Dis 8A EECS16A

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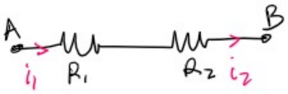
Series Resistors

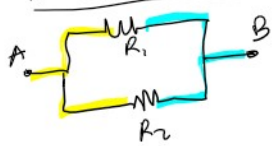


$i_1 = i_2$

same current goes through resistors

$R_{eq} = R_1 + R_2 + \cdots$

Parallel Resistors

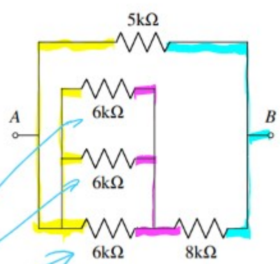


same nodes on either side of the resistors → same voltage

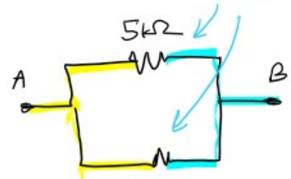
$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots\right)^{-1}$$

1. Series and Parallel Combinations

For the resistor network shown below, find an equivalent resistance between the terminals A and B using the resistor combination rules for series and parallel resistors.



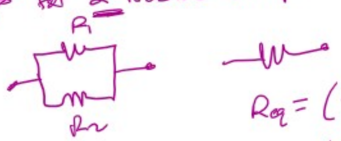
$$R_{eq} = \left(\frac{1}{6k\Omega} + \frac{1}{6k\Omega} + \frac{1}{6k\Omega}\right)^{-1} = \left(\frac{3}{6k\Omega}\right)^{-1} = 2k\Omega$$



$R_{eq} = 2k\Omega + 8k\Omega = 10k\Omega$

$$R_{eq} = \left(\frac{1}{5k\Omega} + \frac{1}{10k\Omega}\right)^{-1} = \left(\frac{3}{10k\Omega}\right)^{-1} = \frac{10k\Omega}{3} = \boxed{3.33\,k\Omega}$$

→ for 2 resistors in parallel



$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \left(\frac{R_1 + R_2}{R_1 R_2}\right)^{-1} = \frac{R_1 R_2}{R_1 + R_2}$$

Superposition

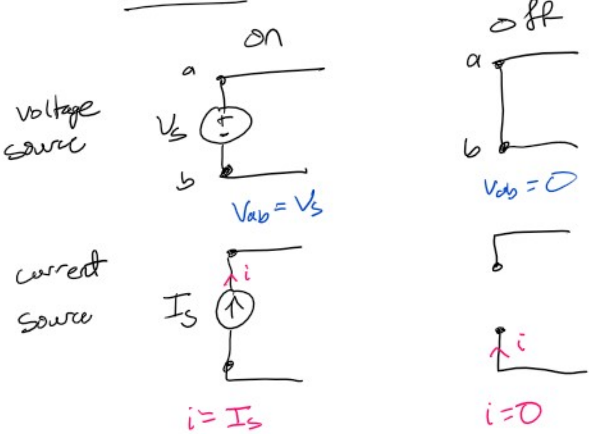
* useful for circuits w/ multiple sources

$V_{out} = V_{out, s_1 \text{ only}} + V_{out, s_2 \text{ only}} + \cdots$

desired voltage (could be a current as well)

desired voltage with only one source on

sources



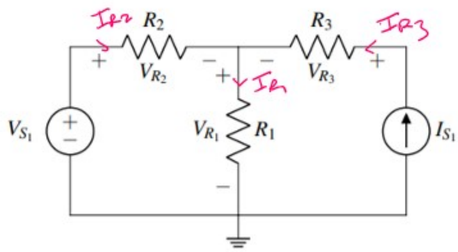
2. Superposition

For the following circuits:

i. Use the superposition theorem to solve for the voltages across the resistors.

ii. For parts (b) and (c) only, find the power dissipated/generated by all components. Is power conserved?

