Logistical Notes

1. MT 2 next Monday, logistics post coming out on Piazza this week
2. Circuits Review Session 3 moved to Friday 2PM-4PM PT (10/30/20)

Learning Objectives

1. Using notion of steady state in looking at charges and voltages in capacitive circuits, using equivalence to solve for charges and voltages.
2. Capacitors discharging/charging by current sources
3. Change in capacitors, what does \( Q = CV \) mean?

Playlist

1. Jamie Maestre - Fly (FKJ Remix)
2. Prince Mini Kid
   Hiatus Kaiyote
3. Tom Misch - It Runs Through Me (feat. De La Soul)
1. Voltages Across Capacitors

For the circuits given below, determine the voltage across each capacitor and calculate the charge and energy stored on each capacitor (assume all capacitors start uncharged, and then we’ve let the system reach steady state). We are also given $C_1 = 1 \mu F$, $C_2 = 3 \mu F$, and $V_s = 1 V$.

Recall charge has units of Coulombs (C), and capacitance is measured in Farads (F) = \( \frac{\text{Coulomb}}{\text{Volt}} \).

It may also help to note metric prefix examples: $3 \mu F = 3 \times 10^{-6} F$.

(a) \[ V_s = 1 \text{Volt} \]
\[ V_s = \frac{Q}{C_1} = 1 \text{Volt} \]
\[ I = \frac{dQ}{dt} = C \frac{dV}{dt} \]
\[ I = 0 \text{A} \]

(at steady state caps have zero current)
\[ \Rightarrow \text{constant voltage} \Rightarrow 0 \text{current} \]
\[ \Rightarrow Q = CV = (+1 \mu F)(1V) = \frac{1 \mu C}{\mu} = 10^{-6} \]
\[ E = \frac{1}{2} CV^2 = \frac{1}{2} (1 \mu F)(1V)^2 = \frac{1}{2} \mu J \]

Units: \( [F][V]^2 = [C] \frac{V}{V} = [C][V] = [\frac{C}{V}] = [J] \)

Joules (energy)
Helpful diagrams for considering the charges capacitors linked in series:
(without any initial charges)

Left: Our series capacitors may be modeled as one equivalent capacitor $C_{eq}$, which after some time is charged up by $V_s$ to have $+q$ on the top plate and $-q$ on the bottom plate.

Middle: We return to the 2-capacitor picture, but carry this insight of equivalent charge with us. Now the charge $+q$ is on the top plate of capacitor $C_1$, and $-q$ is on the bottom plate of capacitor $C_2$.

Right: Since capacitor plates have opposite & equal charges, we attain this final right diagram.

As another conceptual check, we notice that the node between $C_1$ and $C_2$ is isolated from any other connections and should always remain charge neutral. From the diagram right we see this is maintained since $(+q) + (-q) = 0.$
2. Current Sources And Capacitors

Given the circuit below, find an expression for $v_{out}(t)$ in terms of $I_s$, $C$, $V_0$, and $t$, where $V_0$ is the initial voltage across the capacitor at $t = 0$.

Then plot the function $v_{out}(t)$ over time on the graph below for the following conditions detailed below. Use the values $I_s = 1\text{mA}$ and $C = 2\mu\text{F}$.

- (a) Capacitor is initially uncharged $V_0 = 0$ at $t = 0$.
- (b) Capacitor has been charged with $V_0 = +1.5\text{V}$ at $t = 0$.
- (c) Practice: Swap this capacitor for one with half the capacitance $C = 1\mu\text{F}$, which is initially uncharged $V_0 = 0$ at $t = 0$.

HINT: Recall the calculus identity $\int_a^b f'(x)dx = f(b) - f(a)$, where $f'(x) = \frac{df}{dx}$.