
EECS 16A Designing Information Devices and Systems I
Fall 2020 Discussion 3B

1. Matrix Multiplication

Consider the following matrices:

$$\mathbf{A} = [1 \ 4] \quad \mathbf{B} = \begin{bmatrix} 3 \\ 2 \end{bmatrix} \quad \mathbf{C} = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} \quad \mathbf{D} = \begin{bmatrix} 3 & 2 \\ 2 & 1 \end{bmatrix}$$
$$\mathbf{E} = \begin{bmatrix} 1 & 9 & 5 & 7 \\ 4 & 3 & 2 & 2 \end{bmatrix} \quad \mathbf{F} = \begin{bmatrix} 5 & 5 & 8 \\ 6 & 1 & 2 \\ 4 & 1 & 7 \\ 3 & 2 & 2 \end{bmatrix} \quad \mathbf{G} = \begin{bmatrix} 8 & 1 & 6 \\ 3 & 5 & 7 \\ 4 & 9 & 2 \end{bmatrix} \quad \mathbf{H} = \begin{bmatrix} 5 & 3 & 4 \\ 1 & 8 & 2 \\ 2 & 3 & 5 \end{bmatrix}$$

For each matrix multiplication problem, *if the product exists*, find the product by hand. Otherwise, explain why the product does not exist.

(a) $\mathbf{A B}$

(b) $\mathbf{C D}$

(c) $\mathbf{D C}$

(d) **C E**

(e) **F E**

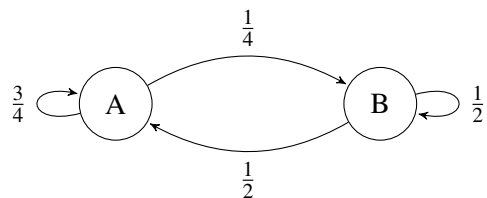
(f) **E F**

(g) **G H** (Practice on your own)

(h) **H G** (Practice on your own)

2. Transition Matrix

Suppose we have a network of pumps as shown in the diagram below. Let us describe the state of A and B using a state vector $\vec{x}[n] = \begin{bmatrix} x_A[n] \\ x_B[n] \end{bmatrix}$ where $x_A[n]$ and $x_B[n]$ are the states at time-step n .



(a) Find the state transition matrix S , such that $\vec{x}[n+1] = S \vec{x}[n]$.

(b) Let us now find the matrix S^{-1} such that we can recover the previous state $\vec{x}[n-1]$ from $\vec{x}[n]$. Specifically, solve for S^{-1} such that $\vec{x}[n-1] = S^{-1} \vec{x}[n]$.

(c) Now draw the state transition diagram that corresponds to the S^{-1} that you just found.

- (d) Redraw the diagram from the first part of the problem, but now with the directions of the arrows reversed. Let us call the state transmission matrix of this "reversed" state transition diagram T . Does $T = S^{-1}$?

- (e) Suppose we start in the state $\vec{x}[1] = \begin{bmatrix} 12 \\ 12 \end{bmatrix}$. Compute the state vector after 2 time-steps $\vec{x}[3]$.

- (f) **(Challenge practice problem)** Given our starting state from the previous problem, what happens if we look at the state of the network after a lot of time steps? Specifically which state are we approaching, as defined below?

$$\vec{x}_{final} = \lim_{n \rightarrow \infty} \vec{x}[n]$$

Note that the final state needs to be what we call a *steady state*, meaning $S \vec{x}_{final} = \vec{x}_{final}$.

Also what can you say about $x_A[n] + x_B[n]$?

Use information from both of these properties to write out a new system of equations and solve for \vec{x}_{final} .