Recall from lecture the way to compute a determinant of any $2 \times 2$ matrix is by using the following formula:
\[
\begin{bmatrix}
  a & b \\
  c & d
\end{bmatrix}
\]
\[
\det(A) = ad - bc
\]

1. **Mechanical Eigenvalues and Eigenvectors**

   **Definition:** For some matrix $A$, the polynomial function of $\lambda$, $f(\lambda) = \det(A - \lambda I)$, is known as the *characteristic polynomial* of $A$.

   Find the eigenvalues (which are the roots of the characteristic polynomial) of each matrix $M$ and their associated eigenvectors. State if the inverse of $M$ exists.

   (a) $M = \begin{bmatrix}
   0 & 1 \\
   -2 & -3
\end{bmatrix}$

   (b) $M = \begin{bmatrix}
   -2 & 4 \\
   -4 & 8
\end{bmatrix}$
2. Steady State Reservoir Levels

We have 3 reservoirs: A, B and C. The pumps system between the reservoirs is depicted in Figure 1.
(a) Write out the transition matrix $T$ representing the pumps system.

(b) You are told that $\lambda_1 = 1$, $\lambda_2 = \frac{-\sqrt{2} - 1}{10}$, $\lambda_3 = \frac{\sqrt{2} - 1}{10}$ are the eigenvalues of $T$. Find a steady state vector $\vec{x}$, i.e. a vector such that $T\vec{x} = \vec{x}$. 

Figure 1: Reservoir pumps system.