1. Vectors

A vector is an ordered list of numbers. For instance, a point on a plane \((x, y)\) is a vector! We label vectors using an arrow overhead \(\vec{v}\), and since vectors can live in ANY dimension of space we’ll need to leave our notation general \(\vec{v} = (v_1, v_2, \ldots)\). Below are few more examples (the left-most form is the general definition):

\[
\vec{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^n \\
\vec{a} = \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} \in \mathbb{R}^3 \\
\vec{b} = \begin{bmatrix} 2.4 \\ 5.3 \end{bmatrix} \in \mathbb{R}^2
\]

Just to unpack this a bit more, \(\vec{b} \in \mathbb{R}^3\) in english means "vector \(\vec{b}\) lives in 3-Dimensional space".

- The \(\in\) symbol literally means "in"
- The \(\mathbb{R}\) stands for "real numbers" (FUN FACT: \(\mathbb{Z}\) means "integers" like \(-2, 4, 0, \ldots\))
- The exponent \(\mathbb{R}^n \leftarrow\) indicates the dimension of space, or the number of elements in the vector.

One last thing: it is standard to write vectors in column-form, like seen with \(\vec{a}, \vec{b}, \vec{x}\) above. We call these column vectors, in contrast to horizontally written vectors which we call row vectors.

Okay, let’s dig into a few examples:

(a) Which of the following vectors live in \(\mathbb{R}^2\) space?

\[
i. \begin{bmatrix} 3 \\ 6 \end{bmatrix} \\
ii. \begin{bmatrix} 5 \\ 0 \\ 3 \\ 5 \end{bmatrix} \\
iii. \begin{bmatrix} -4.76 \\ 1.32 \\ 0.01 \end{bmatrix} \\
iv. \begin{bmatrix} -20 \\ 100 \end{bmatrix}
\]

(b) Graphically show the vectors (either in a sketch with axes, or a plot on a computer):

\[
i. \begin{bmatrix} 2 \\ 5 \end{bmatrix} \\
ii. \begin{bmatrix} 5 \\ 2 \end{bmatrix}
\]

(c) Compute the sum \(\vec{a} + \vec{b} = \vec{c}\) from the vectors below, and then graphically sketch or plot these vectors. (show them in a way that forms a triangle; also is there only one possible triangle?)

\[
\vec{a} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\
\vec{b} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}
\]
2. Computations: matrix-vector multiplication

For each matrix vector multiplication problem, find the product by hand

(a) \[ A = \begin{bmatrix} 1 & 6 \\ 2 & -7 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \]

(b) \[ A = \begin{bmatrix} 1 & 9 & 2 \\ 7 & 10 & -7 \\ -1 & 2 & -8 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix} \]

3. Matrix Multiplication

Consider the following matrices:

\[ A = \begin{bmatrix} 1 & 4 \\ \end{bmatrix} \quad B = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \quad D = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix} \]
\[ E = \begin{bmatrix} 1 & 9 & 5 & 7 \\ 4 & 3 & 2 & 2 \end{bmatrix} \quad F = \begin{bmatrix} 5 & 5 & 8 \\ 6 & 1 & 2 \\ 4 & 1 & 7 \\ 3 & 2 & 2 \end{bmatrix} \quad G = \begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix} \quad H = \begin{bmatrix} 5 & 3 & 4 \\ 1 & 8 & 2 \\ 2 & 3 & 5 \end{bmatrix} \]

For each matrix multiplication problem, if the product exists, find the product by hand. Otherwise, explain why the product does not exist.

(a) \( A \ B \)

(b) \( C \ D \)

(c) \( D \ C \)

(d) \( C \ E \)

(e) \( F \ E \) (only note whether or not the product exists)

(f) \( E \ F \) (only note whether or not the product exists)

(g) \( G \ H \) (Practice on your own)

(h) \( H \ G \) (Practice on your own)