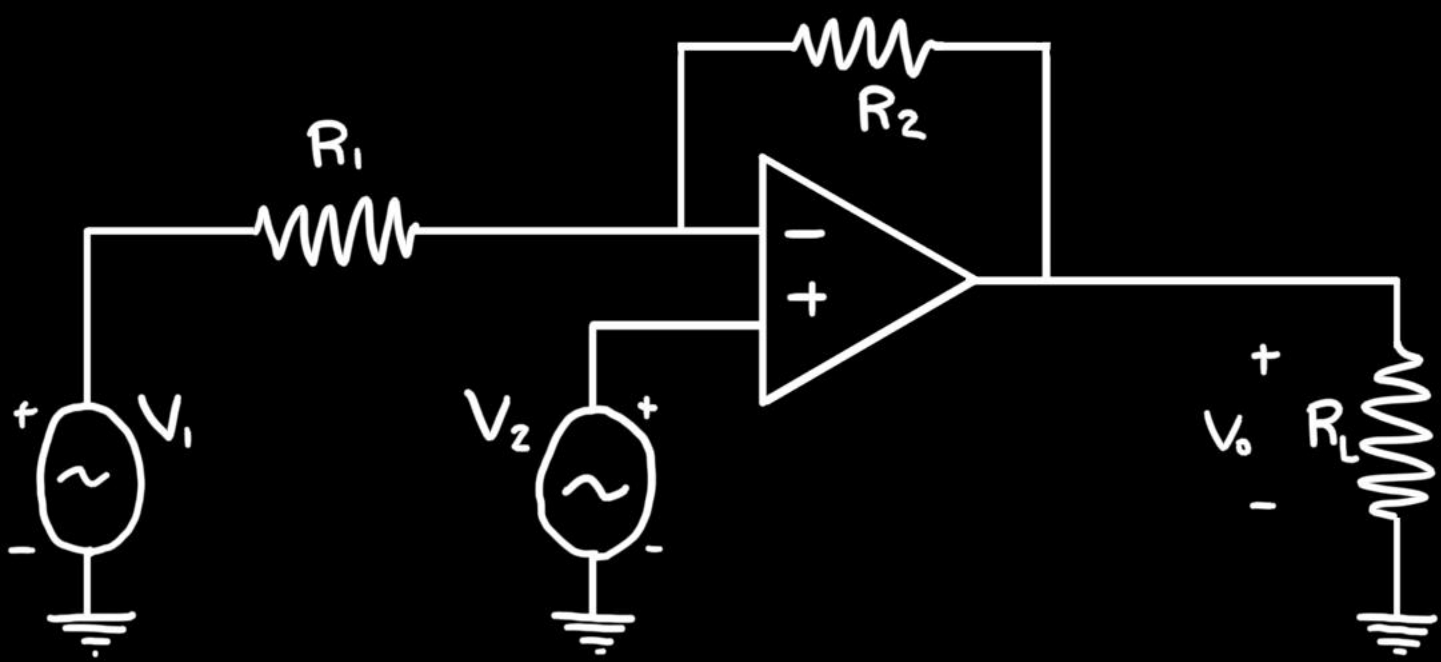
Discussion 12A Notes

① Multi-Input Op-Amp Circuits

Use superposition to solve for the output of this circuit:



