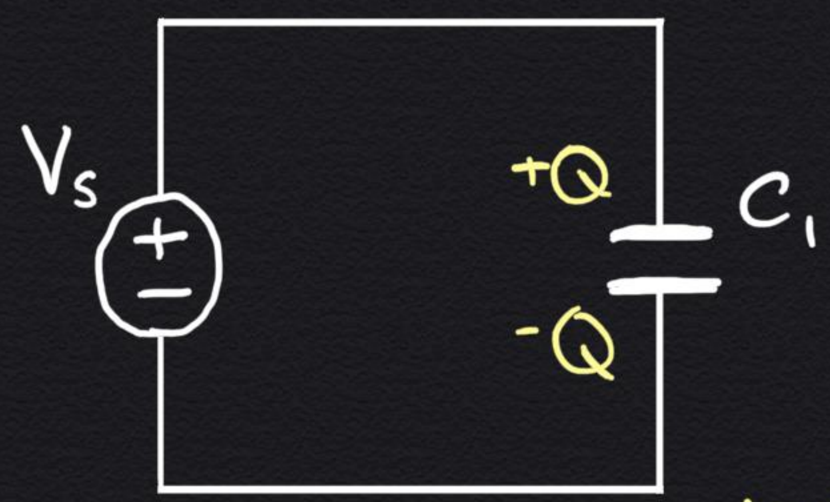
(1) Given the circuits below, find the voltage across each capacitor and find the charge and energy on each as well:

$C_1 = 1\mu F \quad C_2 = 3\mu F \quad V_s = 1V$

a]



Recall:

* $C = \frac{Q}{V}$
* $E = \frac{1}{2}CV^2$
* Metric: $m \sim 10^{-3}$, $\mu \sim 10^{-6}$, $n \sim 10^{-9}$
* Units:
  - Farad = $\frac{\text{Coulomb}}{\text{Voltage}}$
  - Joule = Coulomb × Voltage
* $C_{EQ} = \frac{C_1 C_2}{C_1 + C_2}$

