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EECS 16A    Designing Information Devices and Systems I    Homework 6  
 Spring 2022

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**This homework is due Friday, March 4, 2021, at 23:59.**

**Self-grades are due Monday, March 7, 2021, at 23:59.**

**Submission Format**

Your homework submission should consist of **one** file.

- `hw5.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned) as well as your IPython notebook saved as a PDF.  
 If you do not attach a PDF “printout” of your IPython notebook, you will not receive credit for problems that involve coding. Make sure that your results and your plots are visible. Assign the IPython printout to the correct problem(s) on Gradescope.

Submit the file to the appropriate assignment on Gradescope.

**1. Reading Assignment**

For this homework, please review and read Note 11A/B, which introduces the basics of circuit analysis and node voltage analysis. You are always welcome and encouraged to read beyond this as well.

**2. Page Rank**

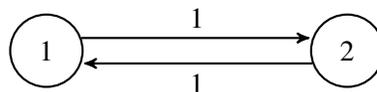
***Learning Goal:** This problem highlights the use of transition matrices in modeling dynamical linear systems. Predictions about the steady state of a system can be made using the eigenvalues and eigenvectors of this matrix.*

In homework and discussion, we have discussed the behavior of water flowing in reservoirs and the people flowing in social networks. We now consider the setting of web traffic where the dynamical system can be described with a directed graph, also known as state transition diagram.

As we have seen in lecture and discussion the “transition matrix”,  $\mathbf{T}$ , can be constructed using the state transition diagram, as follows: entries  $t_{ji}$  represent the *proportion* of the people who are at website  $i$  that click the link for website  $j$ .

The **steady-state frequency** (i.e. fraction of visitors in steady-state) for a graph of websites is related to the eigenspace associated with eigenvalue 1 for the “transition matrix” of the graph. Once computed, an eigenvector with eigenvalue 1 will have values which correspond to the steady-state frequency for the fraction of people for each webpage. When the elements of this eigenvector are made to **sum to one** (to conserve population), the  $i^{th}$  element of the eigenvector will correspond to the fraction of people on the  $i^{th}$  website.

- (a) For graph A shown below, what are the steady-state frequencies (i.e. fraction of visitors in steady-state) for the two webpages? Graph A has weights in place to help you construct the transition matrix. Remember to ensure that your steady state-frequencies sum to 1 to maintain conservation.



Graph A

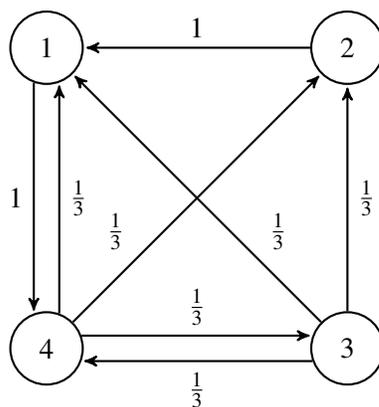
(b) For graph B, what are the steady-state frequencies for the webpages? You may use IPython and the Numpy command `numpy.linalg.eig` for this. Graph B is shown below, with weights in place to help you construct the transition matrix.

**Hint:** `numpy.linalg.eig` returns eigenvectors and eigenvalues. The eigenvectors are arranged in a matrix in *column-major* order. In other words, given eigenvectors

$$\vec{v}_1 = \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} \text{ and } \vec{v}_2 = \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix}$$

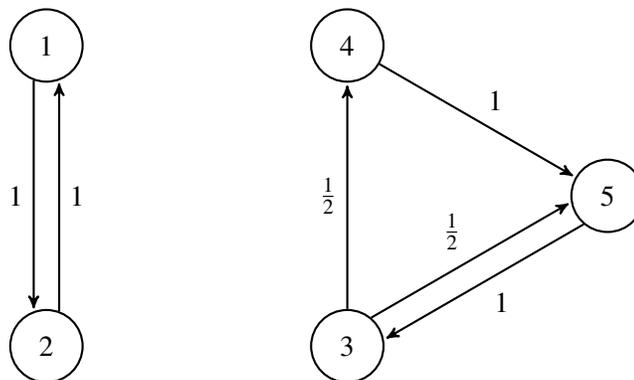
NumPy will return:

