1. Orthonormal Matrices and Projections

An orthonormal matrix, $A$, is a matrix whose columns, $\vec{a}_i$, are:

- Orthogonal (ie. $\langle \vec{a}_i, \vec{a}_j \rangle = 0$ when $i \neq j$)
- Normalized (ie. vectors with length equal to 1, $\| \vec{a}_i \| = 1$). This implies that $\| \vec{a}_i \|^2 = \langle \vec{a}_i, \vec{a}_i \rangle = 1$.

(a) Suppose that the matrix $A \in \mathbb{R}^{N \times M}$ has linearly independent columns. The vector $\vec{y}$ in $\mathbb{R}^N$ is not in the subspace spanned by the columns of $A$. What is the projection of $\vec{y}$ onto the subspace spanned by the columns of $A$?

(b) Show if $A \in \mathbb{R}^{N \times N}$ is an orthonormal matrix then the columns, $\vec{a}_i$, form a basis for $\mathbb{R}^N$. 

(c) When $A \in \mathbb{R}^{N \times M}$ and $N \geq M$ (i.e. tall matrices), show that if the matrix is orthonormal, then $A^T A = I_{M \times M}$.

(d) Again, suppose $A \in \mathbb{R}^{N \times M}$ where $N \geq M$ is an orthonormal matrix. Show that the projection of $\vec{y}$ onto the subspace spanned by the columns of $A$ is now $AA^T \vec{y}$.

(e) Given $A \in \mathbb{R}^{N \times M} = \begin{bmatrix} 0 & 0 & \sqrt{2}/2 \\ 0 & 0 & \sqrt{2}/2 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and the columns of $A$ are orthonormal, find the least squares solution to $A\hat{x} = \vec{y}$ where $\vec{y} = [5 \quad 12 \quad 7 \quad 8]^T$. 
2. Identifying satellites and their delays

We are given the following two signals, $\vec{s}_1$ and $\vec{s}_2$ respectively, that are signatures for two satellites.

(a) Your cellphone antenna receives the following signal $r[n]$. You know that there may be some noise present in $r[n]$ in addition to the transmission from the satellite.

Which satellites are transmitting? What is the delay between the satellite and your cellphone? Use cross-correlation to justify your answer. You can use iPython to compute the cross-correlation.
(b) Now your cellphone receives a new signal $r[n]$ as below. What the satellites that are transmitting and what is the delay between each satellite and your cellphone?