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# EECS 16A    Designing Information Devices and Systems I

## Spring 2021    Homework 9

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**This homework is due April 2, 2021, at 23:59.**

**Self-grades are due April 5, 2021, at 23:59.**

### Submission Format

Your homework submission should consist of **one** file.

- `hw9.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

### 1. Reading Assignment

For this homework, please read Notes 16, 17 (17.1 - 17.2, specifically) and 17B. Note 16 will provide an introduction to capacitors (a circuit element which stores charge), capacitive equivalence, and the underlying physics behind them. Sections 17.1 - 17.2 in Note 17 will provide an overview of the capacitive touchscreen and how to measure capacitance. Note 17B will provide a walkthrough of the charge-sharing algorithm.

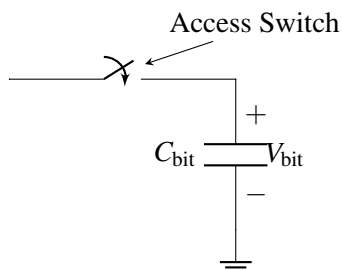
- (a) Describe the key ideas behind how a capacitor works. How are capacitor equivalences calculated? Contrast this with how we calculate resistor equivalences.
- (b) Consider the capacitive touchscreen. Describe how it works, and compare and contrast it to the resistive touchscreens we have seen in previous lectures and homeworks.
- (c) What property of charge is applied in connecting phase 1 calculations to phase 2 calculations in the charge sharing algorithm?

### 2. Dynamic Random Access Memory (DRAM)

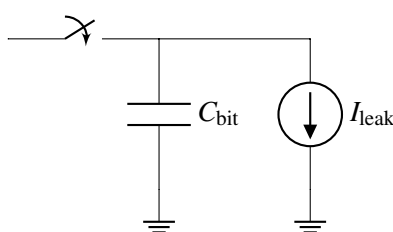
Nearly all devices that include some form of computational capability (phones, tablets, gaming consoles, laptops, ...) use a type of memory known as Dynamic Random Access Memory (DRAM). DRAM is where the “working set” of instructions and data for a processor is typically stored, and the ability to pack an ever increasing number of bits on to a DRAM chip at low cost has been critical to the continued growth in computational capability of our systems. For example, a single DRAM chip today can store  $> 8$  billion bits and is sold for  $\approx$  \$3-\$5.

At the most basic level and as shown below, every bit of information that DRAM can store is associated with a capacitor. The amount of charge stored on that capacitor (and correspondingly, the voltage across the capacitor) determines whether a “1” or a “0” is stored in that location.

## Single DRAM Bit Cell



In any real capacitor, there is always a path for charge to “leak” off the capacitor and cause it to eventually discharge. In DRAMs, the dominant path for this leakage to happen is through the access switch, which we will **model as a leakage to ground**. The figure below shows **a model of this leakage**:



*Fun Fact: This leakage is actually responsible for the “D” in “DRAM” – the memory is “dynamic” because after a cell is “written” by storing some charge onto its capacitor, if you leave the cell alone for too long, the value you wrote in will disappear because the charge on the capacitor leaked away.*

Let’s now try to use some representative numbers to compute how long a DRAM cell can hold its value before the information leaks away. Let  $C_{\text{bit}} = 28 \text{ fF}$  (note that  $1 \text{ fF} = 1 \times 10^{-15} \text{ F}$ ) and the capacitor be initially charged to  $1.2 \text{ V}$  to store a “1.”  $V_{\text{bit}}$  must be  $> 0.9 \text{ V}$  in order for the circuits outside of the column to properly read the bit stored in the cell as a “1.”

**What is the maximum value of  $I_{\text{leak}}$  that would allow the DRAM cell retain its value for  $> 1 \text{ ms}$ ?**

*Hint: Start by writing out the equations you know about charge and current related to the capacitor. Note here that the current source is discharging the capacitor.*

### 3. Capacitive Touchscreen

The model for a capacitive touchscreen can be seen in Figure 1. See Table 1 for values of the dimensions. The green area represents the contact area of the finger with the top insulator. It has dimensions  $w_2 \times d_1$ , where  $w_2$  is the horizontal width of the finger contact area and  $d_1$  is the depth (into the page) of the finger contact area. The top metal (red area) has dimensions  $w_1 \times d_1$ . The bottom metal (grey area) has dimensions  $w \times d_2$ , where  $w$  is larger than both  $w_1$  and  $w_2$ . The vertical distance between the top metal (red) and bottom plate (grey) is  $t_1$ , and the vertical distance between the finger (green) and the bottom plate (grey) is  $t_2$ .

