both of these cross-correlations should have only zeros outside the range: $-2 \leq n \leq 2$.

$\text{corr}_{\vec{s}_1}(\vec{s}_2)[k]$

$\vec{s}_1$	0		0		1		2		3		0		0	
$\vec{s}_2[n+2]$	2		4		3		0		0		0		0	
$\langle \vec{s}_1, \vec{s}_2[n+2] \rangle$	0	+	0	+	3	+	0	+	0	+	0	+	0	= 3

$\vec{s}_1$	0		0		1		2		3		0		0	
$\vec{s}_2[n+1]$	0		2		4		3		0		0		0	
$\langle \vec{s}_1, \vec{s}_2[n+1] \rangle$	0	+	0	+	4	+	6	+	0	+	0	+	0	= 10

$\vec{s}_1$	0		0		1		2		3		0		0	
$\vec{s}_2[n]$	0		0		2		4		3		0		0	
$\langle \vec{s}_1, \vec{s}_2[n] \rangle$	0	+	0	+	2	+	8	+	9	+	0	+	0	$= 19$

$\vec{s}_1$	0		0		1		2		3		0		0	
$\vec{s}_2[n-1]$	0		0		0		2		4		3		0	
$\langle \vec{s}_1, \vec{s}_2[n-1] \rangle$	0	+	0	+	0	+	4	+	12	+	0	+	0	$= 16$

$\vec{s}_1$	0		0		1		2		3		0		0	
$\vec{s}_2[n-2]$	0		0		0		0		2		4		3	
$\langle \vec{s}_1, \vec{s}_2[n-2] \rangle$	0	+	0	+	0	+	0	+	6	+	0	+	0	$= 6$

$$\text{corr}_{\vec{s}_2}(\vec{s}_1)[k]$$

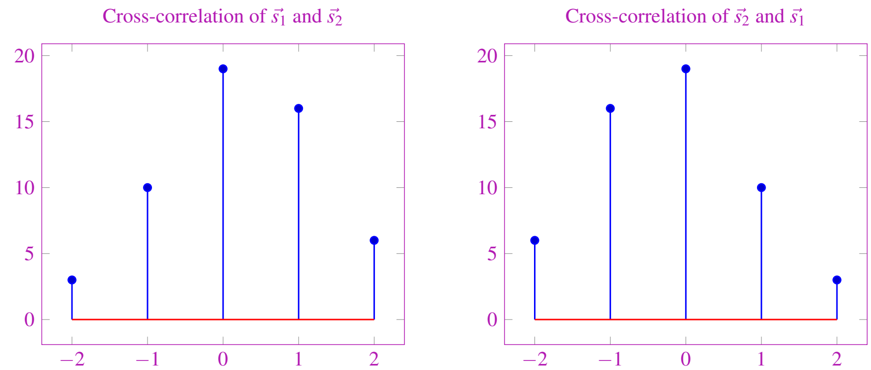
$\vec{s}_2[n]$	0		0		2		4		3		0		0	
$\vec{s}_1[n+2]$	1		2		3		0		0		0		0	
$\langle \vec{s}_2, \vec{s}_1[n+2] \rangle$	0	+	0	+	6	+	0	+	0	+	0	+	0	$= 6$

$\vec{s}_2[n]$	0		0		2		4		3		0		0	
$\vec{s}_1[n+1]$	0		1		2		3		0		0		0	
$\langle \vec{s}_2, \vec{s}_1[n+1] \rangle$	0	+	0	+	4	+	12	+	0	+	0	+	0	$= 16$

$\vec{s}_2[n]$	0		0		2		4		3		0		0	
$\vec{s}_1[n]$	0		0		1		2		3		0		0	
$\langle \vec{s}_2, \vec{s}_1[n] \rangle$	0	+	0	+	2	+	8	+	9	+	0	+	0	$= 19$

$\vec{s}_2[n]$	0		0		2		4		3		0		0	
$\vec{s}_2[n-1]$	0		0		0		1		2		3		0	
$\langle \vec{s}_2, \vec{s}_1[n-1] \rangle$	0	+	0	+	0	+	4	+	6	+	0	+	0	$= 10$

$\vec{s}_2[n]$	0		0		2		4		3		0		0	
$\vec{s}_2[n-2]$	0		0		0		0		1		2		3	
$\langle \vec{s}_2, \vec{s}_1[n-2] \rangle$	0	+	0	+	0	+	0	+	3	+	0	+	0	$= 3$



Notice that $\text{corr}_{\vec{s}_1}(\vec{s}_2)[k] = \text{corr}_{\vec{s}_2}(\vec{s}_1)[-k]$, i.e. changing the order of the signals reverses the cross-correlation sequence.