* $Q = C_1 V_s = (1\mu F)(1V)$ — $(1\times10^{-6} F)$, C/V

$= 1\times10^{-6} \frac{C}{V} V$

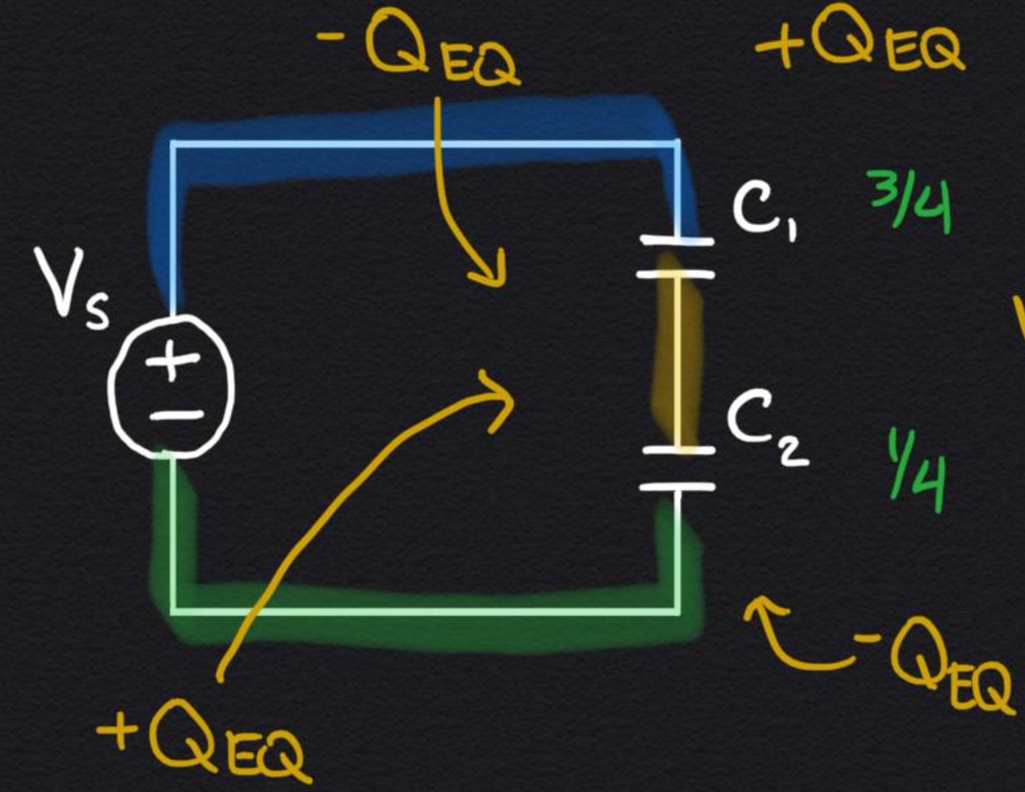
$= 1\mu C$

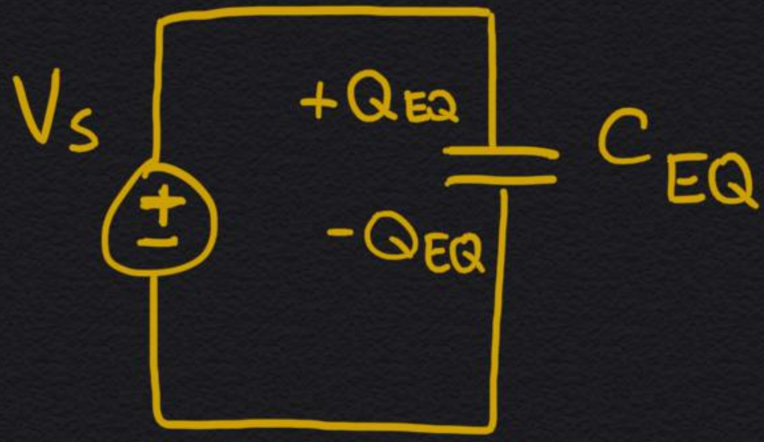
* $E = \frac{1}{2}CV^2 = \frac{1}{2}(1\mu F)(1V)^2$ — $\frac{1\mu C}{1V}$

$= \frac{1}{2} 1\mu C \frac{1V^2}{1V} = \frac{1}{2}\mu C \cdot V = \frac{1}{2}\mu J$

$= 500\,nJ$

b)





- $C_{EQ} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(1\mu F)(3\mu F)}{(4\mu F)} = \frac{3}{4}\mu F$

- $Q_{EQ} = C_{EQ} V_s = \left(\frac{3}{4}\mu F\right)(1\,V) = \frac{3}{4}\mu C$ $\quad \left(\frac{3}{4} \cdot \frac{\mu C}{V}\right)$

$$V_1 = \frac{Q_{EQ}}{C_1} = \frac{3/4\,\mu C}{1\,\mu F} = \frac{3}{4} V$$

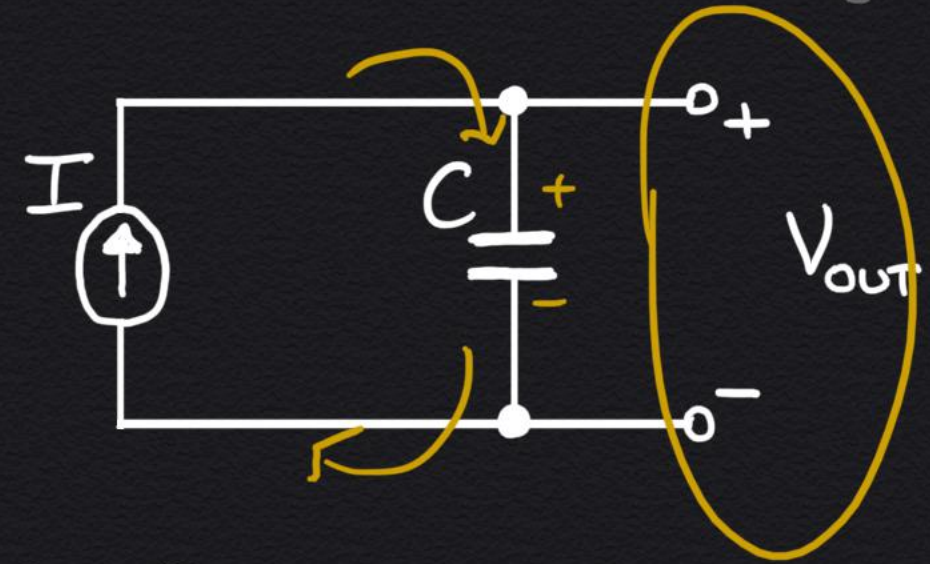
$$V_2 = \frac{Q_{EQ}}{C_2} = \frac{3/4\,\mu C}{3\mu F} = \frac{1}{4} V$$

