1. Resistive Touchscreen

In this question we will be re-examining the 2-dimensional resistive touchscreen. This touchscreen is slightly different to the one shown in lecture and more like the one we will be examining in lab.

The touchscreen has length $L$ and width $W$ and is composed of a rigid bottom-layer and a flexible top-layer. Instead of having a two continuous resistive sheets on the top and bottom layers, this is a simpler implementation with $N$ vertical strips of conductive material in the top layer and $N$ horizontal strips of conductive material in the bottom layer. The strips of a single layer are all connected by an ideal conducting plate on each side. All strips have resistivity $\rho$, and cross-sectional area $A$.

Assume that all top layer resistive strips and bottom layer resistive strips are spaced apart equally, and that the upper left touch point in Figure 1(b) is position $(1,1)$, and the upper right touch point is $(W,1)$. The spacing between the strips in the top layer is $\frac{W}{N}$, and the spacing between the strips in the bottom layer is $\frac{L}{N}$.

(a) Find the resistance $R_x$ for a single vertical strip and $R_y$ for a single horizontal red strip, as a function of the screen dimensions $W$ and $L$, the strip resistivity $\rho$, and the cross-sectional area $A$.

\[ R_x = \rho \frac{W}{A} \]

\[ R_y = \rho \frac{L}{A} \]
(b) Consider a 2 x 2 example for the touchscreen circuit shown in Figure 2.
Assume that we connect a voltage source $V_s$, between the top and bottom terminals of the blue strips, and a voltmeter $V_m$ to one of the left or right terminals as depicted in the diagram. If $V_s = 3V$, $R_y = 2000\Omega$, and $R_y = 2000\Omega$, draw the equivalent circuit for when the point (2, 2) is pressed and solve for the measured voltage $V_m$ with respect to ground.
Reminder: all top layer resistive strips and bottom layer resistive strips are spaced apart equally, and that the upper left touch point is position (1, 1). The spacing between the strips in the top layer is $\frac{2L}{N^2}$, and the spacing between the strips in the bottom layer is $\frac{2L}{N}$.

\[ V_m = \frac{V_s}{3} \]

(c) Suppose a touch occurs at coordinates $(i, j)$ for an arbitrary $N \times N$ touchscreen, and the voltage source and meter are connected as in the figures. A 3 x 3 example is shown in Figure 1(b). Find an expression for $V_m$ as a function of $V_s$, $N$, $i$, and $j$. Again, the upper left corner is the coordinate $(1, 1)$ and the upper right coordinate is $(N, 1)$.

\[ V_m = \frac{N+1-j}{N+1} \frac{R_y}{N} \]

(d) Optional / Fun: Experiment with the TinkerCad models below to validate the theoretical results you just derived.
TinkerCad model of 2 x 2 equivalent circuit: https://www.tinkercad.com/things/56DxZ3MD7B
TinkerCad model of 3 x 2 equivalent circuit: https://www.tinkercad.com/things/koOjuJ/n/UE
2. Volt and Ammeter

(a) For the voltage divider below, how would we connect a voltmeter to the circuit to measure the voltage \( V_{R_2} \)?

\[
V_{\text{meas}} = V_a - V_b = V_{R_2}
\]

(b) What would happen if we accidentally connected an ammeter in the same configuration instead? Assume our ammeter is ideal.

\[
\begin{align*}
V_a &= V_b \\
V_{R_2} &= V_a - V_b = 0 \\
I_{R_2} &= \frac{V_{R_2}}{R_2} = \frac{0}{R_2} = 0
\end{align*}
\]

KCL @ \( V_a \): \( I_{R_1} = I_{\text{meter}} \)

KCL @ \( V_b \): \( I_{\text{meter}} = I_{R_2} \)

(c) For the current divider below, how would we connect an ammeter to the circuit to measure the current \( I_{R_2} \)?

\[
I_{\text{meas}} = \frac{V_b}{R_2} = \frac{V_3}{R_2}
\]

(d) What would happen if we accidentally connected a voltmeter in that configuration instead? Assume the voltmeter is ideal.

\[
\begin{align*}
V_a &= V_0 \\
V_{R_2} &= V_a - V_b = 0 \\
I_{R_2} &= \frac{V_{R_2}}{R_2} = \frac{0}{R_2} = 0
\end{align*}
\]

KCL @ \( V_0 \): \( I_{R_2} = I_{\text{meter}} \)

\[
V_{\text{meas}} = V_a - V_b = V_3 = I_{R_2} R_2 = I_3 R_2
\]