1. Charge Sharing

Consider the circuit shown below. In phase \( \phi_1 \), the switches labeled \( \phi_1 \) are on while the switches labeled \( \phi_2 \) are off. In phase \( \phi_2 \), the switches labeled \( \phi_2 \) are on while the switches labeled \( \phi_1 \) are off.

(a) Draw the polarity of the voltage (using \( + \) and \( - \) signs) across the two capacitors \( C_1 \) and \( C_2 \). (It doesn’t matter which terminal you label \( + \) or \( - \); just remember to keep these consistent through phase 1 and 2.)

(b) Redraw the circuit in phase \( \phi_2 \) and phase \( \phi_1 \). Keep your polarity from part (a) in mind.

(c) Find \( V_{in} \) in phase \( \phi_2 \) as a function of \( V_{in}, C_1 \), and \( C_2 \).

4. \( Q_{u_2}^{\phi_2} = Q_{u_2}^{\phi_1} + Q_{e_2}^{\phi_2} = C_1 V_1 + C_2 V_2 \\
   = Q_{u_2}^{\phi_1} + Q_{e_2}^{\phi_2} = 2Q_{e_2}^{\phi_2} \\
   = 2 \frac{C_1 C_2}{C_1 + C_2} V_{in} \)

5. \( Q_{u_2}^{\phi_2} = Q_{u_2}^{\phi_1} + Q_{e_2}^{\phi_2} = C_1 V_1 + C_2 V_2 = C_1 V_{out} + C_2 V_{out} \\
   = (C_1 + C_2) V_{out} \)

6. \( Q_{u_2}^{\phi_2} = Q_{u_2}^{\phi_1} \\
   \frac{2 C_1 C_2 V_{in}}{C_1 + C_2} = (C_1 + C_2) V_{out} \\
   \rightarrow \ V_{out} = \frac{2 C_1 C_2}{(C_1 + C_2)^2} V_{in} \)

4. How will the charges be distributed in phase \( \phi_2 \) if we assume \( C_1 \gg C_2 \)?

\( Q_{u_2}^{\phi_2} = C_1 V_{out} + \) \( C_1 \gg C_2 \) \( \rightarrow \ Q_{u_2}^{\phi_2} \gg Q_{e_2}^{\phi_2} \)

\( Q_{e_2}^{\phi_2} = C_2 V_{out} \)

**Charge Sharing Algorithm**

1. Label voltages across caps, choose polarity and stick with it.
2. Draw the circuit in both phases (label nodes differently).
3. Identify floating nodes in phase 2 (nodes where charge is trapped)

For each floating node:

4. Calculate total charge on floating node plates in phase 1 \( Q_{u_2}^{\phi_1} \).
5. Find total charge on floating node plates in phase 2 \( Q_{u_2}^{\phi_2} \).
6. Charge conservation: \( Q_{u_2}^{\phi_2} = Q_{u_2}^{\phi_1} \) (solve for unknown).
2. Voltage Booster

We have made extensive use of resistive voltage dividers to reduce voltage. What about a circuit that boosts voltage to a value greater than the supply $V_s = 5V$? We can do this with capacitors!

(a) In the circuit above switches $\phi_1$ are initially closed and switch $\phi_2$ is initially open. Calculate the value of the output voltage, $V_{out}$, with respect to ground, and the amount of charge stored on capacitor $C_1$ at that state (phase 1).

\[ Q_1 = C V_{C1} = C V_{out} = C V_s \]

\[ V_{out} = V_s \]

(b) Now, after the capacitors are charged, switches $\phi_1$ are opened and switch $\phi_2$ is closed. Calculate the new voltage output voltage, $V_{out}$, at steady state.

\[ Q_{\phi_2}^{\prime} = Q_2 \]

by charge conservation:

\[ Q_{\phi_2}^{\prime} = +Q_2 = C V_s \]

\[ Q_{\phi_2} = -Q_2 = C V_{C2} = C (V_2 - V_1) = C (V_{out} - V_s) \]

\[ C V_s = C (V_{out} - V_s) \]

\[ \Rightarrow V_{out} = 2V_s \]
Comparators

\[ V_{out} = \begin{cases} V_{DD} & \text{if } V^+ > V^- \\ V_{SS} & \text{otherwise } (V^+ \leq V^-) \end{cases} \]

\[ \text{T}^+ = \text{T}^- = 0 \]

3. Comparators

For each of the circuits shown below, find \( V_{out} \) for \( V_{in} \) ranging from \(-10\text{V}\) to \(10\text{V}\) for part (a) and from \(9\text{V}\) to \(10\text{V}\) for part (b).

(a)

\[ V^+ = V_{in} \]
\[ V^- = \frac{2kR}{1kR + 2kR} V_{in} = \frac{2}{3} V_{in} \]

\[ \begin{align*}
&\text{if } V_{in} > 0 \quad V^+ > V^- \quad V_{out} = 5\text{V} \\
&\text{if } V_{in} < 0 \quad V^+ < V^- \quad V_{out} = -5\text{V}
\end{align*} \]

(b)

\[ V^+ = \frac{2}{3} V_{in} \]
\[ V^- = 2V \]

\[ \begin{align*}
&\text{if } V_{in} > 3\text{V} \quad V^+ > V^- \quad V_{out} = 5\text{V} \\
&\text{if } V_{in} < 3\text{V} \quad V^+ < V^- \quad V_{out} = -5\text{V}
\end{align*} \]