EECS 16A Designing Information Devices and Systems I
Fall 2020 Homework 11

This homework is due November 13, 2020, at 23:59.
Self-grades are due November 16, 2020, at 23:59.

Submission Format

Your homework submission should consist of **one** file.

- `hw11.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned)

Submit each file to its respective assignment on Gradescope.

1. **Reading Assignment**

For this homework, please read Notes 18 and 19. They will provide an overview of operational amplifiers (op-amps), negative feedback, the "golden rules" of op-amps, and various op-amp configurations (non-inverting, inverting, buffers, etc). You are always encouraged to read beyond this as well.

(a) What are the two "golden rules" of ideal op amps? When do these rules hold true?

(b) What is the internal gain of an op-amp, A? What is its value for an ideal op-amp? For non-ideal?

2. **Op-Amp in Negative Feedback**

(Contributors: Adhyyan Narang, Ava Tan, Aviral Pandey, Deepshika Dhanasekar, Lam Nguyen, Panos Zarkos, Titan Yuan, Vijay Govindarajan, Urmita Sikder)

In this question, we analyze op amp circuits that have finite op amp gain A. We replace the op amp with its circuit model with parameterized gain and observe the gain's effect on terminal and output voltages as the gain approaches infinity. Figure 1 shows the equivalent model of the op-amp. **Note here that** $V_{SS} = -V_{DD}$**.**

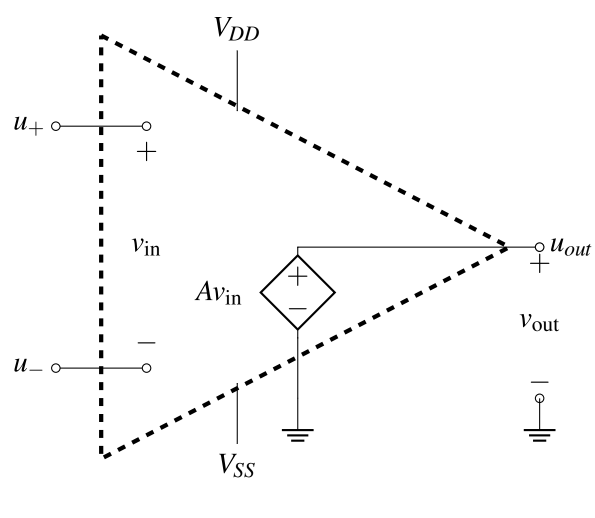


Figure 1: Op-amp model

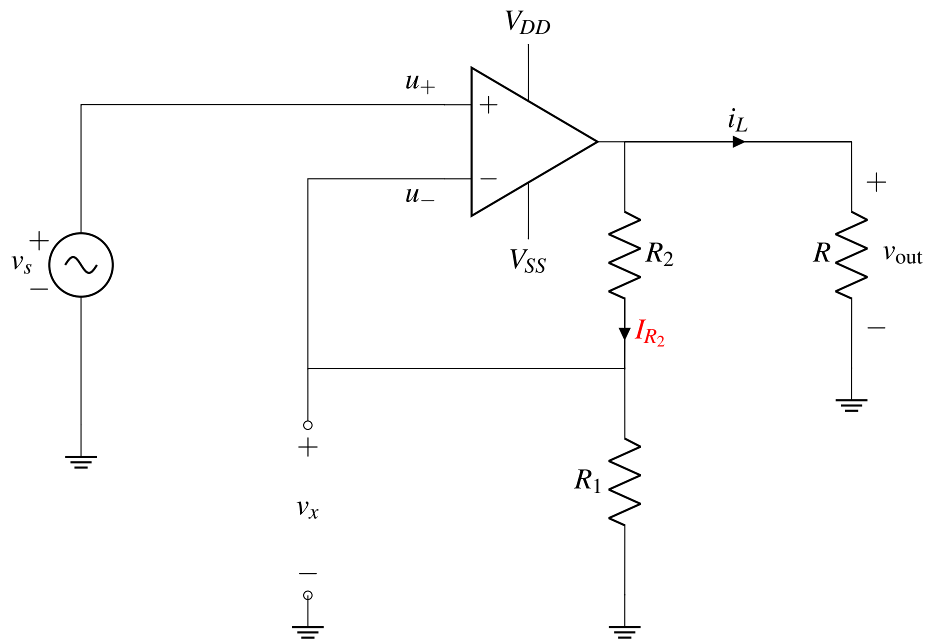


Figure 2: Non-inverting amplifier circuit

For parts (a) - (e) only, assume that the op amp is ideal (i.e., $A \to \infty$). We will consider the case of finite gain A in parts (f) - (h).