2. Search and Rescue Dogs

Berkeley's Puppy Pound needs your help! While Mr. Muffin was being walked, the volunteer let go of his leash and he is now running wild in the streets of Berkeley (which are quite dangerous)! Thankfully, all of the puppies at the pound have a collar that sends a bluetooth signal to receiver towers, which are spread throughout the streets (pictured below). If the puppy/collar is within range of the receiver tower, the collar will send the tower a message: the distance of the collar to the tower. Each cell tower has a range of 5 city blocks. Can you help the pound locate their lost puppy?

Note: A city block is defined as the middle of an intersection to the middle of an adjacent intersection (scale provided on map.) Mr. Muffin is constrained to running wild in the streets, meaning he won't be found in any buildings. If your TA asks 'Where is Mr. Muffin?' it is sufficient to answer with his intersection or 'between these two intersections.'

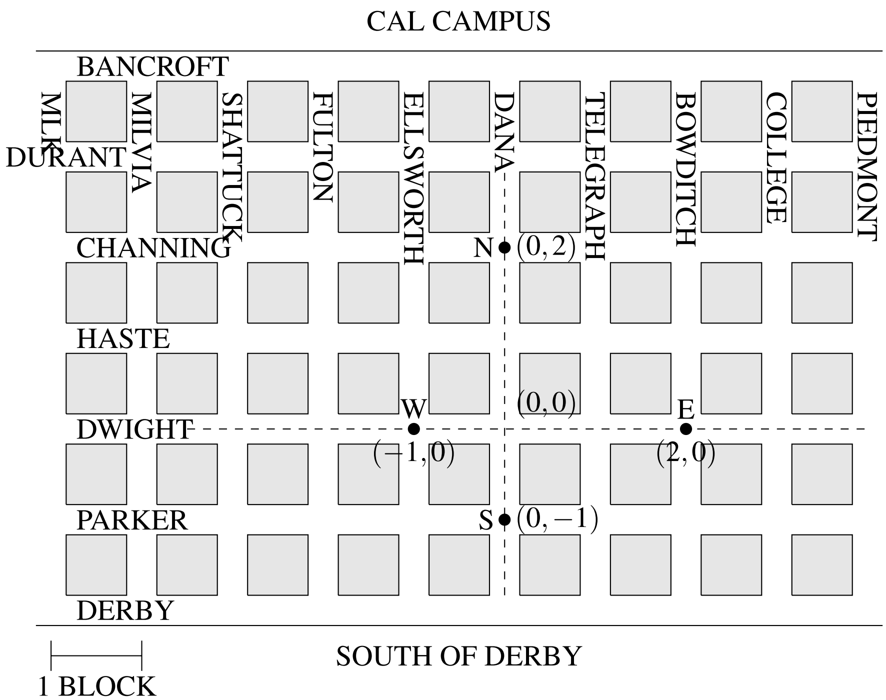

[1]

(a) You check the logs of the cell towers, and they have received the following messages:

