
EECS 16A Designing Information Devices and Systems I Discussion 4A
 Spring 2022

1. Mechanical Inverses

For each sub-part below, determine whether or not the inverse of \mathbf{A} exists.
 If it exists, compute the inverse using the Gauss-Jordan method.

(a) $\mathbf{A} = \begin{bmatrix} 1 & 0 \\ 0 & 9 \end{bmatrix}$

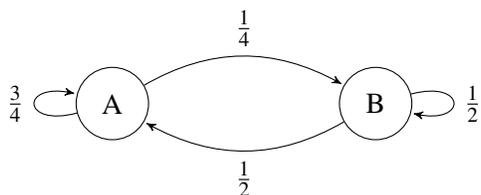
(b) $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

(c) $\mathbf{A} = \begin{bmatrix} 1 & 5 & 3 \\ 2 & -2 & 4 \end{bmatrix}$

(d) $\mathbf{A} = \begin{bmatrix} 5 & 5 & 15 \\ 2 & 2 & 4 \\ 1 & 1 & 4 \end{bmatrix}$

2. Transition Matrix

Suppose we have a network of pumps as shown in the diagram below. Let us describe the state of A and B using a state vector $\vec{x}[n] = \begin{bmatrix} x_A[n] \\ x_B[n] \end{bmatrix}$ where $x_A[n]$ and $x_B[n]$ are the states at time-step n .



- (a) Find the state transition matrix S , such that $\vec{x}[n+1] = S\vec{x}[n]$.
Separately find the sum of the terms for each column vector in S . Do you notice any pattern?
- (b) Let us now find the matrix S^{-1} such that we can recover the previous state $\vec{x}[n-1]$ from $\vec{x}[n]$.
Specifically, solve for S^{-1} such that $\vec{x}[n-1] = S^{-1}\vec{x}[n]$.
- (c) Now draw the state transition diagram that corresponds to the S^{-1} that you just found.
Also find the sum of the terms for each column vector in S^{-1} . Do you notice any pattern?
- (d) Redraw the diagram from the first part of the problem, but now with the directions of the arrows reversed. Let us call the state transmission matrix of this "reversed" state transition diagram T . Does $T = S^{-1}$?
- (e) Suppose we start in the state $\vec{x}[1] = \begin{bmatrix} 12 \\ 12 \end{bmatrix}$. Compute the state vector after two time-steps, $\vec{x}[3]$.