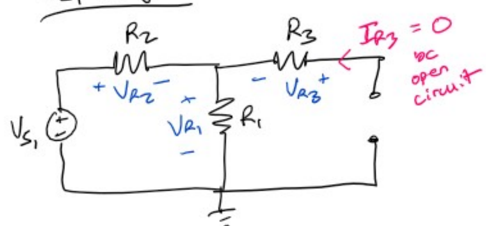
(a)



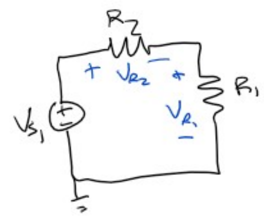
superposition

$$\begin{cases} V_{R1} = V_{R_1, V_{S_1} \text{ only}} + V_{R_1, I_{S_1} \text{ only}} \\ V_{R2} = V_{R_2, V_{S_1} \text{ only}} + V_{R_2, I_{S_1} \text{ only}} \\ V_{R3} = V_{R_3, V_{S_1} \text{ only}} + V_{R_3, I_{S_1} \text{ only}} \end{cases}$$

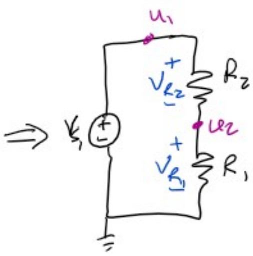
V_{S_1} only



$\Rightarrow$



$\Rightarrow$

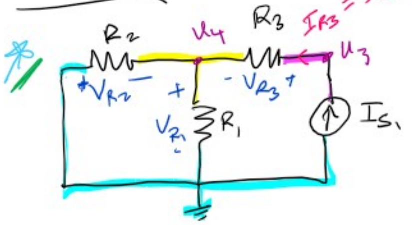


$$V_{R_1, V_{S_1} \text{ only}} = \frac{R_1}{R_1 + R_2} V_{S_1}$$

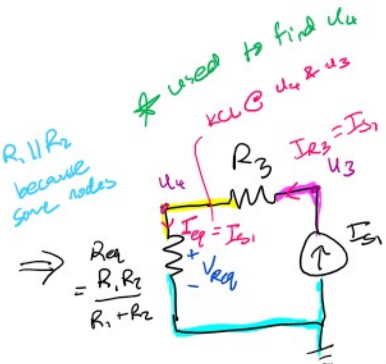
$$V_{R_2, V_{S_1} \text{ only}} = u_1 - \underbrace{u_2}_{V_{R_1}} = V_{S_1} - \frac{R_1}{R_1 + R_2} V_{S_1} = \frac{R_2}{R_1 + R_2} V_{S_1}$$

$$V_{R_3, V_{S_1} \text{ only}} = I_{R_3} R_3 = 0$$

I_{S_1} only



$\Rightarrow$



$$V_{Req} = u_4 - 0 = I_{eq} R_{eq} = I_{S_1} \frac{R_1 R_2}{R_1 + R_2}$$

$$u_4 = I_{S_1} \frac{R_1 R_2}{R_1 + R_2}$$

$$V_{R_3, I_{S_1} \text{ only}} = I_{R_3} R_3 = I_{S_1} R_3$$

from original circuit

$$V_{R_1, I_{S_1} \text{ only}} = u_4 - 0 = I_{S_1} \frac{R_1 R_2}{R_1 + R_2}$$

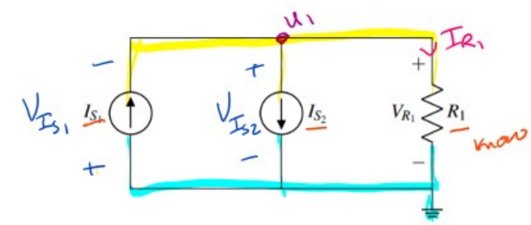
$$V_{R_2, I_{S_1} \text{ only}} = 0 - u_4 = -I_{S_1} \frac{R_1 R_2}{R_1 + R_2}$$

$$\boxed{V_{R_1} = \frac{R_1}{R_1 + R_2} V_S + I_{S_1} \frac{R_1 R_2}{R_1 + R_2}}$$

