1. Inner Product Properties

For this question we will verify our coordinate definition of the inner product

\[ \langle \vec{x}, \vec{y} \rangle = x_1y_1 + x_2y_2 + \ldots + x_ny_n, \quad \text{for any } \vec{x}, \vec{y} \in \mathbb{R}^n \]

indeed satisfies the key properties required for all inner products, but presently for the 2-dimensional case. Suppose \( \vec{x}, \vec{y}, \vec{z} \in \mathbb{R}^2 \) for the following parts:

(a) Show symmetry \( \langle \vec{x}, \vec{y} \rangle = \langle \vec{y}, \vec{x} \rangle \):

(b) Show linearity \( \langle \vec{x}, c\vec{y} + d\vec{z} \rangle = c\langle \vec{x}, \vec{y} \rangle + d\langle \vec{x}, \vec{z} \rangle \), where \( c \in \mathbb{R} \) is a real number.

(c) Show non-negativity \( \langle \vec{x}, \vec{x} \rangle \geq 0 \), with equality if and only if \( \vec{x} = \vec{0} \):

2. Geometric Interpretation of the Inner Product

In this problem we explore the geometric interpretation of the Euclidean inner product, restricting ourselves to vectors in \( \mathbb{R}^2 \).

(a) Derive a formula for the inner product of two vectors in terms of their magnitudes and the angle between them. The figure below may be helpful:

(b) For each sub-part, identify any two (nonzero) vectors \( \vec{x}, \vec{y} \in \mathbb{R}^2 \) that satisfy the stated condition and compute their inner product.

   i. Identify a pair of parallel vectors:
ii. Identify a pair of anti-parallel vectors (vectors that point in opposite directions):

iii. Identify a pair of perpendicular vectors:

3. Correlation

You are given the following two signals:

![Signal 1](image1.png)

![Signal 2](image2.png)

(a) Sketch the linear cross-correlation of signal 1 with signal 2, that is find \( corr(\vec{s}_1, \vec{s}_2) \). Do not assume the signals are periodic.

(b) Assume signal \( \vec{s}_2 \) is periodic with period 5. Find the linear cross correlation \( corr(\vec{s}_1, \vec{s}_2) \) of the two signals.