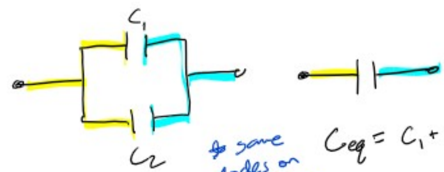
Dis 8B EECS16A

tinyurl.com/miyuki-feedback

Parallel




* same nodes on either side

$C_{eq} = C_1 + C_2 + \cdots$

Series

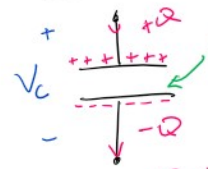

* for resistors have same current

* for capacitors have same charge

$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \cdots\right)^{-1}$

different than formula for series resistors

in physics




physical gap

+Q charge builds on top plate

-Q charge builds on bottom plate

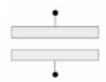
* inducing a current on the bottom

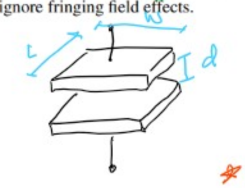
## 1. Capacitance Equivalence

For the structures shown below, assume that the plates have a depth $L$ into the page and a width $W$ and are always a distance $d$ apart. The dielectric between the plates has absolute permitivity $\varepsilon$. For the following calculations, assume the capacitance is purely parallel plate, i.e. ignore fringing field effects.

(a) What is the capacitance of the structure shown below?

$\varepsilon$ ← permittivity

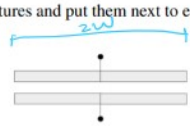





$C_a = \varepsilon \frac{A}{d} = \varepsilon \frac{LW}{d}$

* $R = \rho \frac{L}{A}$

(b) Suppose that we take two such structures and put them next to each other as shown below. What is the capacitance of this new structure?

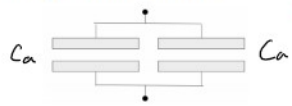



$C_b = \varepsilon \frac{A}{d} = \varepsilon \frac{L(2W)}{d} = 2C_a$

same

(c) Now suppose that rather than connecting the together as shown above, we connect them with an ideal wire as shown below. What is the capacitance of this structure?

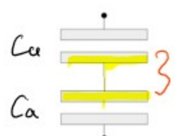
* parallel




$C_c = C_a + C_a = 2C_a = 2\varepsilon \frac{LW}{d} = C_b$

(d) Suppose that we now take two capacitors and connect them as shown below. What is the capacitance of the structure?

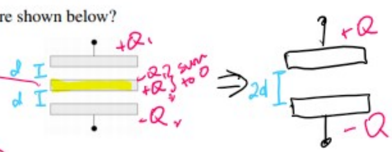
* series




$C_d = \left(\frac{1}{C_a} + \frac{1}{C_a}\right)^{-1} = \frac{C_a C_a}{C_a + C_a} = \frac{C_a^{\cancel{2}}}{2\cancel{C_a}} = \frac{C_a}{2} = \varepsilon \frac{LW}{2d}$

same

(e) What is the capacitance of the structure shown below?




net charge on middle plate is zero

$Q_1 = Q_2$

$C_e = \varepsilon \frac{LW}{2d} = C_d$

**2. Voltages Across Capacitors**

For the circuits given below, determine the voltage across each capacitor and calculate the charge and energy stored on each capacitor (assume all capacitors start ***uncharged***, and then we've let the system reach steady state). We are also given $C_1 = 1\,\mu\text{F}$, $C_2 = 3\,\mu\text{F}$, and $V_s = 1\,\text{V}$.

Recall charge has units of Coulombs (C), and capacitance is measured in Farads (F) $= \frac{\text{Coulomb}}{\text{Volt}}$.
It may also help to note metric prefix examples: $3\mu\text{F} = 3\times10^{-6}\text{F}$.

* Energy stored in a capacitor

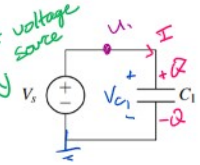
$$E = \tfrac{1}{2} C V^2$$

↑ voltage across cap

$$Q = CV$$

↑ charge ↑ capacitance ↑ voltage

(a)




$$V_{C_1} = u_1 - 0 = V_s$$

$$Q = C_1 V_{C_1} = (1\mu\text{F})(1\text{V}) = 1\mu\text{C}$$

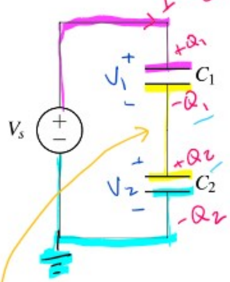
$$E = \tfrac{1}{2} C_1 V_{C_1}^2 = \tfrac{1}{2}(1\mu\text{F})(1\text{V})^2 = \tfrac{1}{2}\mu\text{J}$$

↙ Joule unit for energy

$$I = \frac{dQ}{dt} = C_1 \cancelto{0}{\frac{dV_{C_1}}{dt}} = 0$$

* steady state: current is no longer flowing
→ no time dependence

(b)

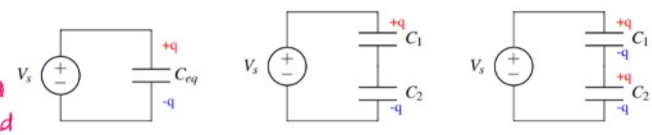



* capacitors are initially uncharged

← know net charge on floating node is 0 before $V_s$ is connected because caps are initially uncharged
→ net charge on floating node must be 0 after $V_s$ is connected b/c charge is conserved

floating node: node where charge is trapped

**Helpful diagrams** for considering the charges capacitors linked in series:
(without any initial charges)




**Left:** Our series capacitors may be modeled as one equivalent capacitor $C_{eq}$, which after some time is charged up by $V_s$ to have $+q$ on the top plate and $-q$ on the bottom plate.