$$E_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2}(1\mu F)\left(\frac{3}{4}V\right)^2 = \frac{9}{32}\mu C \cdot V = \frac{9}{32}\mu J$$

$$E_2 = \frac{1}{2} C_2 V_2^2 = \frac{1}{2}(3\mu F)\left(\frac{1}{4}V\right)^2 = \frac{3}{32}\mu C \cdot V = \frac{3}{32}\mu J$$

② Find $V_{OUT}(t)$ in terms of I, C, V_0, and t.

(Note: V_0 is the initial voltage $V_{OUT}(0)$)



$I = 1\,mA$

$C = 2\,\mu F$

Hint:

$$I = \frac{dQ}{dt} \Longleftrightarrow V = \frac{Q}{C}$$

Plot this function for the following conditions:

a) $V_0 = 0\,V$

b) $V_0 = 1.5\,V$

c) $V_0 = 0\,V$ $C = 1\,\mu F$

$$V = \frac{Q}{C} \longrightarrow \frac{dV}{dt} = \left(\frac{1}{C}\right) \cdot \frac{dQ}{dt} = \left[\frac{I}{C}\right]$$

$$\left\{ V(t) = V_0 + \left(\frac{I}{C}\right) t \right\}$$

$$\int_0^t \left(\frac{dV}{dt'}\right) dt' = V(t) - V(0)$$

($V(0) \rightarrow V_0$)

$$= \int_0^t \frac{I}{C}\, dt' = \frac{I}{C}\int_0^t 1\, dt'$$

($\int_0^t 1\, dt' \rightarrow t$)