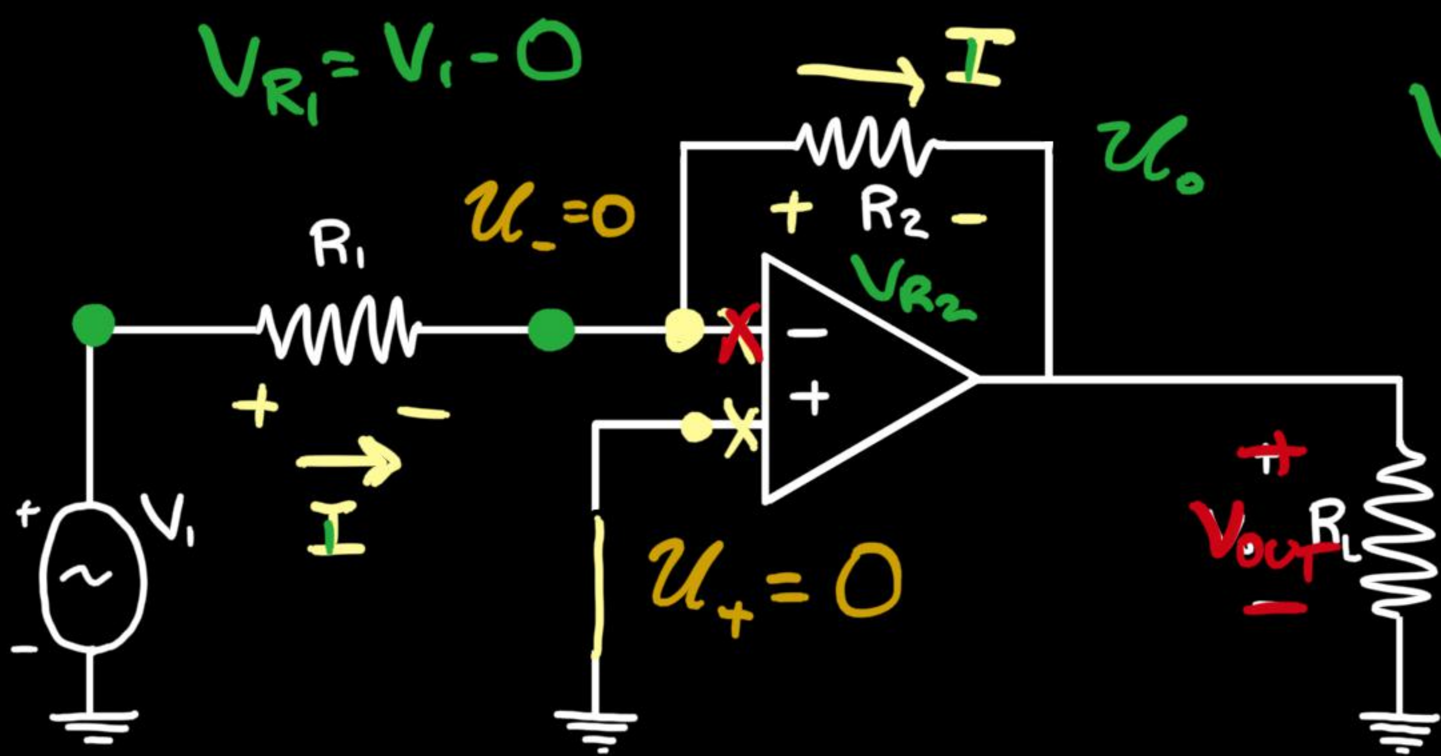
a) Suppose $V_2 = 0\,V$.

b) Suppose $V_1 = 0\,V$.

c) Superimpose results from 'a' and 'b'.

