1. Modular Circuit Buffer
   “How to combine circuits”
   Can we build a circuit that computes the following arithmetic? \( V_x = \frac{1}{2} V_{in} \quad V_y = \frac{1}{3} V_x \)

   Circuits we'd like to implement
   (these are the "modules")

a) Draw a voltage divider for each operation:

   \[ V_x = \left( \frac{R_x}{R_x + R_y} \right) V_{in} = \frac{1}{2} V_{in} \]

   \[ V_y = \left( \frac{R_y}{R_y + 2R_y} \right) V_{in} = \frac{1}{3} V_{in} \]

   While the ratio of resistor values within \[ \frac{1}{2} \] and \[ \frac{1}{3} \] circuits are fixed (\( R_i = R_2 \) and \( R_i = 2R_2 \) respectively), there is no relation of these values between circuits. Thus they've been left as \( R_x \) and \( R_y \) in general.
b) Link the two circuits as initially stated. Does it behave as we hoped?

Now that a load has been added to the \( V_2 \) module, its behavior is altered by an alternate route for current:

\[
R_{EQ} = \left( \frac{1}{R_x} + \frac{1}{2R_y + R_j} \right)^{-1} = \frac{3R_y R_x}{3R_y + R_x}
\]

\[
V_x = \left( \frac{R_{EQ}}{R_x + R_{EQ}} \right) V_{in} = \left( \frac{3R_y R_x / (R_x + 3R_y)}{R_x + 3R_x R_y / (R_x + 3R_y)} \right) V_{in} = \left( \frac{3R_x R_y}{R_x^2 + 3R_x R_y + 3R_x R_y} \right) V_{in} = \frac{1}{2 + \frac{R_x}{3R_y}} V_{in}
\]

\[
V_y = \frac{1}{3} \left( \frac{1}{2 + \frac{R_x}{3R_y}} \right) V_{in} \neq \frac{1}{3} V_{in}
\]

Since the latter \( V_3 \) still has no load, \( V_y = \frac{1}{3} V_x \).

Oh no! I guess just slapping 2 voltage dividers together...

Notice though that for \( R_y \ll R_x \) we find \( V_x \approx \frac{1}{2} V_{in} \) and \( V_y \approx \frac{1}{3} V_{in} \), but we want to be picky and have the circuits work exactly like in their isolated modules regardless of \( R_x, R_y \). We need op-amps...

This is because effectively no current gets into the \( V_3 \) circuit and it still "looks open" to the \( V_2 \) circuit.
Try including an op-amp (in negative feedback) within the circuit to circumvent the loading issue!

Try inserting a unity gain op-amp circuit between them, so the output of \( \frac{1}{2} \) feeds to an op-amp input terminal:

Since the inputs to an op-amp act like open circuits, the \( \frac{1}{2} \) preserves its behavior!

Quick aside...

Review of unity gain op-amp circuit

Golden rule: \( I_+ = I_- = 0 \)

Gain: \( V_b = A(u_+ - u_-) \)

where \( A \) is HUGE (\( A \approx 10^6 \))

Now \( u_+ = V_a \) and \( u_- = V_b \) (since they're the same node), so we find:

\[
V_b = A(V_a - V_b) \quad \Rightarrow \quad (1 + A)V_b = AV_a
\]

\[
V_b = \left( \frac{1}{1 + \frac{1}{A}} \right)V_a \approx V_a
\]
2. Modular Op-Amp Circuits

Perform the following operations using op-amps:

(a) \( V_{\text{out}} = +5V_{\text{in}} \)
(b) \( V_{\text{out}} = -2V_{\text{in}} \)
(c) \( V_{\text{out}} = V_1 + V_2 \)

Can these circuits be combined while maintaining their function?

(a) We need a non-inverting amplifier:

\[
\text{Given that } V_{\text{in}} \text{ leads into an op-amp input terminal (no current), we can safely connect this circuit to others without issue.}
\]
(b) We need an inverting amplifier: \[ \frac{V_{in}}{-2} \rightarrow V_{out} \]

Since \( u_- = u_+ = 0 \), we know \( I = \frac{V_{in} - 0}{R_1} \)

and so \( V_{out} = V_{in} + I \cdot R_2 \)

\[ = V_{in} \left( 1 + \frac{R_2}{R_1} \right) \]

Now we need \( 1 + \frac{R_2}{R_1} = 5 \) \( \Rightarrow \) \( R_2 = 4R_1 \)

Given that 'Vin' does have a current connection to 'Vout', we would not be able to attach a voltage divider before this circuit without messing up that divider. However, the gain \([-2]\) works regardless!

\[ \text{we'd need a buffer} \swarrow \]

\[ \text{(c) } V_{out} = V_1 + V_2 \]

\[ u_+ = \frac{1}{2}(V_1 + V_2) \]

\[ I = \frac{V_1 + V_2}{2R} \]

\[ V_{out} = 2u_- \]

\[ = \frac{2}{2}(V_1 + V_2) \]

\[ = V_1 + V_2 \swarrow \]