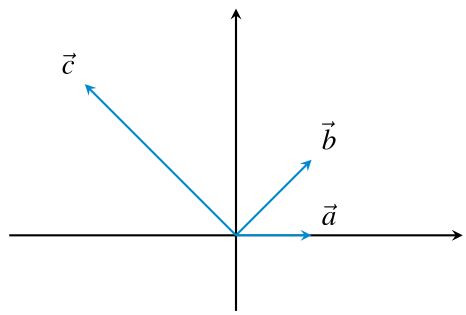
EECS 16A Designing Information Devices and Systems I
Spring 2022 Discussion 2B

1. Visualizing Span

We are given a point $\vec{c}$ that we want to get to, but we can only move in two directions: $\vec{a}$ and $\vec{b}$. We know that to get to $\vec{c}$, we can travel along $\vec{a}$ for some amount α, then change direction, and travel along $\vec{b}$ for some amount β. We want to find these two scalars α and β, such that we reach point $\vec{c}$. That is, $\alpha\vec{a}+\beta\vec{b}=\vec{c}$.



(a) Formulate the system of equations as a matrix to find the unknowns, α, β, in terms of the vectors $\vec{a}, \vec{b}, \vec{c}$.

(b) First, consider the case where $\vec{a} = \begin{bmatrix}1\\0\end{bmatrix}$, $\vec{b} = \begin{bmatrix}1\\1\end{bmatrix}$, and $\vec{c} = \begin{bmatrix}-2\\2\end{bmatrix}$. Draw these vectors on a sheet of paper. Now find the two scalars α and β, such that we reach point $\vec{c}$. What are these scalars if we use $\vec{a} = \begin{bmatrix}1\\1\end{bmatrix}$ and $\vec{b} = \begin{bmatrix}2\\1\end{bmatrix}$ instead?

2. Span Basics

(a) What is span $\left\{ \begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\1\\0\end{bmatrix} \right\}$?

(b) Is $\begin{bmatrix}5\\5\\0\end{bmatrix}$ in span $\left\{ \begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\1\\0\end{bmatrix} \right\}$?

(c) What is a possible choice for $\vec{v}$ that would make span$\left\{ \begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}2\\1\\0\end{bmatrix}, \vec{v} \right\} = \mathbb{R}^3$?

(d) For what values of b_1, b_2, b_3 is the following system of linear equations consistent? ("Consistent" means there is at least one solution.)

$$\begin{bmatrix}1 & 2\\2 & 1\\0 & 0\end{bmatrix}\vec{x} = \begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$$

3. Span Proofs

Given some set of vectors $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$, show the following:

(a)
$$\text{span}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\} = \text{span}\{\alpha\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}, \text{ where } \alpha \text{ is a non-zero scalar}$$

In other words, we can scale our spanning vectors and not change their span.

(b) (for practice)
$$\text{span}\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\} = \text{span}\{\vec{v}_2, \vec{v}_1, \ldots, \vec{v}_n\}$$

In other words, we can swap the order of our spanning vectors and not change their span.