EECS 16A Designing Information Devices and Systems I
Spring 2021 Discussion 11B

Reference: Inner products

For this course we will use a standard inner product definition from matrix-vector multiplication:

$$\langle \vec{x}, \vec{y} \rangle \;=\; x_1 y_1 + x_2 y_2 + \ldots + x_n y_v \;, \quad \text{for any } \; \vec{x}, \vec{y} \in \mathbb{R}^n.$$

In general, any inner product $\langle \cdot, \cdot \rangle$ on a real vector space $\mathbb{V}$ is a bilinear function that satisfies the following three properties:

(a) **Symmetry:** $\langle \vec{x}, \vec{y} \rangle = \langle \vec{y}, \vec{x} \rangle$.

(b) **Linearity:** $\langle \vec{x}, \vec{y} + \vec{z} \rangle = \langle \vec{x}, \vec{y} \rangle + \langle \vec{x}, \vec{z} \rangle$ and $\langle c\vec{x}, \vec{y} \rangle = c\langle \vec{x}, \vec{y} \rangle$, where $c \in \mathbb{R}$ is a real number.

(c) **Non-negativity:** $\langle \vec{x}, \vec{x} \rangle \geq 0$, with equality if and only if $\vec{x} = \vec{0}$.

Here $\vec{x}$, $\vec{y}$, and $\vec{z}$ can be any vectors in the vector space $\mathbb{V}$.
The norm (or length) of a vector $\vec{x} = [x_1, x_2, \ldots, x_n]^T$ is defined using the inner product as

$$\|\vec{x}\| \;=\; \sqrt{\langle \vec{x}, \vec{x} \rangle} \;\equiv\; \sqrt{x_1^2 + x_2^2 + \ldots + x_n^2} \;.$$

1. Inner Product Properties

For this question we will verify our coordinate definition of the inner product

$$\langle \vec{x}, \vec{y} \rangle \;=\; x_1 y_1 + x_2 y_2 + \ldots + x_n y_v \;, \quad \text{for any } \; \vec{x}, \vec{y} \in \mathbb{R}^n$$

indeed satisfies the key properties required for all inner products, but presently for the 2-dimensional case. Suppose $\vec{x}, \vec{y}, \vec{z} \in \mathbb{R}^2$ for the following parts:

(a) Show symmetry $\langle \vec{x}, \vec{y} \rangle = \langle \vec{y}, \vec{x} \rangle$:

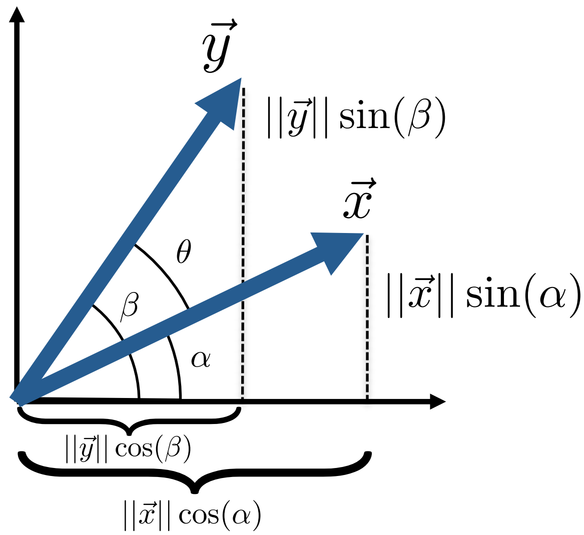
(b) Show linearity $\langle \vec{x}, c\vec{y} + d\vec{z} \rangle = c\langle \vec{x}, \vec{y} \rangle + d\langle \vec{x}, \vec{z} \rangle$, where $c \in \mathbb{R}$ is a real number.

(c) Show non-negativity $\langle \vec{x}, \vec{x} \rangle \geq 0$, with equality if and only if $\vec{x} = \vec{0}$:

2. Geometric Interpretation of the Inner Product

In this problem we explore the geometric interpretation of the Euclidean inner product, restricting ourselves to vectors in $\mathbb{R}^2$.

