EECS 16A Designing Information Devices and Systems I
Fall 2020 Discussion 5A

Recall from lecture the way to compute a determinant of any 2×2 matrix is by using the following formula:

$$\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad \det(\mathbf{A}) = ad - bc$$

1. Mechanical Eigenvalues and Eigenvectors

Definition: For some matrix $\mathbf{A}$, the polynomial function of λ, $f(\lambda) = \det(\mathbf{A} - \lambda\mathbf{I})$, is known as the *characteristic polynomial* of $\mathbf{A}$.

Find the eigenvalues (which are the roots of the characteristic polynomial) of each matrix $\mathbf{M}$ and their associated eigenvectors. State if the inverse of $\mathbf{M}$ exists.

(a) $\mathbf{M} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$

(b) $\mathbf{M} = \begin{bmatrix} -2 & 4 \\ -4 & 8 \end{bmatrix}$

(c) $\mathbf{M} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

(d) $\mathbf{M} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

2. Steady State Reservoir Levels

We have 3 reservoirs: A, B and C. The pumps system between the reservoirs is depicted in Figure 1.

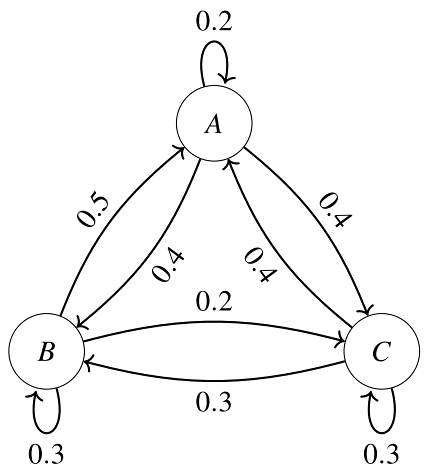

Figure 1: Reservoir pumps system.

(a) Write out the transition matrix **T** representing the pumps system.

(b) You are told that $\lambda_1 = 1$, $\lambda_2 = \frac{-\sqrt{2}-1}{10}$, $\lambda_3 = \frac{\sqrt{2}-1}{10}$ are the eigenvalues of **T**. Find a steady state vector $\vec{x}$, i.e. a vector such that $T\vec{x} = \vec{x}$.