- (a) Draw the equivalent circuit of the touchscreen that contains the nodes  $F$ ,  $E_1$ , and  $E_2$  when: (i) there no finger present; and (ii) when there is a finger present. Express the capacitance values in terms of  $C_0$ ,  $C_{F-E_1}$ , and  $C_{F-E_2}$ .

*Hint: Note that node  $F$  represents the finger. When there is no touch node  $F$  would be non-existent.  
Hint: Treat  $E_1$  as the “top node”,  $E_2$  as the “bottom node”, and the finger  $F$  as an intermediate node when present.*

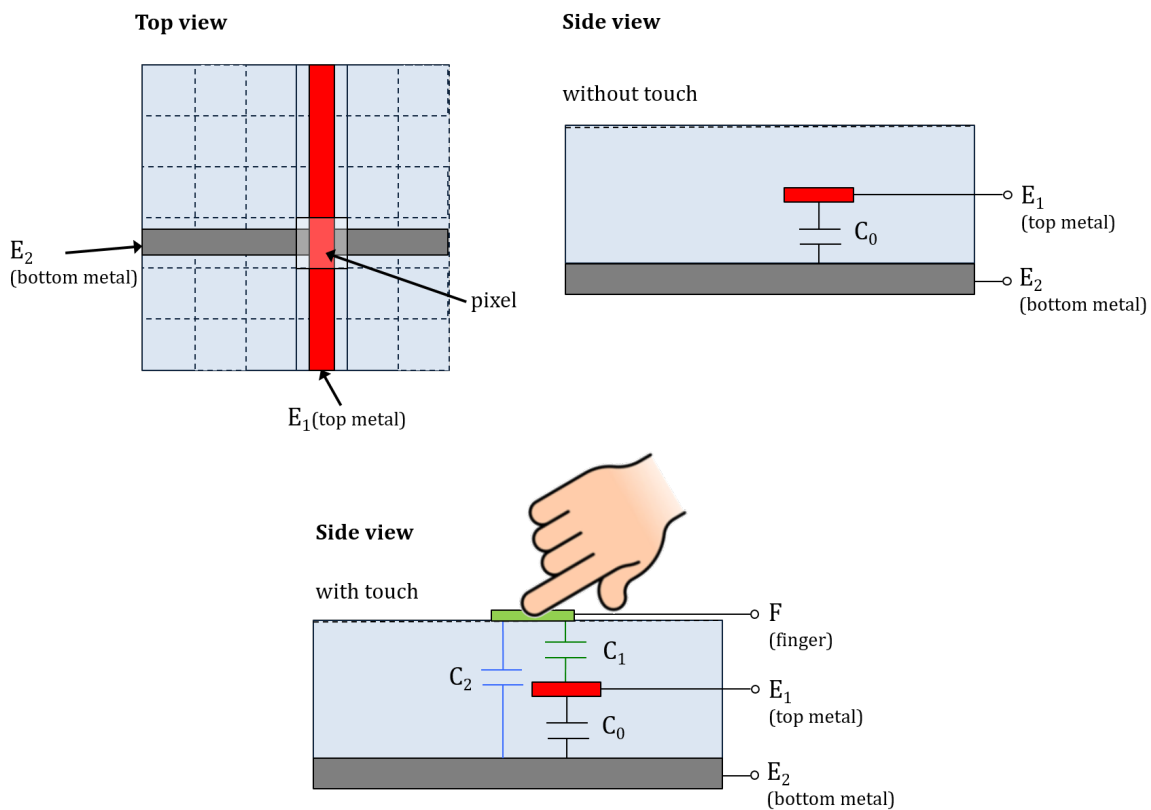


Figure 1: Model of capacitive touchscreen.

