### Reference Circuits

<table>
<thead>
<tr>
<th>Voltage Divider</th>
<th>Inverting Amplifier</th>
<th>Noninverting Amplifier</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram" /></td>
<td><img src="image2.png" alt="Diagram" /></td>
<td><img src="image3.png" alt="Diagram" /></td>
</tr>
<tr>
<td>$V_{R2} = V_S \left( \frac{R_2}{R_1+R_2} \right)$</td>
<td>$v_{out} = v_{in} \left( -\frac{R_f}{R_t} \right)$</td>
<td>$v_{out} = v_{in} \left( 1 + \frac{R_{top}}{R_{bottom}} \right)$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Current Divider</th>
<th>Inverting Amplifier with Reference</th>
<th>Noninverting Amplifier with Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image4.png" alt="Diagram" /></td>
<td><img src="image5.png" alt="Diagram" /></td>
<td><img src="image6.png" alt="Diagram" /></td>
</tr>
<tr>
<td>$I_1 = I_S \left( \frac{R_2}{R_1+R_2} \right)$</td>
<td>$v_{out} = v_{in} \left( -\frac{R_f}{R_t} \right) + V_{REF} \left( \frac{R_f}{R_t} + 1 \right)$</td>
<td>$v_{out} = v_{in} \left( 1 + \frac{R_{top}}{R_{bottom}} \right) - V_{REF} \left( \frac{R_{top}}{R_{bottom}} \right)$</td>
</tr>
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<table>
<thead>
<tr>
<th>Voltage Summer</th>
<th>Unity Gain Buffer</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image7.png" alt="Diagram" /></td>
<td><img src="image8.png" alt="Diagram" /></td>
</tr>
<tr>
<td>$V_{out} = V_1 \left( \frac{R_2}{R_1+R_2} \right) + V_2 \left( \frac{R_1}{R_1+R_2} \right)$</td>
<td>$v_{out} = v_{in}$</td>
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</table>
Analyzing Ckts

Equations
1. KVL, KCL
2. Ohm's Law
3. Q = CV, I_c = C \frac{dV}{dt}
4. Golden Rules: I^+ = I^- = 0
   V^+ = V^- in steady state

Do algebra

Design: using the blocks we have seen to solve problems
1. Review: Design Procedure

Now that we’ve analyzed many circuits, we are ready to focus on designing interesting circuits to perform specific tasks. Circuit design is generally more challenging than circuit analysis. Usually, we are given a prompt about what type of circuit is desired and some specifications, and then we must decide what components to use and how to connect them. Circuit design problems are very similar to proof problems, which we looked at earlier in this course: Both cases are open ended, and we frequently have to integrate many ideas and explore several different possibilities to reach a solution.

When faced with a design problem, a good place to start is to follow the design procedure outlined here:

**Step 1 (Specification):** Concretely restate the goals for the design.

Frequently, a design prompt will include a lot of text, so we’d like to restate all of the most important features of our design. We’ll refer to these specifications later to determine if our design is complete.

**Step 2 (Strategy):** Describe your strategy (often in the form of a block diagram) to achieve your goal.

To do this, start by thinking about what you can measure vs. what you want to know. For example, in our capacitive touchscreen, we want to know if there is a touch and we can measure voltage. Since we know that a touch can change the capacitance, we break this down into the following block diagram:

```
| touch/no touch | Convert touch to capacitance | Convert capacitance to voltage | voltage |
```

**Step 3 (Implementation):** Implement the components described in your strategy.

This is where pattern matching is useful: remind yourself of blocks you know, (ex. voltage divider, inverting amplifier) and check if any of these can be used to implement steps of your strategy. If you don’t know of a block that does what you want, think about how to modify or extend the blocks you know.