$V_{R_1} = V_1 - 0$ $\qquad$ $V_{R_2} = u_- - u_o$

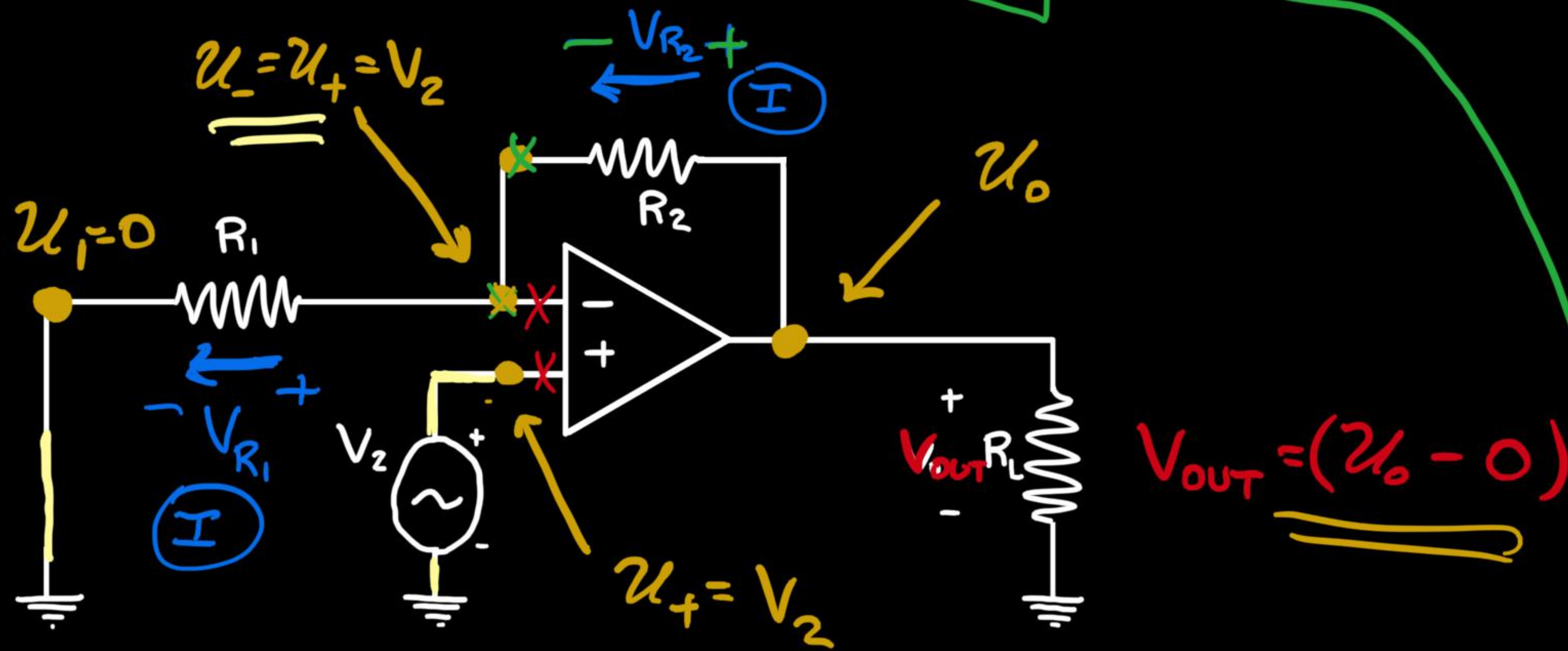
a)



$$\left[\, I = \frac{V_1 - \cancel{u_-}^{\,0}}{R_1} \,\right] \quad u_-$$

$$V_{OUT} = 0 - (\underbrace{I R_2}_{V_{R_2}}) = \boxed{-V_1 \frac{R_2}{R_1}} \checkmark$$

b)



$$I = \frac{V_{R_1}}{R_1} = \frac{u_- - u_1}{R_1} = \frac{V_2 - 0}{R_1} = \frac{V_2}{R_1}$$

$$u_0 = u_- + \underbrace{I R_2}_{V_{R_2}} = V_2 + \frac{V_2}{R_1} R_2 = \boxed{V_2 \left(1 + \frac{R_2}{R_1}\right)}$$

c)

$$\boxed{V_{OUT} = -V_1 \left(\frac{R_2}{R_1}\right) + V_2 \left(1 + \frac{R_2}{R_1}\right)}$$

② Find the inner-product of $\langle x, y\rangle$ for the following:
$= \vec{x}^T\vec{y}$

a) $\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ $\quad \vec{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$$\langle \vec{x}, \vec{y}\rangle = \underline{1\cdot 1} + \underline{0\cdot 2} + \underline{1\cdot 3}$$
$$= 1 + 0 + 3 = 4$$
$$= \vec{x}^T\vec{y} = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Ex: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$ $\quad A^T = \begin{bmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{bmatrix}$

b) $\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $\quad \vec{y} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$$\langle \vec{x}, \vec{y}\rangle = \underline{1\cdot 1} + \underline{2\cdot 0} + \underline{3\cdot 1}$$
$$= 1 + 3 = 4 \checkmark$$

