1. Energy Disaggregation

Suppose you live in a home with just these three appliances: an air conditioning unit (AC), a television (TV), and a refrigerator (R). Now, say you want to find the amount of electricity these appliances use individually, but the only measurement you can take is of the total power your home draws using your meter outside (this is often mounted on the side of the house and shows a running total of your electricity usage).

To do this, you will turn some appliances on and off and then read different total measurements. You can turn off the TV at any time, but you can’t unplug the fridge since the food would spoil. We keep the air conditioner off throughout the morning, but then it must stay on during the afternoon. However, the breaker trips (meaning the electricity suddenly shuts off) if all three are running, so the TV and AC cannot run at the same time.

(a) Can you design a way to calculate how much power each appliance uses? What type of measurements will you need to make, and how many?

Let $x_R$ be the power consumed by the refrigerator, $x_{TV}$ by the TV, and $x_{AC}$ by the AC, and let $m_i$ represent the power measured in measurement $i$. To find out the values of three variables, we somehow need three equations/measurements.

(b) Write out a system of equations that describes your measurements. Can you solve this system so that each appliance’s power is written in terms of measurements $m_i$?

For example: if you measure the power $m_1$ in the afternoon with the AC and refrigerator on but the TV is off, then the equation might look like $x_{AC} + x_R = m_1$.

(c) Let us say the breaker is fixed, so now we can safely run the AC and TV at the same time. Is there another way (or ways) you could create a new system of equations to solve? If so, see if you can solve your new system!

(d) Lastly, suppose as a busy Berkeley student, you only get a chance to take two measurements. Can you determine how much power each of the three appliances draw? If not, what combinations of power consumption can you find out?
2. **Linear or Nonlinear**

Determine whether the following functions \( f: \mathbb{R}^2 \to \mathbb{R} \) are linear or nonlinear.

(a) 
\[
f(x) = \sin \left( \frac{\pi}{2} \right) x + \cos \left( \frac{\pi}{2} \right)
\]

(b) 
\[
f(x_1, x_2) = 3x_1 + 4x_2
\]

(c) 
\[
f(x_1, x_2) = e^{x_2} + x_1^2
\]

(d) 
\[
f(x_1, x_2) = x_2 - x_1 + 3
\]