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EECS 16A    Designing Information Devices and Systems I  
Fall 2020    Discussion 14B

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### 1. Orthonormal Matrices and Projections

An orthonormal matrix,  $\mathbf{A}$ , is a matrix whose columns,  $\vec{a}_i$ , are:

- Orthogonal (ie.  $\langle \vec{a}_i, \vec{a}_j \rangle = 0$  when  $i \neq j$ )
- Normalized (ie. vectors with length equal to 1,  $\|\vec{a}_i\| = 1$ ). This implies that  $\|\vec{a}_i\|^2 = \langle \vec{a}_i, \vec{a}_i \rangle = 1$ .

(a) Suppose that the matrix  $\mathbf{A} \in \mathbb{R}^{N \times M}$  has linearly independent columns. The vector  $\vec{y}$  in  $\mathbb{R}^N$  is not in the subspace spanned by the columns of  $\mathbf{A}$ . What is the projection of  $\vec{y}$  onto the subspace spanned by the columns of  $\mathbf{A}$ ?

(b) Show if  $\mathbf{A} \in \mathbb{R}^{N \times N}$  is an orthonormal matrix then the columns,  $\vec{a}_i$ , form a basis for  $\mathbb{R}^N$ .

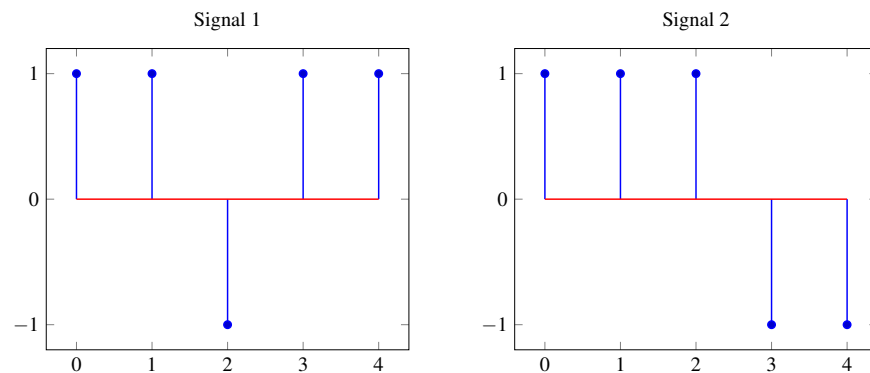
(c) When  $\mathbf{A} \in \mathbb{R}^{N \times M}$  and  $N \geq M$  (i.e. tall matrices), show that if the matrix is orthonormal, then  $\mathbf{A}^T \mathbf{A} = \mathbf{I}_{M \times M}$ .

(d) Again, suppose  $\mathbf{A} \in \mathbb{R}^{N \times M}$  where  $N \geq M$  is an orthonormal matrix. Show that the projection of  $\vec{y}$  onto the subspace spanned by the columns of  $\mathbf{A}$  is now  $\mathbf{A} \mathbf{A}^T \vec{y}$ .

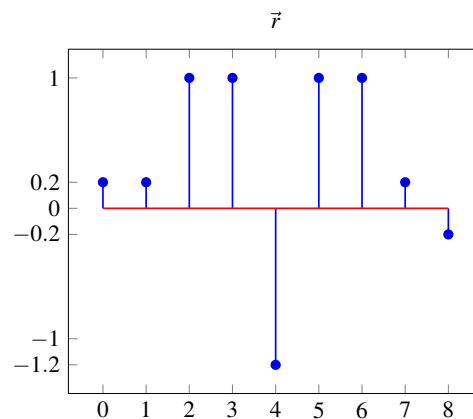
(e) Given  $\mathbf{A} \in \mathbb{R}^{N \times M} = \begin{bmatrix} 0 & 0 & \frac{\sqrt{2}}{2} \\ 0 & 0 & \frac{\sqrt{2}}{2} \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$  and the columns of  $\mathbf{A}$  are orthonormal, find the least squares solution to  $\mathbf{A} \hat{\mathbf{x}} = \vec{y}$  where  $\vec{y} = [5 \ 12 \ 7 \ 8]^T$ .

## 2. Identifying satellites and their delays

We are given the following two signals,  $\vec{s}_1$  and  $\vec{s}_2$  respectively, that are signatures for two satellites.



- (a) Your cellphone antenna receives the following signal  $r[n]$ . You know that there may be some noise present in  $r[n]$  in addition to the transmission from the satellite.



Which satellites are transmitting? What is the delay between the satellite and your cellphone? Use cross-correlation to justify your answer. You can use iPython to compute the cross-correlation.

- (b) Now your cellphone receives a new signal  $r[n]$  as below. What the satellites that are transmitting and what is the delay between each satellite and your cellphone?

