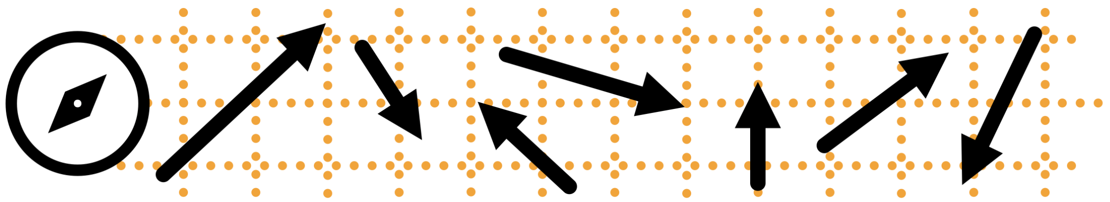
EECS 16A Fall 2020 — Designing Information Devices and Systems I — Discussion 1B

1. Vectors



A vector is an ordered list of numbers. For instance, a point on a plane (x,y) is a vector! We label vectors using an arrow overhead $\vec{v}$, and since vectors can live in ANY dimension of space we'll need to leave our notation general $(x,y) \rightarrow \vec{v} = (v_1, v_2, \ldots)$. Below are few more examples (the left-most form is the general definition):

$$\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n \qquad \vec{a} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \in \mathbb{R}^3 \qquad \vec{b} = \begin{bmatrix} 2.4 \\ 5.3 \end{bmatrix} \in \mathbb{R}^2$$

Just to unpack this a bit more, $\vec{b} \in \mathbb{R}^3$ in english means "vector $\vec{b}$ lives in 3-Dimensional space".

- The $\in$ symbol literally means "in"
- The $\mathbb{R}$ stands for "real numbers" (FUN FACT: $\mathbb{Z}$ means "integers" like $-2, 4, 0, \ldots$)
- The exponent $\mathbb{R}^{n \leftarrow}$ indicates the dimension of space, or the amount of numbers in the vector.

One last thing: it is standard to write vectors in column-form, like seen with $\vec{a}, \vec{b}, \vec{x}$ above. We call these *column vectors*, in contrast to horizontally written vectors which we call *row vectors*.

Okay, let's dig into a few examples:

(a) Which of the following vectors live in $\mathbb{R}^2$ space?

$$i. \begin{bmatrix} 3 \\ 6 \end{bmatrix} \qquad ii. \begin{bmatrix} 5 \\ 0 \\ 3 \\ 5 \end{bmatrix} \qquad iii. \begin{bmatrix} -4.76 \\ 1.32 \\ 0.01 \end{bmatrix} \qquad iv. \begin{bmatrix} -20 \\ 100 \end{bmatrix}$$

(b) Graphically show the vectors (either in a sketch with axes, or a plot on a computer):

$$i. \begin{bmatrix} 2 \\ 5 \end{bmatrix} \qquad ii. \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

(c) Compute the sum $\vec{a} + \vec{b} = \vec{c}$ from the vectors below, and then graphically sketch or plot these vectors. (show them in a way that forms a triangle; also is there only one possible triangle?)

$$\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \vec{b} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

2. Solving Systems of Equations A system of linear equations can either have one solution, an infinite number of solutions, or no solution at all. For the following systems of equations, state whether there is a unique solution, no solution, or an infinite number of solutions. If there are an infinite number of solutions give one possible solution.

(a) Solve the following system. How many solutions does it have?

$$\begin{aligned} x \quad + \quad y \quad &= 4 \\ x \quad - \quad y \quad &= 2 \end{aligned}$$

(b) Now write the system in augmented matrix form:

$$x + y = 4 \tag{1}$$

$$x - y = 2 \tag{2}$$

(c) Once in augmented matrix form we can use a systematic procedure called Gaussian Elimination to solve the system of equations. See what solution you get using Gaussian elimination.

(d) Now consider the system

$$7x + y = 7 \tag{3}$$
$$42x + 6y = 42. \tag{4}$$

How many solutions does it have? Solve it first using any method, then write it as an augmented matrix and try to solve it.

(e) Now consider the system

$$7x + y = 7 \tag{5}$$
$$42x + 6y = 42 \tag{6}$$
$$7x + y = 6 \tag{7}$$

How many solutions does it have? Solve it first using any method, then write it as an augmented matrix and try to solve it.