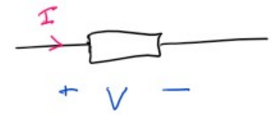
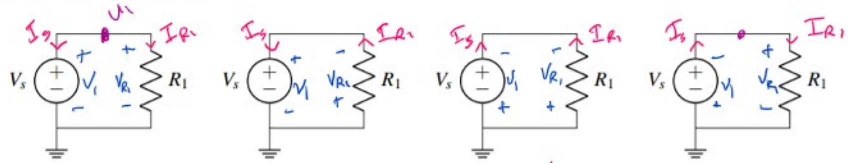
tinyurl.com/miyuki-feedback

1. Passive Sign Convention and Power

passive sign convention



(a) We have made four copies of a circuit below. Following passive sign convention, there are four different possible labelings of current directions and voltage polarities for the circuit. For each copy, label each circuit's voltage source and resistor with current direction and voltage polarity labelings, keeping with passive sign convention.

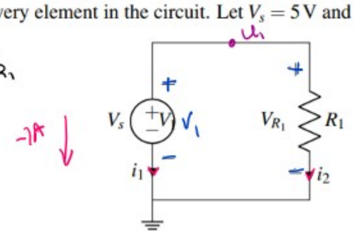


all are equally valid

(b) Suppose we consider one of the possible labelings you have found above. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5\,V$ and let $R_1 = 5\,\Omega$.

Power: rate of change of energy

find i_1, i_2, V_1, V_{R_1}



power dissipated by a circuit element → $P = IV$ $\quad$ * $V = IR$

$= I^2 R$
$= \frac{V^2}{R}$ } only applies to resistors

KCL @ u_1: $0 = i_1 + i_2 \quad i_1 = -i_2$

element eqns: $V_{R_1} = i_2 R_1 = u_1 - 0$

$V_1 = u_1 - 0 = V_s$

solve: $u_1 = V_s = 5V \qquad i_2 = \frac{V_{R_1}}{R_1} = \frac{5V}{5\Omega} = 1A$

$V_1 = u_1 = 5V$ $\qquad i_1 = -i_2 = -1A$

$V_{R_1} = 5V$

Power

$P_{R_1} = IV = i_2 V_{R_1} = (1A)(5V) = 5W$

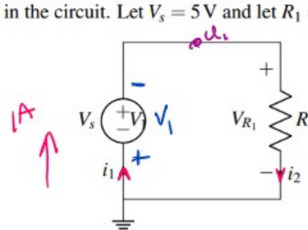
* resistors can't supply power

$P_{V_s} = IV = i_1 V_1 = (-1A)(5V) = -5W$

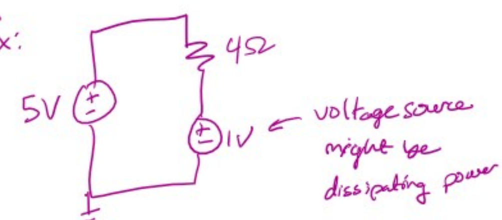
power is supplied

* negative value means power is supplied
positive value means power is dissipated

(c) Suppose we choose a second labeling of the circuit as shown below. Calculate the power dissipated or supplied by every element in the circuit. Let $V_s = 5\,V$ and let $R_1 = 5\,\Omega$.



Ex:



KCL @ u_1: $i_1 = i_2$

element eqns:

$V_1 = 0 - u_1 = -V_s$

$V_s = u_1 - 0$

$V_{R_1} = i_2 R_1 = u_1 - 0$

solve:

$V_1 = -5V$

$V_{R_1} = 5V$

$i_2 = \frac{V_{R_1}}{R_1} = \frac{5V}{5\Omega} = 1A$

$i_1 = i_2 = 1A$

Power

$P_{R_1} = IV = i_2 V_{R_1} = (1A)(5V) = 5W$

$P_{V_s} = IV = i_1 V_1 = (1A)(-5V) = -5W$

(d) Did the values of the element voltages and element currents change with the different labeling? Did the power for each circuit element change? Did the node voltages change? If a quantity didn't change with a difference in labeling, discuss what would have to change for quantity to change.