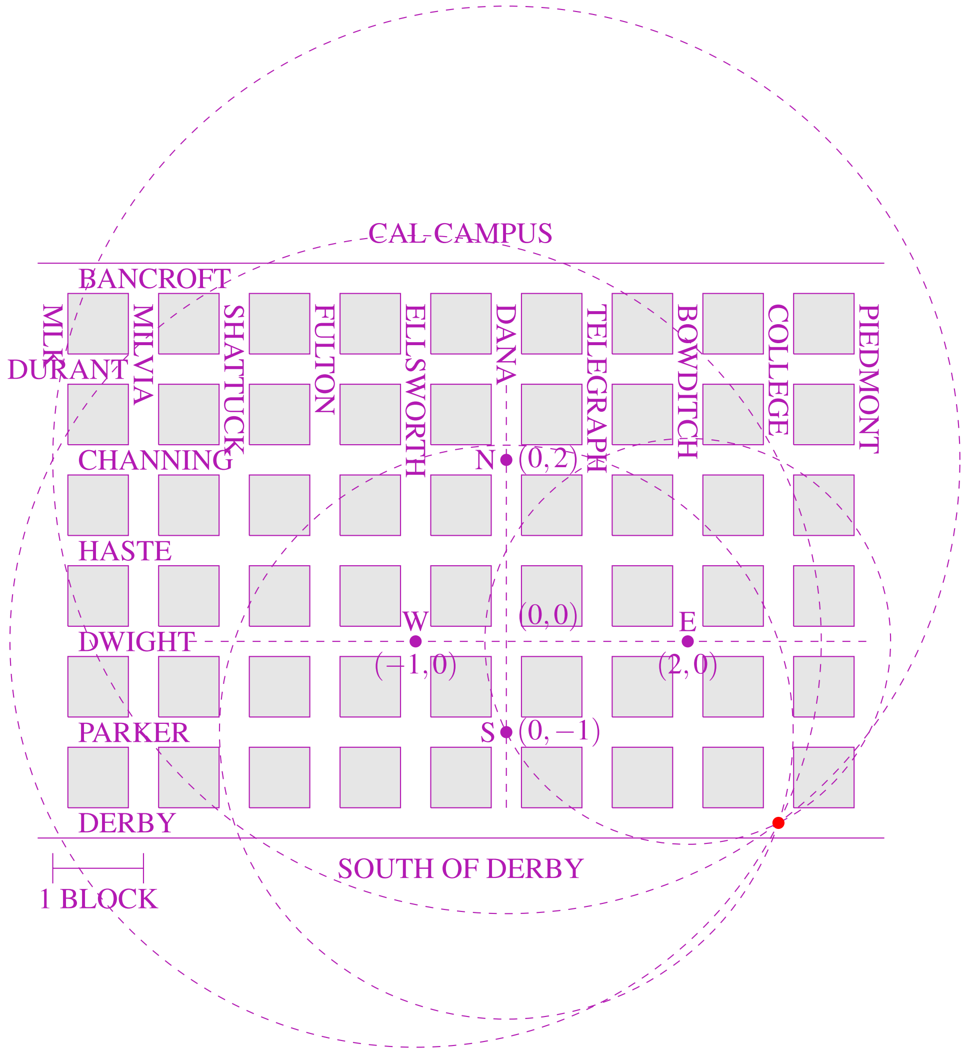
Sensor	Distance
N	5
W	$\sqrt{20}$
E	$\sqrt{5}$
S	$\sqrt{10}$

On the map provided, identify where Mr. Muffin is!

[1] http://www.pupsmile.com/wp-content/uploads/2012/11/running_happy_dog-1024x684.jpeg


CAL CAMPUS
BANCROFT
DURANT
CHANNING
HASTE
DWIGHT
PARKER
DERBY
MLK
MILVIA
SHATTUCK
FULTON
ELLSWORTH
DANA
TELEGRAPH
BOWDITCH
COLLEGE
PIEDMONT
N (0,2)
(0,0)
W (−1,0)
E (2,0)
S (0,−1)
SOUTH OF DERBY
1 BLOCK

Answer:


CAL CAMPUS
BANCROFT
DURANT
CHANNING
HASTE
DWIGHT
PARKER
DERBY
SOUTH OF DERBY
MLK
MILVIA
SHATTUCK
FULTON
ELLSWORTH
DANA
TELEGRAPH
BOWDITCH
COLLEGE
PIEDMONT
N
(0,2)
(0,0)
W
(−1,0)
E
(2,0)
S
(0,−1)
1 BLOCK

(b) Can you set this up as a system of equations? Is it linear? If it's not linear, can you think of a way to make it linear? Now, how do you set this up in matrix form?

Hint: Set $(0,0)$ to be Dwight and Dana.

Hint 2: Distance $= \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

Hint 3: You don't need all 4 equations. You have two unknowns, x and y. You know from lecture that you need three circles to uniquely find a point. How can you use the third circle/equation to get two equations and two unknowns?

Note: Remember to check for consistency for all nonlinear equations after finding the coordinates.

Answer:

First, set up the system of equations:

$$(x-0)^2 + (y-2)^2 = 5^2$$
$$(x+1)^2 + (y-0)^2 = \sqrt{20}^2$$
$$(x-2)^2 + (y-0)^2 = \sqrt{5}^2$$

Simplify out:

$$x^2 + y^2 - 4y + 4 = 25$$
$$x^2 + 2x + 1 + y^2 = 20$$
$$x^2 - 4x + 4 + y^2 = 5$$

Then subtract equation (1) from equations (2) and (3):

