EECS 16A Designing Information Devices and Systems I
Spring 2023 Exam Prep 4B

1. Matrix Madness (Fall 2022 Midterm 1 Question 5)

For the following subparts, consider the matrix $\mathbf{A} = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$.

(a) What is the value of a that satisfies the expression below?

$$Null(\mathbf{A}) = \text{span}\left(\begin{bmatrix} a \\ 2 \end{bmatrix}\right)$$

(b) What is the value of b that satisfies the new expression below?

$$Col(\mathbf{A}) = \text{span}\left(\begin{bmatrix} b \\ 1 \end{bmatrix}\right)$$

(c) An arbitrary matrix $\mathbf{B}$ satisfies the following equations:

$$\mathbf{B}\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$$

$$\mathbf{B}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

What is $\mathbf{B}\begin{bmatrix} 5 \\ -2 \end{bmatrix}$?

2. Nullspace (Spring 2022 Midterm Question 8)

(a) Consider the matrix below. What is the set of vectors that span the nullspace of A?

$$\mathbf{A} = \begin{bmatrix} 1 & -2 \\ -1 & -6 \end{bmatrix}$$

(b) Consider the matrix below. What is the set of vectors that span the nullspace of A?

$$\mathbf{A} = \begin{bmatrix} 3 & -6 & 2 \\ -2 & 4 & 2 \end{bmatrix}$$

(c) Consider the following matrix:

$$\mathbf{A} = \begin{bmatrix} (1-x) & 2 \\ 0 & (6+x) \end{bmatrix}$$

Find all values of x for which A has a non-trivial nullspace.

3. Matrix Multiplication Proof (Spring 2022 Midterm 1 Question 10)

(a) Given that Matrix A is square and has linearly independent columns, which of the following is true?

i. A is full rank

ii. A has a trivial nullspace

iii. $A\vec{x} = \vec{b}$ has a unique solution for all $\vec{b}$

iv. A is invertible

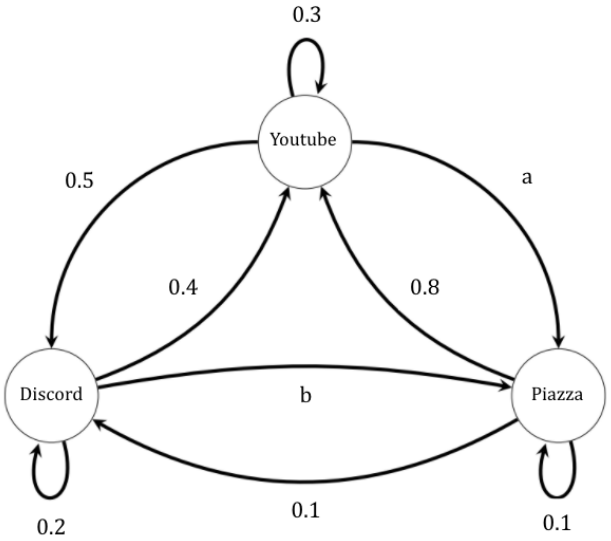
v. The determinant of A is non-zero

(b) Let two square matrices $M_1, M_2 \in \mathbb{R}^{2x2}$ each have linearly independent columns. Prove that $G = M_1 M_2$ also has linearly independent columns.

4. Transition matrix Part I (Fall 2021 Midterm 1 Question 16)

Prof. Arias decides to study the internet behavior of EECS 16A students on a typical weekday night. The number of students on the top three websites (YouTube, Piazza, and Discord) at time t can be expressed as follows: $x_y[t], x_p[t], x_d[t]$, respectively.

She finds that the flow of students across the three websites can be shown as follows:



(a) Let $\vec{x}[t] = \begin{pmatrix} x_y[t] \\ x_p[t] \\ x_d[t] \end{pmatrix}$ where $x_y[t], x_p[t], x_d[t]$ represent the number of students on Youtube, Piazza, and Discord at time t. Determine $\mathbf{A}$ such that $\vec{x}[t+1] = \mathbf{A}\vec{x}[t]$.

(b) Determine a and b such that the system is conservative.