Power stayed the same

node voltages stayed the same

element voltages/currents did change

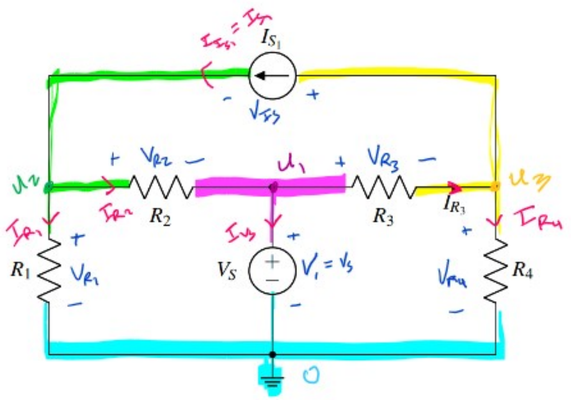
2. Circuit Analysis

Provided the circuit below...

(a) ...use nodal analysis to solve for all node voltages.

(b) ...find the current I_{R_3} flowing through resistor R_3.

- **Step 1:** Select a reference (ground) node. Any node can be chosen for this purpose. We will measure all of the voltages in the rest of the circuit relative to this point.
- **Step 2:** Label all nodes with voltage set by voltage sources.
- **Step 3:** Label remaining nodes.
- **Step 4:** Label element voltages and currents, following **Passive Sign Convention** (discussed below).



* purpose of NVA is to find all node voltages

→ once we know these we can find any other unknowns pretty easily

* $V_S = V_1 = u_1 - 0 \quad u_1 = V_S$

Step 5: Write KCL equations for all nodes with unknown voltages.

KCL @ u_2: $I_{S_1} = I_{R_1} + I_{R_2}$ ①

u_3: $I_{R_3} = I_{S_1} + I_{R_4}$ ②

Step 6: Find expressions for all element currents in terms of element voltages and characteristics.

Ohm's Law:

$V_{R_1} = I_{R_1} R_1 \qquad V_{R_2} = I_{R_2} R_2 \qquad V_{R_3} = I_{R_3} R_3 \qquad V_{R_4} = I_{R_4} R_4$

$I_{R_1} = \frac{V_{R_1}}{R_1} \qquad I_{R_2} = \frac{V_{R_2}}{R_2} \qquad I_{R_3} = \frac{V_{R_3}}{R_3} \qquad I_{R_4} = \frac{V_{R_4}}{R_4}$

Step 7: Substitute all element voltages with node voltages found in your step 6 equations.

$V_{R_1} = u_2 - 0 \qquad V_{R_2} = u_2 - u_1 = u_2 - V_S \qquad V_{R_3} = u_1 - u_3 = V_S - u_3 \qquad V_{R_4} = u_3 - 0$

$I_{R_1} = \frac{u_2}{R_1} \qquad I_{R_2} = \frac{u_2 - V_S}{R_2} \qquad * \; I_{R_3} = \frac{V_S - u_3}{R_3} \qquad I_{R_4} = \frac{u_3}{R_4}$

Step 8: Substitute expressions found in step 7 into the KCL equations from step 5.

① $I_{S_1} = I_{R_1} + I_{R_2} \rightarrow I_{S_1} = \frac{u_2}{R_1} + \frac{u_2 - V_S}{R_2}$

② $I_{R_3} = I_{S_1} + I_{R_4} \rightarrow \frac{V_S - u_3}{R_3} = I_{S_1} + \frac{u_3}{R_4}$

} 2 eqns, 2 unknowns (u_2, u_3)

Step 9: Solve for the node voltage values.

$$u_2 = \frac{V_S R_1 + I_{S_1} R_1 R_2}{R_1 + R_2}$$

$$u_3 = \frac{V_S R_4 - I_{S_1} R_4 R_3}{R_4 + R_3}$$

(b) ...find the current I_{R_3} flowing through resistor R_3.

$$I_{R_3} = \frac{V_S - u_3}{R_3} = \frac{V_S - \frac{V_S R_4 - I_{S_1} R_4 R_3}{R_4 + R_3}}{R_3} \longrightarrow I_{R_3} = \frac{V_S + I_S R_4}{R_3 + R_4}$$