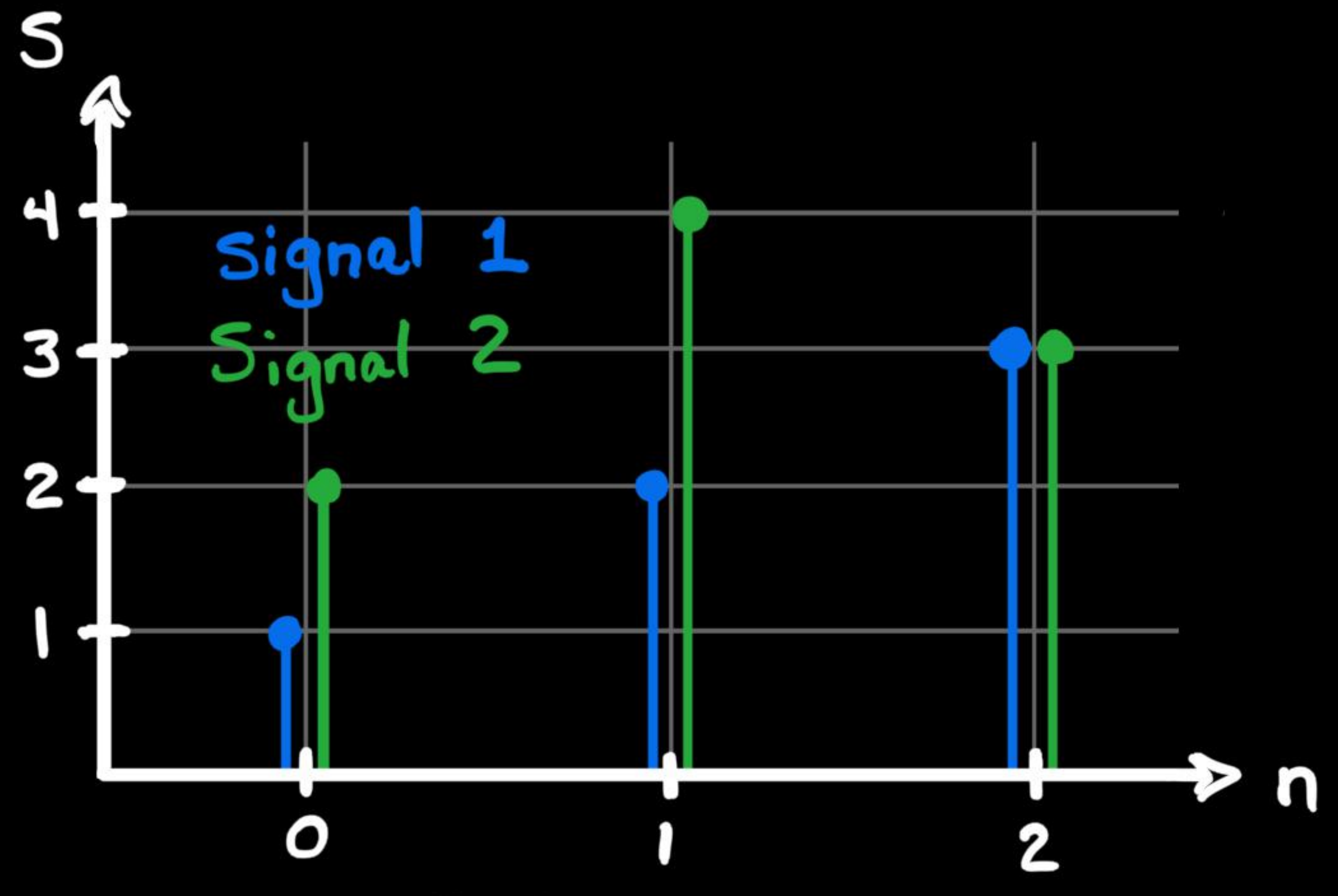
- Inner-product is symmetric!!!

c) $\vec{x} = \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$ $\quad \vec{y} = \begin{bmatrix} -3 \\ 2 \\ 1 \end{bmatrix}$

$$\langle \vec{x}, \vec{y}\rangle = 1\cdot -3 + 0\cdot 2 + 3\cdot 1$$
$$= -3 + 0 + 3$$
$$= 0 \checkmark$$

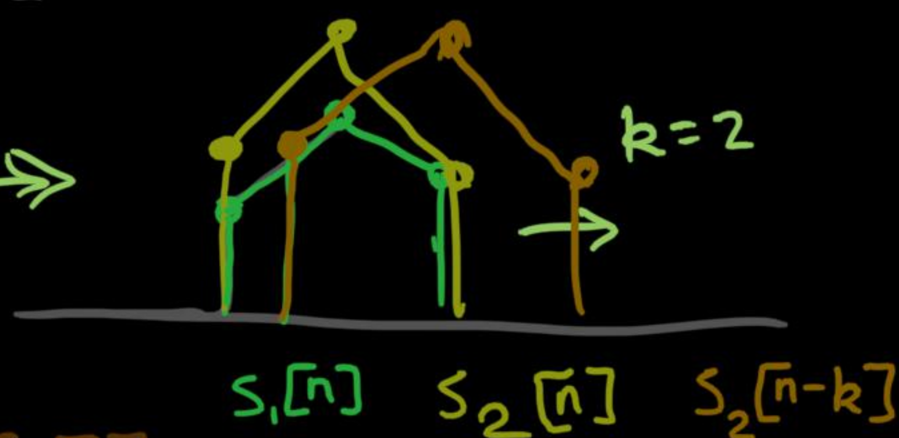
③ Correlation

$$\mathrm{Corr}_x(y)[k] = \sum_{i=-2}^{4} x[i] \cdot y[i-k]$$



* $S_1[n] = S_2[n] = 0$ if $n<0$ or $n>2$.