**Step 4 (Verification):** Check that your design from Step 3 does what you specified in Step 1.

It’s tempting to think that you’re done after implementation, but verification is critical! In particular, check block-to-block connections, as these are the most common point for problems. Does one block load another block causing it to behave differently than expected? Are there any contradictions (ex. a voltage source with both ends connected by a wire, or a current source directed into an open circuit)? Repeat previous steps if necessary to make sure that your final circuit meets the specifications.
2. Noise Cancelling Headphones

The basic goal of a noise cancelling headphones is for the user to hear only the desired audio signal and not any other sounds from external sources. In order to achieve this goal, noise cancelling headphones include at least one microphone that listens to what you might have otherwise heard from external sources, and then feeds a signal into your speakers that cancels (subtracts out) that externally-generated sound.

If you feel like you are getting lost in this problem, try to return to the design procedure steps. Have you (re)written down the goal of your circuit? Can you describe from a high level what blocks you may need? Have you checked the reference circuits to see what you may be able to build?

(a) Let’s start by looking at the most basic part of the headphones: driving the speaker itself with the audio stream we would like to hear. In our system, the source of the audio comes from a digital-to-analog converter (DAC) that translates digital bits to analog voltages. It can be modeled as a voltage source with min/max values of 0V and 1V and a 50Ω source resistance. The speaker can be modeled as an 8Ω resistor, but in order to produce loud enough sounds and not damage the speaker (driving the speaker with non-zero average voltage can damage the transducer within the speaker), it needs to be driven with a range of −1.5V to 1.5V (relative to the ground connected to the DAC, which is the same ground used throughout the system).

You are provided two voltage sources with values −1.5V and 1.5V, an op-amp, and any resistors you would like. Design a circuit that could drive the speaker while meeting the specifications above. **Hint:** **shifting a signal is the same as adding a fixed value to it.**

Suggested procedure:

i. Specification: restate the goals for this circuit.
ii. Strategy: Draw a block diagram to achieve your goals. Think about what you can measure and what circuits you have seen, e.g. adders and multipliers. Try to break down the goals into multiple steps.
iii. Implementation: Build each of the blocks you described above. Use the reference circuits and pattern match.
iv. Verification: Put your circuit together and make sure it works. Especially look at when you put blocks together, and watch out for loading effects.
1. Specification in out
   
   **Input:** 0 - 1V (04C)  
   **Output:** -1.5V - 1.5V (speaker)

![Block diagram]

2. Strategy ~ Block diagram
   
   2 steps:  - amplify range  
   - shift center
   
   Does the order matter?
   
   If we amplify first.
   
   0 - 1V \rightarrow 0 - 3V
   
   Can't do this. we are limited to \pm 1.5V

   \[
   V_{in} \xrightarrow{\text{shift}} \xrightarrow{\text{amplify}} V_{out}
   \]

   Will we have the same range as before \([-0.5, 0.5]\)? see below
Implementation:

i) Level-shift
\[ V_x = \frac{R_2}{R_1+R_2} V_1 + \frac{R_1}{R_1+R_2} V_2 \]

\[ V_x = \frac{R_2}{R_0+R_2} \times 0.5V + \frac{R_0}{R_0+R_2} \times (-1.5V) \]

\[ 0 = R_2 (0.5V) + R_0 (-1.5V) \]

\[ \Rightarrow R_2 = \frac{3 \times 0.5V}{0.5} = 150.52 \]

New range?

\[ V_{in} = 0V \quad V_x = \frac{150}{80+150} \times 0V + \frac{80}{80+150} \times (-1.5V) \]

\[ = 0V + \frac{1}{4} \times (-1.5V) \]

\[ = -375mV = \frac{3}{8} V \]

\[ V_{in} = 1V \quad V_x = \frac{3}{8} \times (1V) + \frac{5}{8} \times (-1.5V) \]

\[ = 375mV = \frac{3}{8} V \]

