EECS 16A Designing Information Devices and Systems I
Spring 2023 Exam Prep 3A

1. Campfire Smores (Fall 2019 Midterm 1 Question 3)

Patrick and SpongeBob are making smores.

There are three ingredients: **Graham Crackers, Marshmallows, and Chocolate**. To make a smore, SpongeBob needs: s_g Graham Crackers, s_m number of Marshmallows, and s_c Chocolate.

Ingredients	**Amount Needed**
Graham Crackers (s_g)	10
Marshmallows (s_m)	14
Chocolate (s_c)	20

Table 1: SpongeBob's smore

They find out that these ingredients are only stored in bundles as below:

Lobster Pack (p_l)
6 graham crackers
4 marshmallows
2 chocolates

Mr. Krabs Pack (p_k)
2 graham crackers
2 marshmallows
1 chocolates

Squidward Pack (p_s)
3 graham crackers
3 marshmallows
5 chocolates

Gary Pack (p_g)
1 graham crackers
4 marshmallows
5 chocolates

Pearl Pack (p_p)
2 graham crackers
3 marshmallows
2 chocolates

Table 2: Amount of Ingredients per Bundle

Spongebob and Patrick need to know how many of each bundle to buy: number of "Lobster" Packs, p_l, number of "Mr. Krabs" Packs, p_k, number of "Squidward" Packs, p_s, number of "Gary" Packs, p_g, and number of "Pearl" Packs, p_p.

(a) How many equations/constraints does the information in the problem provide you with?

(b) Based on the information provided in Tables 1 and 2, **write** an equation of the form $\mathbf{A}\vec{p} = \vec{s}$ that SpongeBob can use to decide how many of each pack to buy. Here, $\vec{p} = \begin{bmatrix} p_l \\ p_k \\ p_s \\ p_g \\ p_p \end{bmatrix}$.

(c) Now, the ingredients in the packets ($\mathbf{A}$) and Spongebob's receipe ($\vec{s}$) change. We have:
$\mathbf{A} = \begin{bmatrix} 1 & 1 & 3 & 2 & 2 \\ 0 & 1 & 3 & 0 & 2 \\ 1 & 3 & 9 & 2 & 6 \end{bmatrix}$, and $\vec{s} = \begin{bmatrix} 3 \\ 2 \\ 10 \end{bmatrix}$.
Find a $\vec{p}$ that satisfies $\mathbf{A}\vec{p} = \vec{s}$. If no solution exists, explain why not.

2. Matrix Multiplications (Spring 2022 Midterm 1 Question 6)

(a) The matrix $A \in \mathbb{R}^{500x501}$ is shown below

$$A = \begin{bmatrix} a_{1,1} & \cdots & a_{1,501} \\ \vdots & \ddots & \vdots \\ a_{500,1} & \cdots & a_{500,501} \end{bmatrix}$$

Given another matrix $B \in \mathbb{R}^{501x500}$, what are the dimensions of the matrix AB?

(b) (4 points) What are the dimensions of $((A^T A)B)^T$?

(c) Given that the elements of matrix A and B follow the pattern:

$$A = \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 \\ 0 & 2 & 0 & \cdots & 0 \\ 0 & 0 & 3 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \end{bmatrix} \qquad a_{i,j} = \begin{cases} i & i = j \\ 0 & i \neq j \end{cases}$$

$$B = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 2 & 1 & \cdots & 1 \\ 1 & 1 & 3 & \cdots & 1 \\ \vdots & \vdots & \vdots & \cdots & \end{bmatrix} \qquad b_{k,l} = \begin{cases} k & k = l \\ 1 & k \neq l \end{cases}$$

Find the element in the 4^{th} row and 4^{th} column of the matrix multiplication (AB).
In other words, what is $(AB)_{4,4}$?

(d) What is $(AB)_{4,5}$?

3. Geometric Transformations (Spring 2022 Midterm 1 Question 7)

(a) Write an expression for the transformation matrix that would reflect a vector across the line $y = -x$ and then rotate them by 45 degrees counterclockwise. Write your answer as some combination of the matrices below (ex: A*B).

$$\mathbf{A} = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}; \mathbf{B} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}; \mathbf{C} = \begin{bmatrix} \cos(-45^\circ) & -\sin(-45^\circ) \\ \sin(-45^\circ) & -\cos(-45^\circ) \end{bmatrix};$$

$$\mathbf{D} = \begin{bmatrix} \cos(45^\circ) & -\sin(45^\circ) \\ \sin(45^\circ) & \cos(45^\circ) \end{bmatrix}$$

(b) Consider a new transformation matrix T shown below.

$$T = \begin{bmatrix} -\cos(-60^\circ) & \sin(-60^\circ)) \\ \sin(-60^\circ) & \cos(-60^\circ) \end{bmatrix}$$

What transformation does T represent? Write your answer in terms of degrees rotated and/or reflection over an axis. Graph how this matrix transforms $\vec{v}_1 = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$. Do your best to approximate when necessary. All reasonable answers will be accepted.