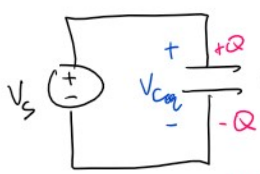
**Middle:** We return to the 2-capacitor picture, but carry this insight of equivalent charge with us. Now the charge $+q$ is on the top plate *of capacitor* $C_1$, and $-q$ is on the bottom plate *of capacitor* $C_2$.

**Right:** Since capacitor plates have opposite & equal charges, we attain this final right digram.

As another conceptual check, we notice that the node between $C_1$ and $C_2$ is isolated from any other connections and should always remain *charge neutral*. From the diagram right we see this is maintained since $(+q)+(-q) = 0$.

$$-Q_1 + Q_2 = 0$$
$$Q_1 = Q_2$$

→ tells us that the caps are in series!!




$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \frac{C_1 C_2}{C_1 + C_2}$$

$$Q = Q_1 = Q_2$$

$$V_{C_{eq}} = V_s$$

$$Q = C_{eq} V_{C_{eq}} = \frac{C_1 C_2}{C_1 + C_2} V_s = \left(\tfrac{3}{4}\mu\text{F}\right)(1\text{V}) = \tfrac{3}{4}\mu\text{C}$$

back to original circuit:

$$Q = Q_1 = Q_2$$

$$Q_1 = C_1 V_1 \qquad V_1 = \frac{Q_1}{C_1} = \frac{\frac{3}{4}\mu\text{C}}{1\mu\text{F}} = \tfrac{3}{4}\text{V}$$

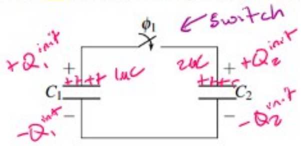
$$Q_2 = C_2 V_2 \qquad V_2 = \frac{Q_2}{C_2} = \frac{\frac{3}{4}\mu\text{C}}{3\mu\text{F}} = \tfrac{1}{4}\text{V}$$

$$E_1 = \tfrac{1}{2} C_1 V_1^2 = \tfrac{1}{2}(1\mu\text{F})\left(\tfrac{3}{4}\text{V}\right)^2$$

$$E_2 = \tfrac{1}{2} C_2 V_2^2 = \tfrac{1}{2}(3\mu\text{F})\left(\tfrac{1}{4}\text{V}\right)^2$$

**3. Capacitors and Charge Conservation**

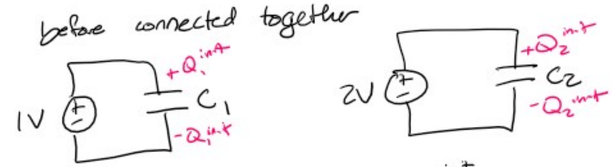
(a) Consider the circuit below with $C_1 = C_2 = 1\,\mu\text{F}$ and an open switch. Suppose that $C_1$ is initially charged to $+1\,\text{V}$ and that $C_2$ is charged to $+2\,\text{V}$. How much charge is on $C_1$ and $C_2$?




open (open circuit)

closed (wire)

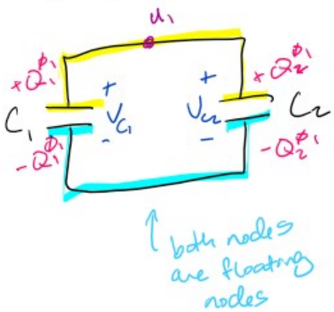
before connected together




$$Q_1^{init} = C_1 V_1 = (1\mu F)(1V) = 1\mu C$$

$$Q_2^{init} = C_2 V_2 = (1\mu F)(2V) = 2\mu C$$

(b) Now the switch is closed (i.e. the capacitors are connected together.) What are the voltages across and the charges on $C_1$ and $C_2$?




charge is conserved on floating node!!

$$Q_{open}^{u_1} = Q_{closed}^{u_1}$$

$$3\mu C = (C_1 + C_2)V$$

$$V = \frac{3\mu C}{C_1 + C_2} = \frac{3\mu C}{2\mu F} = 1.5V$$

$$Q_{open}^{u_1} = Q_1^{init} + Q_2^{init} = 1\mu C + 2\mu C = 3\mu C$$

$$Q_{closed}^{u_1} = Q_1^{\phi_1} + Q_2^{\phi_1} = C_1 V + C_2 V = (C_1 + C_2)V$$

$$Q_1^{\phi_1} = C_1 V_{C_1} = C_1 V$$

$$Q_2^{\phi_1} = C_2 V_{C_2} = C_2 V$$

$$V_{C_1} = V_{C_2} = u_1 - 0$$

$$V_{C_1} = V_{C_2} = V$$

$$Q_1^{\phi_1} = C_1 V = (1\mu F)(1.5V) = \boxed{1.5\mu C}$$

$$Q_2^{\phi_1} = C_1 V = (1\mu F)(1.5V) = \boxed{1.5\mu C}$$