$$\boxed{V_{R_2} = \frac{R_2}{R_1 + R_2} V_{S_1} - I_{S_1} \frac{R_1 R_2}{R_1 + R_2}}$$

$$\boxed{V_{R_3} = I_{S_1} R_3}$$

(b)



i. KCL @ u_1: $I_{S_1} = I_{S_2} + I_{R_1}$

$$I_{R_1} = I_{S_1} - I_{S_2}$$

$$V_{R_1} = I_{R_1} R_1 = \boxed{(I_{S_1} - I_{S_2})R_1} = u_1 - 0$$

$$V_{I_{S_1}} = 0 - u_1 = -V_{R_1}$$

$$V_{I_{S_2}} = u_1 - 0 = V_{R_1}$$

ii. $P_{R_1} = I_{R_1} V_{R_1} = (I_{S_1} - I_{S_2})(I_{S_1} - I_{S_2})R_1$

$$P_{I_{S_1}} = I_{S_1} V_{I_{S_1}} = -I_{S_1}(I_{S_1} - I_{S_2})R_1$$

$$P_{I_{S_2}} = I_{S_2} V_{I_{S_2}} = I_{S_2}(I_{S_1} - I_{S_2})R_1$$

is Power conserved?

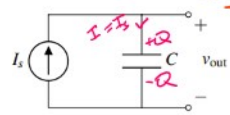
$$P_{R_1} + P_{I_{S_1}} + P_{I_{S_2}} = 0$$

plug in to show $= 0$

3. Current Sources And Capacitors

Given the circuit below, find an expression for $v_{out}(t)$ in terms of I_s, C, V_0, and t, where V_0 is the initial voltage across the capacitor at $t = 0$.

HINT: Recall the calculus identity $\int_a^b f'(x)dx = f(b) - f(a)$, where $f'(x) = \frac{df}{dx}$.



for resistors: $V = IR$

for capacitors: $Q = CV$

charge on a plate — capacitance — voltage across cap

$$I = \frac{dQ}{dt}$$

$$Q = C V_{out}$$

$$I = \frac{dQ}{dt} = \frac{d}{dt}(\overset{\text{const}}{C} V_{out}) = C\frac{dV_{out}}{dt}$$

$$\int_0^t \underbrace{\frac{dV_{out}}{dt}}_{V_{out}'(t)} dt = \int_0^t \frac{I}{C}\, dt \Rightarrow V_{out}(t) - \underbrace{V_{out}(0)}_{V_0} = \frac{I}{C}(t-0)$$

$$\boxed{V_{out} = V_0 + \underbrace{\frac{I_s}{C}}\, t}$$

units

$Q = CV$

coulombs → $[C] = [F \cdot V]$ ← farads

$$[F] = \frac{[C]}{[V]}$$

$I = \frac{dQ}{dt}$

$$[A] = \frac{[C]}{[t]}$$

$$\frac{[A]}{[F]} = \frac{\frac{[C]}{[t]}}{\frac{[C]}{[V]}} = \frac{[V]}{[t]}$$

Then plot the function $v_{out}(t)$ over time on the graph below for the following conditions detailed below. Use the values $I_s = 1\text{mA}$ and $C = 2\mu\text{F}$.

(a) Capacitor is initially uncharged $V_0 = 0$ at $t = 0$.

(b) Capacitor has been charged with $V_0 = +1.5\text{V}$ at $t = 0$.

(c) **Practice:** Swap this capacitor for one with half the capacitance $C = 1\mu\text{F}$, which is initially uncharged $V_0 = 0$ at $t = 0$.

a) $V_{out}(t) = 0 + \frac{1\text{mA}}{2\mu\text{F}} t = \left(\frac{1}{2}\frac{\text{V}}{\text{ms}}\right)t$

b) $V_{out}(t) = 1.5\text{V} + \left(\frac{1}{2}\frac{\text{V}}{\text{ms}}\right)t$

c) $V_{out}(t) = 0 + \left(\frac{1\text{mA}}{1\mu\text{F}}\right)t = \left(1\frac{\text{V}}{\text{ms}}\right)t$