(a) Derive a formula for the inner product of two vectors in terms of their magnitudes and the angle between them. The figure below may be helpful:



(b) For each sub-part, identify any two (nonzero) vectors $\vec{x}, \vec{y} \in \mathbb{R}^2$ that satisfy the stated condition and compute their inner product.

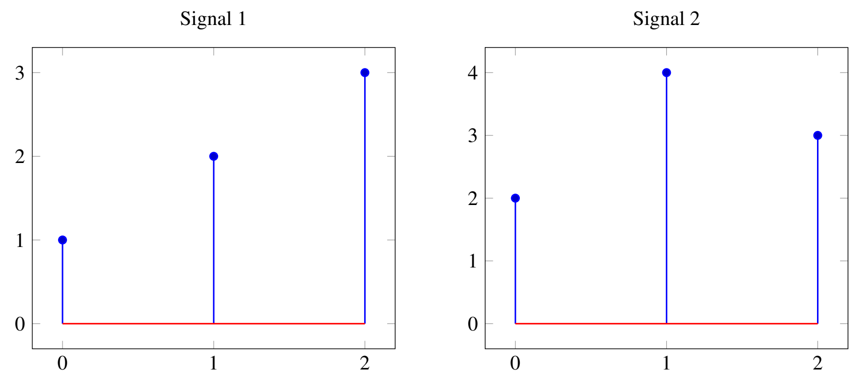
i. Identify a pair of parallel vectors:

ii. Identify a pair of anti-parallel vectors:

iii. Identify a pair of perpendicular vectors:

3. Correlation

We are given the following two signals, $s_1[n]$ and $s_2[n]$ respectively.



Find the cross correlations, $\text{corr}_{s_1}(s_2)$ and $\text{corr}_{s_2}(s_1)$ for signals $s_[n]$ and $s_2[n]$. Recall

$$\text{corr}_x(y)[k] = \sum_{i=-\infty}^{\infty} x[i]y[i-k].$$

$$\text{corr}_{\vec{s}_1}(\vec{s}_2)[k]$$

$\vec{s}_1$	0	0	1	2	3	0	0	
$\vec{s}_2[n+2]$								
$\langle \vec{s}_1, \vec{s}_2[n+2] \rangle$		+	+	+	+	+	+	=

$\vec{s}_1$	0	0	1	2	3	0	0	
$\vec{s}_2[n+1]$								
$\langle \vec{s}_1, \vec{s}_2[n+1] \rangle$		+	+	+	+	+	+	=

$\vec{s}_1$	0	0	1	2	3	0	0	
$\vec{s}_2[n]$								
$\langle \vec{s}_1, \vec{s}_2[n] \rangle$		+	+	+	+	+	+	=

$\vec{s}_1$	0	0	1	2	3	0	0	
$\vec{s}_2[n-1]$								
$\langle \vec{s}_1, \vec{s}_2[n-1] \rangle$		+	+	+	+	+	+	=

$\vec{s}_1$	0	0	1	2	3	0	0	
$\vec{s}_2[n-2]$								
$\langle \vec{s}_1, \vec{s}_2[n-2] \rangle$		+	+	+	+	+	+	=

$$\text{corr}_{\vec{s}_2}(\vec{s}_1)[k]$$

$\vec{s}_2$	0	0	2	4	3	0	0
$\vec{s}_1[n+2]$							
$\langle \vec{s}_2, \vec{s}_1[n+2] \rangle$	+	+	+	+	+	+	=

$\vec{s}_2$	0	0	2	4	3	0	0
$\vec{s}_1[n+1]$							
$\langle \vec{s}_2, \vec{s}_1[n+1] \rangle$	+	+	+	+	+	+	=

$\vec{s}_2$	0	0	2	4	3	0	0
$\vec{s}_1[n]$							
$\langle \vec{s}_2, \vec{s}_1[n] \rangle$	+	+	+	+	+	+	=

$\vec{s}_2$	0	0	2	4	3	0	0
$\vec{s}_1[n-1]$							
$\langle \vec{s}_2, \vec{s}_1[n-1] \rangle$	+	+	+	+	+	+	=

$\vec{s}_2$	0	0	2	4	3	0	0
$\vec{s}_1[n-2]$							
$\langle \vec{s}_2, \vec{s}_1[n-2] \rangle$	+	+	+	+	+	+	=