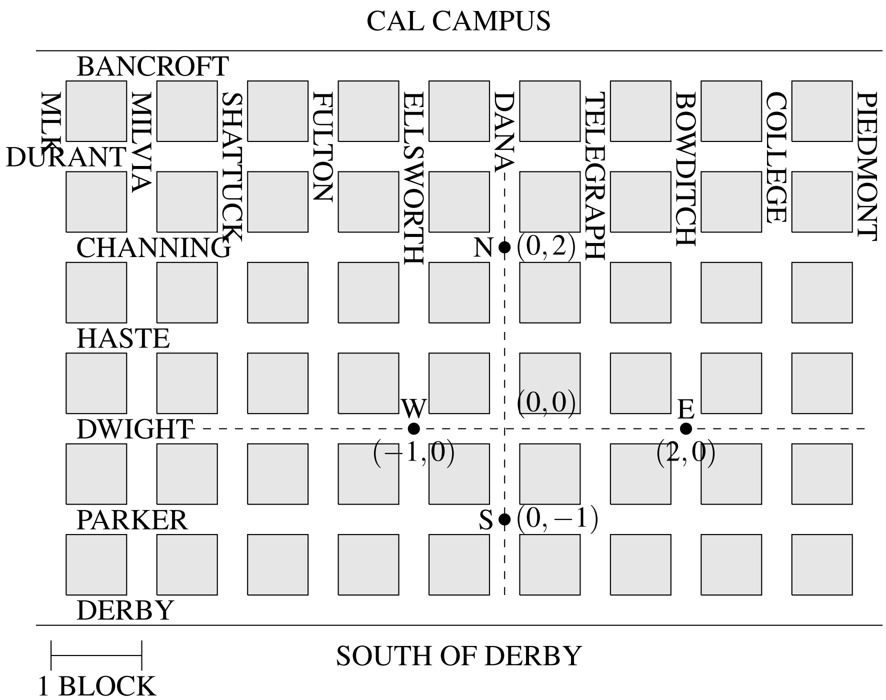
$$2x + 4y - 3 = 20 - 25$$
$$-4x + 4y = 5 - 25$$

This solves to $x = 3, y = -2$ which is roughly College and Derby.

(c) Suppose Mr. Muffin is moving fast, and by the time you get to destination in part (a) he's already run off! You check the logs of the cell towers again, and see the following updated messages:

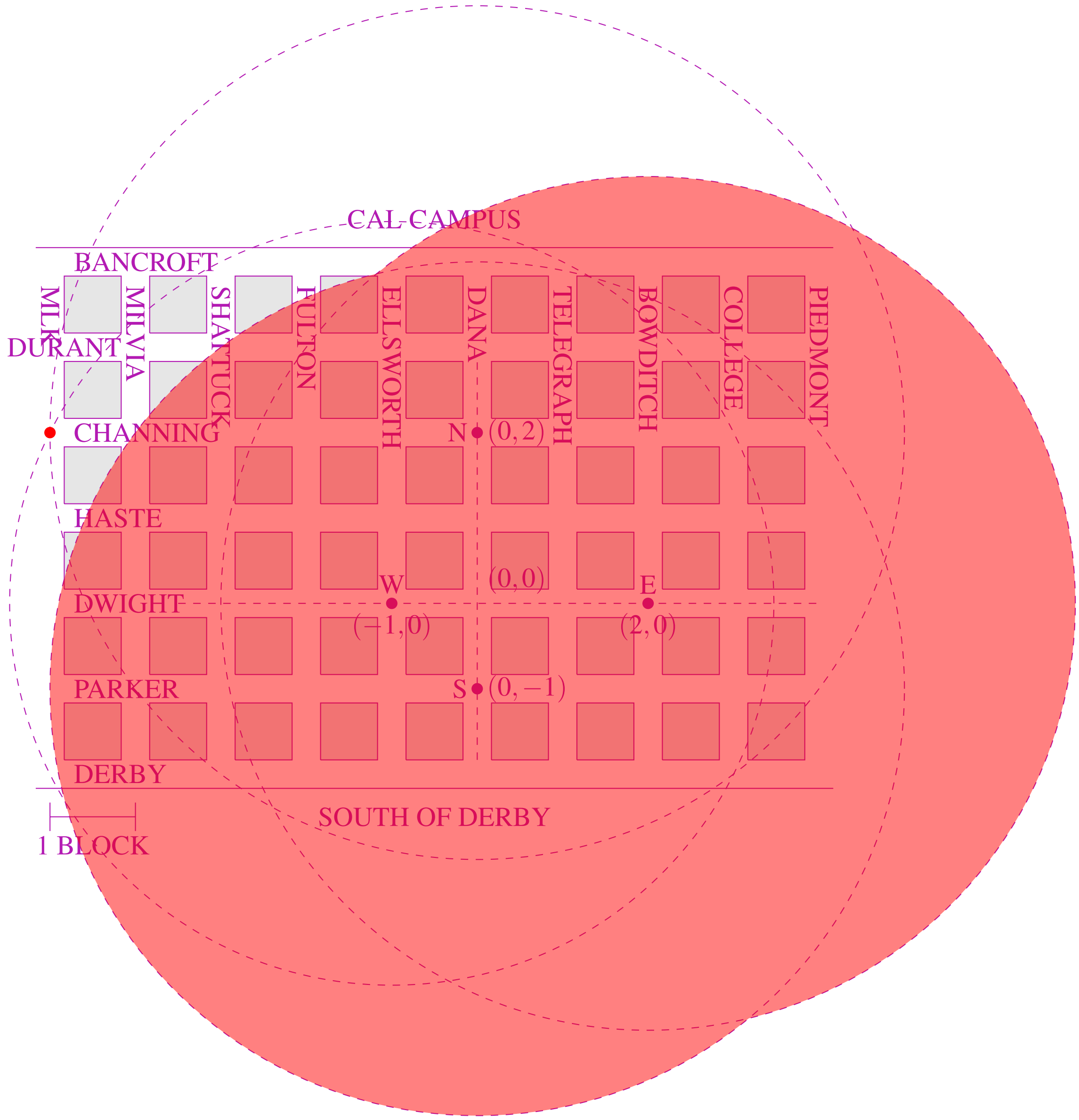
Sensor	Distance
N	5
W	$\sqrt{20}$
E	Out of Range
S	Out of Range