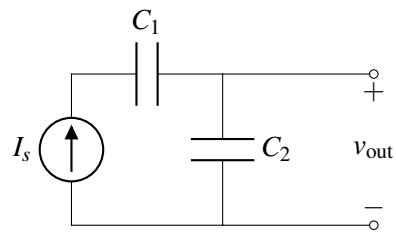
Table 1: Touchscreen Dimension Values

$d_1$	10 mm
$d_2$	1 mm
$t_1$	2 mm
$t_2$	4 mm
$w_1$	1 mm
$w_2$	2 mm

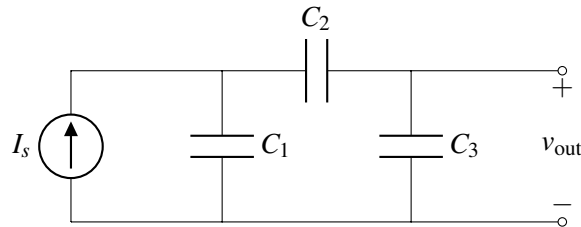
- (b) What are the values of  $C_0$ ,  $C_{E1-F}$ , and  $C_{F-E2}$ ? Assume that the insulating material has a permittivity of  $\epsilon = 4.43 \times 10^{-11} F/m$  and that the thickness of the metal layers is small compared to  $t_1$  (so you can ignore the thickness of the metal layers). Also assume that the right edge of the top metal (red area) in the diagram is aligned with the right edge of the finger (green area) in the diagram.
- (c) What is the difference in effective capacitance between the two metal plates (nodes  $E_1$  and  $E_2$ ) when a finger is present?

#### 4. More Current Sources And Capacitors

- (a) For the circuit given below, give an expression for  $v_{out}(t)$  in terms of  $I_s$ ,  $C_1$ ,  $C_2$ , and time  $t$ . Assume that all capacitors are initially uncharged, i.e. the initial voltage across each capacitor is 0V.



- (b) **(Practice)** For the circuit given below, give an expression for  $v_{\text{out}}(t)$  in terms of  $I_s$ ,  $C_1$ ,  $C_2$ ,  $C_3$ , and  $t$ . Assume that all capacitors are initially uncharged, i.e. the initial voltage across each capacitor is 0V. You may choose to use either NVA or capacitor equivalences to help you solve this problem.



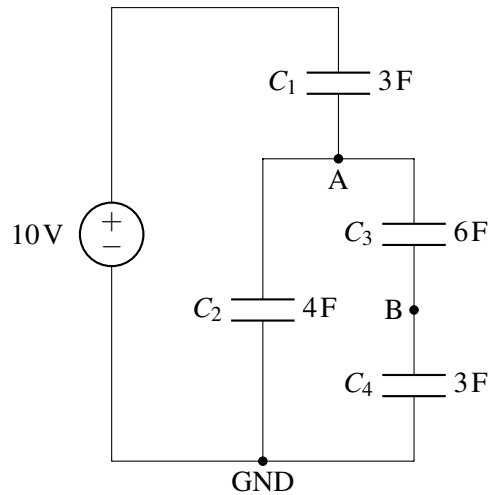
## 5. Circuit with Capacitors

Find the voltages at nodes A and B, and currents flowing through all of the capacitors at steady state. Assume that before the voltage source is applied, the capacitors all initially have a charge of 0 Coulombs.

In general your strategy to solve circuits with capacitors should be similar to solving resistive circuits. For capacitive circuits we often care about steady state (i.e. what happens to the circuit after a long time and no more changes are happening). If we are considering a circuit with capacitors and voltage sources, we will always be thinking about steady state (or the steady state for a phase if we are doing a charge sharing problem with switches). When thinking about steady state you always want to write out the equations for charge that you know, as well as all the KVL type relationships around voltages you know. Then use the key idea that charge is conserved to build out your system of equations. Don't be daunted by the variable names and know that everything just boils down to a system of linear equations.

Here are some principles that are also helpful:

- Charge at a node from which charge cannot escape or enter (floating node) is always conserved. — if the sum of charges is 0 on a floating node, the sum of charges on that floating node at steady state will be zero.
- The charge  $Q$  stored in a capacitor is given by the equation  $Q = CV$ . That is, the plate that corresponds to the "+" terminal, stores  $+Q = +CV$ , and the plate that corresponds to the "-" terminal, stores  $-Q = -CV$ .
- If two capacitors are initially uncharged, and then are connected in series, the charges on both capacitors are equal to each other at steady state.
- The voltage across capacitors in parallel is equal at steady state.