$$\begin{bmatrix} v_{11} & v_{21} \\ v_{12} & v_{22} \end{bmatrix} \tag{1}$$



Graph B

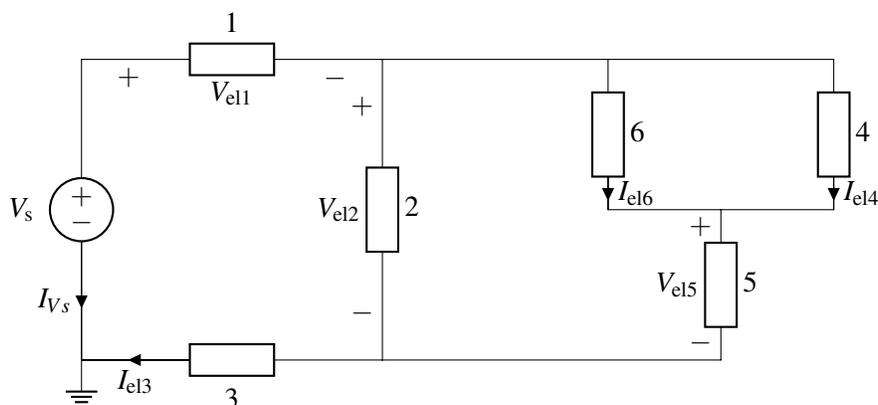
**OPTIONAL** Find the eigenspace that corresponds to the steady-state for graph C. How many independent systems (disjoint sets of webpages) are there in graph C versus in graph B? What is the dimension of the eigenspace corresponding to the steady-state for graph C? Again, graph C with weights in place is shown below. You may use IPython to compute the eigenvalues and eigenvectors again.



Graph C

3. Intro to Circuits

**Learning Goal:** This problem will help you practice labeling circuit elements and setting up KCL and KVL equations.



- (a) How many nodes does the above circuit have? Label them.  
*Note:* The reference/0V (ground) node has been selected for you, so you don't need to label that, but you need to include it in your node count.
- (b) Notice that elements 1 - 6 and the voltage source  $V_s$  have either the *voltage across* or the *current through* them not labeled. Label the missing *voltages across* or *currents through* for elements 1 - 6, and the voltage source  $V_s$ , so that they all follow **passive sign convention**.
- (c) Express all element voltages (including the element voltage across the source,  $V_s$ ) as a function of node voltages. This will depend on the node labeling you chose in part (a).
- (d) Write one KCL equation that involves the currents through elements 1 and 2.  
*Hint:* This will **not** be specific to your node labeling. Your answer may contain currents through other elements too.
- (e) Write a KVL equation for all the loops that contain the voltage source  $V_s$ . These equations should be a function of element voltages and the voltage source  $V_s$ .

#### 4. Reverse Eigenvalues

**Learning Goal:** Understand how to construct a matrix with a particular set of eigenvalues and eigenvectors.

In lecture, homework, and section, we have seen a number of ways to compute eigenvalues and eigenvectors from a particular matrix, and explored what they mean in terms of how the matrix transforms vectors. In this problem, we will explore this in the reverse direction by designing it to have a desired set of eigenvalues. Recall the fundamental eigenvector/eigenvalue equation:

$$A\mathbf{v} = \lambda\mathbf{v} \quad (2)$$

- (a) Suppose you are given the following eigenvalue/eigenvector pairs:

$$\lambda_1 = 1, \vec{v}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \lambda_2 = -1, \vec{v}_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

These eigenvectors/eigenvalues will result in a reflection matrix about the vector  $[1, 1]$ ; in other words every component orthogonal to  $[1, 1]$  will be flipped in sign. Explicitly write out the matrix-vector equations for the two eigenvector/eigenvalue pairs. Make sure to identify each component of the  $\mathbf{A}$  matrix and fill in the relevant values for the eigenvector and eigenvalue. Assume the unknown components of  $\mathbf{A}$  are  $a_{11}, a_{12}, a_{21}, a_{22}$

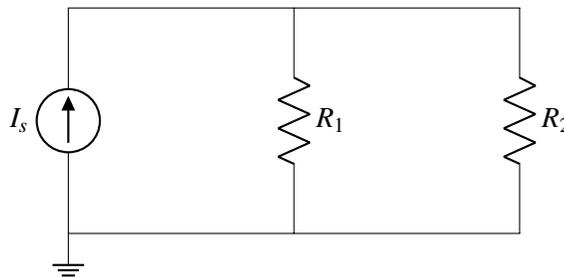
- (b) Reformat the equations you wrote above into a system of linear equations, with  $a_{11}, a_{12}, a_{21}, a_{22}$  as unknowns.
- (c) Now, setup a matrix-vector system of equations and solve for the  $\mathbf{A}$  matrix. Think carefully about what the unknowns in your system are when setting it up.

## 5. Circuit Analysis

**Learning Goal:** This problem will help you practice circuit analysis using the node voltage analysis (NVA) method.

Using the steps outlined in lecture or in Note 11, analyze the following circuits to calculate the currents through each element and the voltages at each node. Use the ground node labelled for you. You may use a numerical tool such as IPython to solve the final system of linear equations.

- (a)  $I_s = 3 \text{ mA}, R_1 = 2 \text{ k}\Omega, R_2 = 4 \text{ k}\Omega$



## 6. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.