Think about 'k' as a sliding delay



a) Find $\mathrm{Corr}_{S_1}(S_2)[k]$

$S_2[0+2] = S_2[2]$

k=-2:

n	(-2	-1)	0	1	2	(3	4)	
$S_1[n]$	0	0	1	2	3	0	0	
$S_2[n+2]$	2	4	3	0	0	0	0	
$\langle S_1, S_2[n+2]\rangle$	0 +	0 +	3 +	0 +	0 +	0 +	0	= 3

k=-1:

n	-2	-1	0	1	2	3	4	
$S_1[n]$	0	0	1	2	3	0	0	
$S_2[n+1]$	0	2	4	3	0	0	0	
$\langle S_1, S_2[n+1]\rangle$	0 +	0 +	4 +	6 +	0 +	0 +	0	= 10

k=0:

n	-2	-1	0	1	2	3	4	
$S_1[n]$	0	0	1	2	3	0	0	
$S_2[n]$	0	0	2	4	3	0	0	
$\langle S_1, S_2[n]\rangle$	0 +	0 +	2 +	8 +	9 +	0 +	0	= 19

k=+1:

n	-2	-1	0	1	2	3	4	
$S_1[n]$	0	0	1	2	3	0	0	
$S_2[n-1]$	0	0	0	2	4	3	0	
$\langle S_1, S_2[n-1]\rangle$	0 +	0 +	0 +	4 +	12 +	0 +	0	= 16

k=+2:

n	-2	-1	0	1	2	3	4	
$S_1[n]$	0	0	1	2	3	0	0	
$S_2[n-2]$	0	0	0	0	2	4	12	
$\langle S_1, S_2[n-2]\rangle$	0 +	0 +	0 +	0 +	6 +	0 +	0	= 6

$$Corr_{S_1}(S_2)[k] = \begin{bmatrix} 3 \\ 10 \\ 19 \\ 16 \\ 6 \end{bmatrix}$$

b) Find $Corr_{S_2}(S_1)[k]$

k=-2:

n	-2	-1	0	1	2	3	4	
$S_2[n]$	0	0	2	4	3	0	0	
$S_1[n+2]$	1	2	3	0	0	0	0	
$\langle S_2, S_1[n+2]\rangle$	0 +	0 +	6 +	0 +	0 +	0 +	0	= 6

k=-1:

n	-2	-1	0	1	2	3	4	
$S_2[n]$	0	0	2	4	3	0	0	
$S_1[n+1]$	0	1	2	3	0	0	0	
$\langle S_2, S_1[n+1]\rangle$	0 +	0 +	4 +	12 +	0 +	0 +	0	= 16

k=0:

n	-2	-1	0	1	2	3	4	
$S_2[n]$	0	0	2	4	3	0	0	
$S_1[n]$	0	0	1	2	3	0	0	
$\langle S_2, S_1[n]\rangle$	0 +	0 +	2 +	8 +	9 +	0 +	0	= 19

k = +1:

n	-2	-1	0	1	2	3	4	
$S_2[n]$	0	0	2	4	3	0	0	
$S_1[n-1]$	0	0	0	1	2	3	0	
$\langle S_2, S_1[n-1] \rangle$	0 +	0 +	0 +	4 +	6 +	0 +	0	= 10

k = +2:

n	-2	-1	0	1	2	3	4	
$S_2[n]$	0	0	2	4	3	0	0	
$S_1[n-2]$	0	0	0	0	1	2	3	
$\langle S_2, S_1[n-2] \rangle$	0 +	0 +	0 +	0 +	3 +	0 +	0	= 3

$$\mathrm{Corr}_{S_2}(S_1)[k] = \begin{bmatrix} 6 \\ 16 \\ 19 \\ 10 \\ 3 \end{bmatrix}$$