Can you find Mr. Muffin? (*Note:* Each cell tower has a range of 5 city blocks)



Answer:

With two out of range sensors, you might think that you will not be able to find a unique solution (you need 3 circles to intersect at a point.) The trick is that out of range still provide information on where Mr. Muffin is NOT. See the diagram below - Mr. Muffin cannot be in the shaded region. Therefore, Mr. Muffin should be in Channing and MLK.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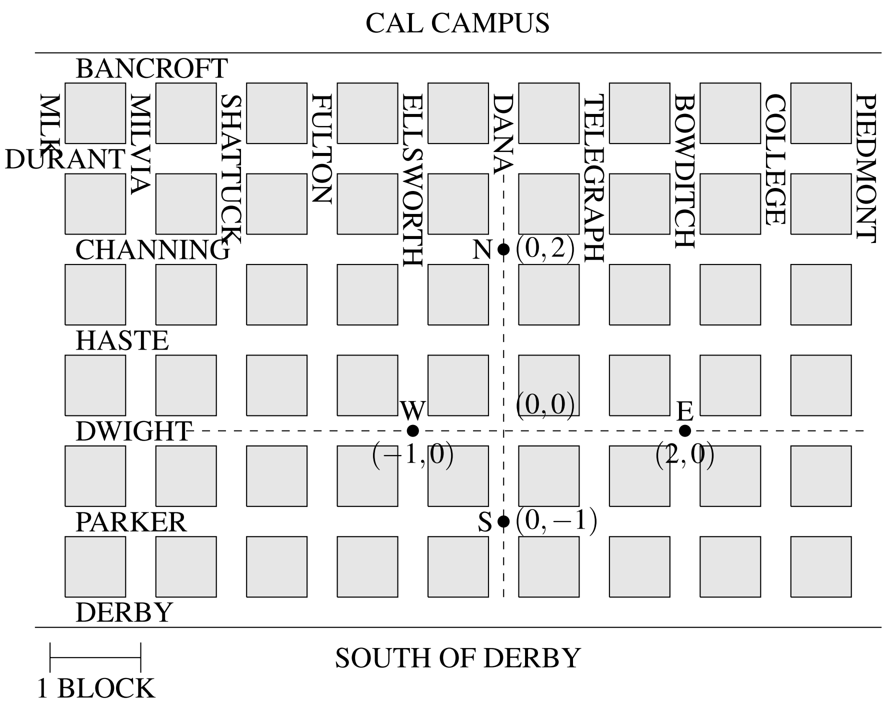
DURANT
CHANNING
N
(0,2)
HASTE
W
(0,0)
E
DWIGHT
(−1,0)
(2,0)
PARKER
S
(0,−1)
DERBY
SOUTH OF DERBY
1 BLOCK