Range: \[ \frac{3}{8} + \frac{3}{8} = \frac{3}{4} V \quad \text{vs} \quad 1V \text{ swing} \]
i) Amplity

\[
\begin{align*}
\text{Scaling by } \# > 1 \Rightarrow \text{non-inverting amplifier}
\end{align*}
\]

\[V_{\text{out}} = (1 + \frac{R_1}{R_2}) V_x\]

\[1 + \frac{R_1}{R_2} = 4\]

\[R_1 = 3R_2\]

\[R_2 = 1k\Omega\]

\[R_1 = 3k\Omega\]

(4) Verification

put it together & check

- Does the math work out?

\[V_x = \frac{3}{4} V_{\text{in}} - \frac{1}{4} \cdot 1.5V\]

\[V_{\text{out}} = 4V_x\]

\[V_{\text{out}} = 3V_{\text{in}} - 1.5V \quad \text{yay}\]

- Do the stages interface correctly? \Rightarrow loading
Now let’s look at implementing the noise cancellation. In this problem, we will assume that we do not have access to software and therefore cannot digitally remove the noise (as do most noise cancelling headphones). We will therefore focus on implementing the cancellation physically, which is to directly take the (analog) voltage produced by the microphone and subtract it out from the voltage we feed to the speaker.

Let’s assume that the microphone can be modeled as a voltage source with min/max values of 0V and 1V (relative to the DAC’s ground) and a 10kΩ source resistance. However, because the materials in the headphones attenuate some of the external sound, the loudest signals picked up by the microphone should correspond to a voltage range of only −125mV to +125mV driven onto the speaker.

Design a circuit that takes in the signal from the microphone and outputs the appropriate signal driven to the speaker. We will assume the following circuit will sum this signal and the one from the previous part to accomplish noise cancellation, so we want this circuit to be invert. You can use op-amps and resistors to do this, but no new voltage sources (except for the model of the microphone of course).
1. Specification

<table>
<thead>
<tr>
<th>In</th>
<th>0 - 1V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Out</td>
<td>-125mV - 125mV, invert</td>
</tr>
</tbody>
</table>

Not told the output of the circuit
Try to put an op-amp at output to prevent loading.

2. Strategy

3 steps:
- Level shift (center at 0)
- Amplify (maybe < 1 ?)
- Invert

3. Implementation

1) Level shift

\[ V_x = \frac{30k}{10k+30k} V_{in} + \frac{10k}{10k+30k} (-1.5V) \]

\[ = \frac{3}{7} V_{in} + \frac{1}{7} (-1.5V) \]

Range: -3/8 V - 3/8 V = -375mV to 375mV
ii) Amplify (+ Invert)

in  -3.75 to 3.75
out +125  -125  Amplify by $\frac{1}{3}$

Scaling + Inverting $\rightarrow$ Inverting amplifier

$$\text{Vout} = \left(-\frac{R_2}{R_1}\right)\text{Vin}$$

$$\frac{R_2}{R_1} = \frac{1}{3}$$
$$R_2 = 1k\Omega$$
$$R_1 = 3k\Omega$$

Verification

Rule of Thumb: b/w blocks, if you don't have the input or the output of an op-amp, add a buffer
(c) (Optional)

We now have a circuit that takes in the DAC voltage and produces the correct voltage for the speaker, and a second circuit that takes in the microphone voltage and produces the correct voltage for the speaker. Now we simply have to add these together.

Design a circuit that adds these two circuits together. We don’t need any gain from this circuit, but since we are driving the 8Ω speaker, we may want to have some isolation between the speaker and these first two circuits.

You may use op-amps and resistors to complete this circuit. Note that since our speaker driver now needs to handle both the cancellation and the desired audio signal, you can assume that the supply voltages fed to the op-amp have sufficiently large magnitude to ensure that they never clip (reach the power rails). In other words, you should continue to assume that you have ±1.5 V voltage sources available to use in the rest of your circuit, but the op-amps are now supplied by a separate set of larger, sufficient voltage sources.