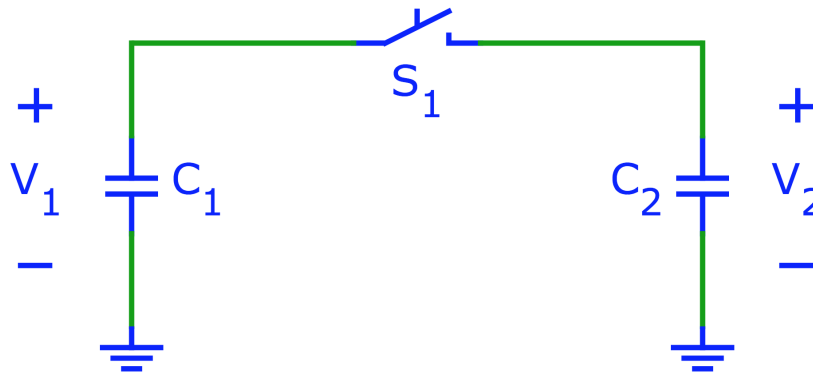


### 6. Charge Sharing

In the circuit below, switch  $S_1$  is initially open. Capacitor  $C_1 = 10^{-3}\text{F}$  is initially charged to  $V_1 = 3\text{V}$  and capacitor  $C_2 = 3 \times 10^{-3}\text{F}$  is initially charged to  $V_2 = 2\text{V}$ .

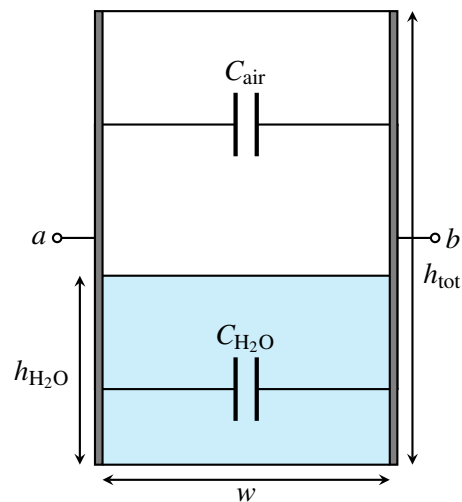
Now  $S_1$  is closed. Calculate the new value of  $V_2$ .

*Hint:* Remember that charge is conserved at floating nodes.



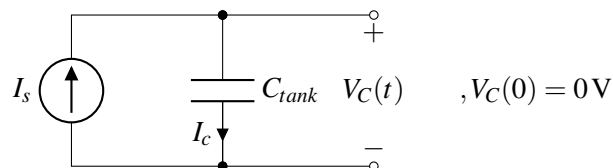
### 7. It's finally raining!

A lettuce farmer in Salinas Valley has grown tired of weather.com's imprecise rain measurements. Therefore, they decided to take matters into their own hands by building a rain sensor. They placed a rectangular tank outside and attached two metal plates to two opposite sides in an effort to make a capacitor whose capacitance varies with the amount of water inside.



The width and length of the tank are both  $w$  (i.e., the base is square) and the height of the tank is  $h_{\text{tot}}$ .

- What is the capacitance between terminals  $a$  and  $b$  when the tank is full? What about when it is empty?  
*Note:* the permittivity of air is  $\epsilon$ , and the permittivity of rainwater is  $81\epsilon$ .
- Suppose the height of the water in the tank is  $h_{\text{H}_2\text{O}}$ . Modeling the tank as a pair of capacitors in parallel, find the total capacitance between the two plates. Call this capacitance  $C_{\text{tank}}$ .
- After building this capacitor, the farmer consults the internet to assist them with a capacitance-measuring circuit. A fellow internet user recommends the following:



In this circuit,  $C_{\text{tank}}$  is the total tank capacitance that you calculated earlier.  $I_s$  is a known current supplied by a current source.

The suggestion is to measure  $V_C$  for a brief interval of time, and then use the difference to determine  $C_{\text{tank}}$ .

Determine  $V_C(t)$ , where  $t$  is the number of seconds elapsed since the start of the measurement. You should assume that before any measurements are taken, the voltage across  $C_{\text{tank}}$ , i.e.  $V_C$ , is initialized to  $0\text{ V}$ , i.e.  $V_C(0) = 0$ .

- Using the equation you derived for  $V_C(t)$ , describe how you can use this circuit to determine  $C_{\text{tank}}$  and  $h_{\text{H}_2\text{O}}$ .

## 8. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.