(d) Mr. Muffin is a very mischievous puppy, and while playing and running around he damaged his collar. The transmitter on his collar will still send a signal to the receiver towers, but the distance sensor has noise. You check the logs of the cell towers, and they have received the following messages:

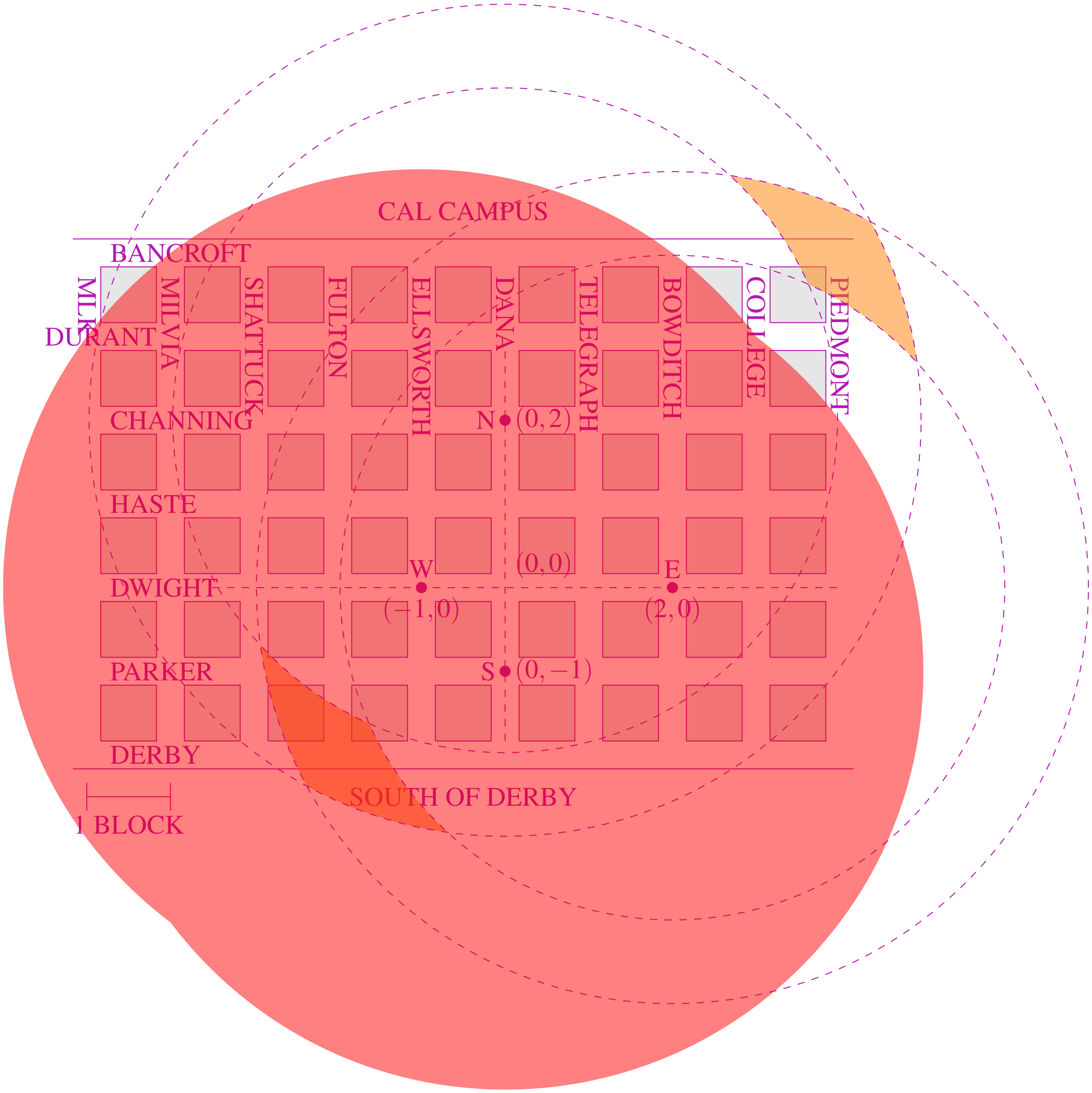
Sensor	Distance
N	4.5 ± 0.5
W	Out of Range
E	4.5 ± 0.5
S	Out of Range

On the map provided, identify where Mr. Muffin is! Can you find exactly where he is? (*Note:* Each cell tower has a range of 5 city blocks)



Answer:

You can't find exactly where he is, but you know he is somewhere between Piedmont/College and Bancroft. See the diagram below.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1 BLOCK