$$\rightarrow \quad V(t) = V_0 + \left(\frac{I}{C}\right) t$$

$\frac{mC}{s} = \frac{\mu C}{ms}$

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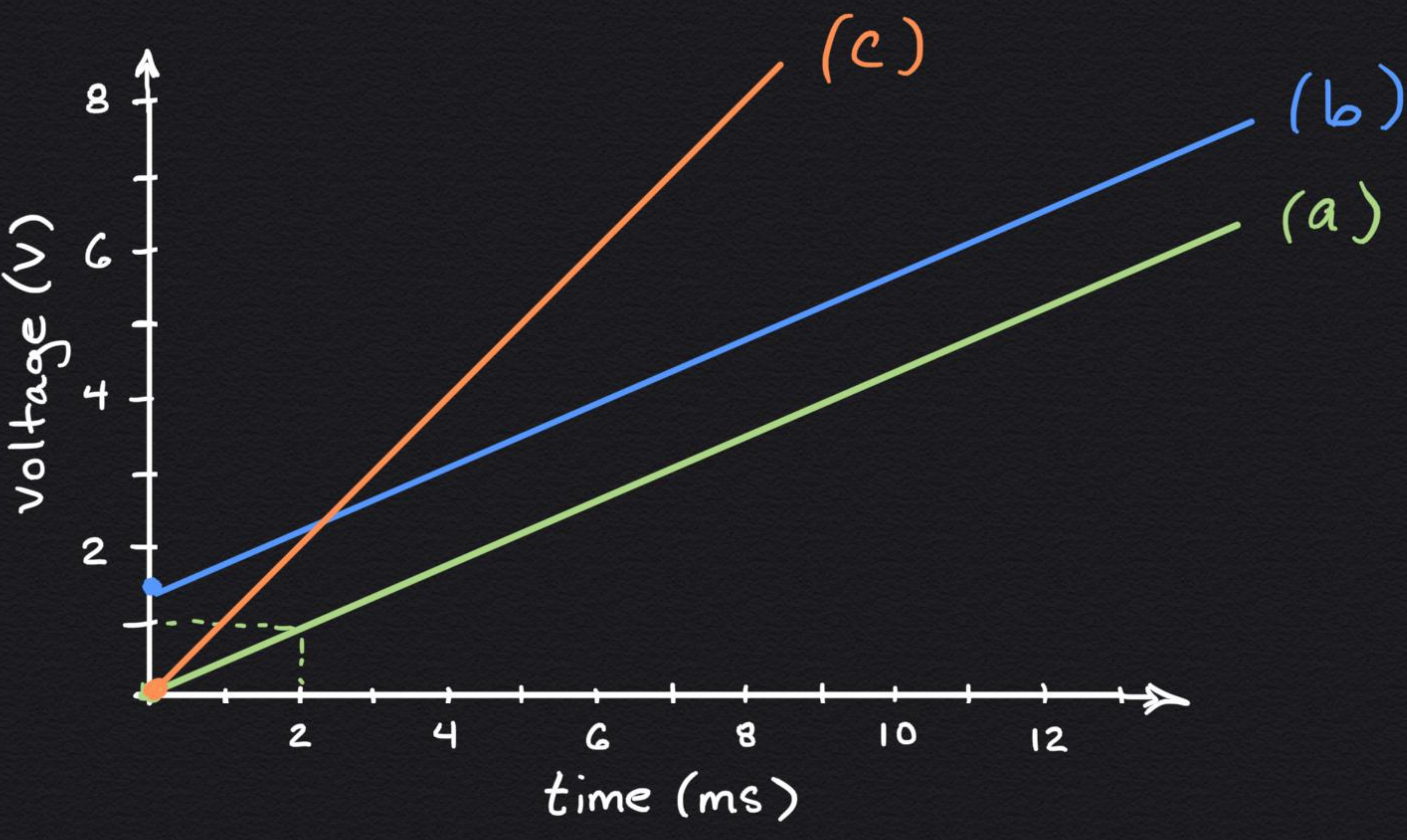
* $y = mx + d$

$$m_{a,b} = \frac{I}{C} = \frac{1\ mA}{(2\ \mu F)} = \frac{1}{2} \cdot \frac{V}{ms}$$

($F \rightarrow \frac{\mu C}{V}$)

$$m_c = \frac{1 mA}{1 \mu F} = 1\ \frac{V}{ms}$$



(Bonus content! Not part of the testable curriculum)

Physics Time:

$$E = k\frac{q_1 q_2}{r} = q_1\left(k\frac{q_2}{r}\right)$$

$$\underbrace{\left(k\frac{q_2}{r}\right)}_{V}$$

Suppose we start "assembing charges"

Energy of 3 charges: $E = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_{23}}{r_{23}}\right)$

Energy of 'N' Charges: $= k \sum_{i}^{N} q_i \sum_{j>i}^{N} \frac{q_j}{r_{ij}}$

$$= k \sum_{i}^{N} q_i \frac{1}{2} \sum_{j \neq i}^{N} \frac{q_j}{r_{ij}}$$

$$= \frac{1}{2} \sum_{i}^{N} q_i \underbrace{\left(k \sum_{j \neq i}^{N} \frac{q_j}{r_{ij}} \right)}_{V_i}$$

$$E = \frac{1}{2} \sum_{i}^{N} q_i V_i$$

Recall... $C = \frac{Q}{V}$

$$E = \frac{1}{2} C V^2$$

$$\equiv \frac{1}{2} Q V$$