(a) Consider the circuit shown in Figure 2, and again $V_{SS} = -V_{DD}$. What is $u_+ - u_-$?

(b) Find v_x as a function of v_{out}.

(c) What is I_{R_2}, i.e. the current flowing through R_2 as a function of v_s? *Hint: Find the current through R_1 first.*

(d) Find v_{out} as a function of v_s.

(e) What is the current i_L through the load resistor R? Give your answer in terms of v_{out}.

(f) We will now examine what happens when A is not ∞. To understand what happens in this case, first draw an equivalent circuit for Figure 2, **by replacing the ideal op-amp in the non-inverting amplifier in Figure 2 with the op-amp model shown in Figure 1.**

Now, using this setup, calculate v_{out} and v_x in terms of A, v_s, R_1, R_2 and R. Is the magnitude of v_x larger or smaller than the magnitude of v_s? Do these values depend on R? *Hint: Note that the first golden rule still applies, i.e. the currents through the input terminals are zero.*

(g) Using your solution to the previous part, calculate the limits of v_{out} and v_x as $A \to \infty$. You should get the same answer as in part (d) for v_{out}.

(h) **[OPTIONAL, CHALLENGE]** Now you want to make a non-inverting amplifier circuit whose gain is nominally $G_{nom} = \frac{v_{\text{out}}}{v_s} = 1 + \frac{R_2}{R_1} = 4$. However, G_{nom} can only be achieved only if the op-amp is ideal, i.e, if its internal gain $A \to \infty$. But, as with most considerations in the physical world, we must account for nonidealities! In reality, because you will be working with an op-amp with finite gain A, your designed circuit gain may come close to but will never quite reach G_{nom} as a result of the real op-amp's finite internal gain A.

Suppose you would like your real op-amp circuit to have a minimum error of 1% (i.e, a minimum circuit gain of 3.96, i.e. $\frac{v_{\text{out}}}{v_s} \geq 3.96$). Remember that only if your op-amp were ideal, you would have a nominal circuit gain of $G_{nom} = \frac{v_{\text{out}}}{v_s} = 1 + \frac{R_2}{R_1} = 4$.

What is the minimum required value of A, called A_{min}, to achieve that specification? *Hint: Use your expression of* v_{out} *in part (f) to find an expression for* $G_{min} = \frac{v_{\text{out}}}{v_s}$ *when* $A \not\to \infty$.

3. Transresistance Amplifier

(Contributors: Ava Tan, Wahid Rahman, Urmita Sikder)

A common use of an op-amp is to convert a current signal into a voltage signal. This configuration is called a *transresistance amplifier*, as shown in Fig. 3. (Note: In the real world, we call this a trans*impedance* amplifier. Impedance is just a fancy word to describe resistance as a function of frequency.) Assume that $V_{SS} = -V_{DD}$ for all the parts of this problem.

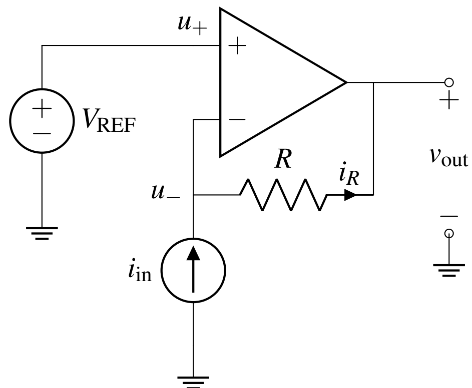


Figure 3: Transresistance amplifier

(a) What is the value of the current i_R in Fig. 3? *Hint: Your answer should be in terms of* i_{in}.

(b) What is the voltage at the negative terminal of the op-amp u_-.? *Hint: You answer should be in terms of* V_{REF}

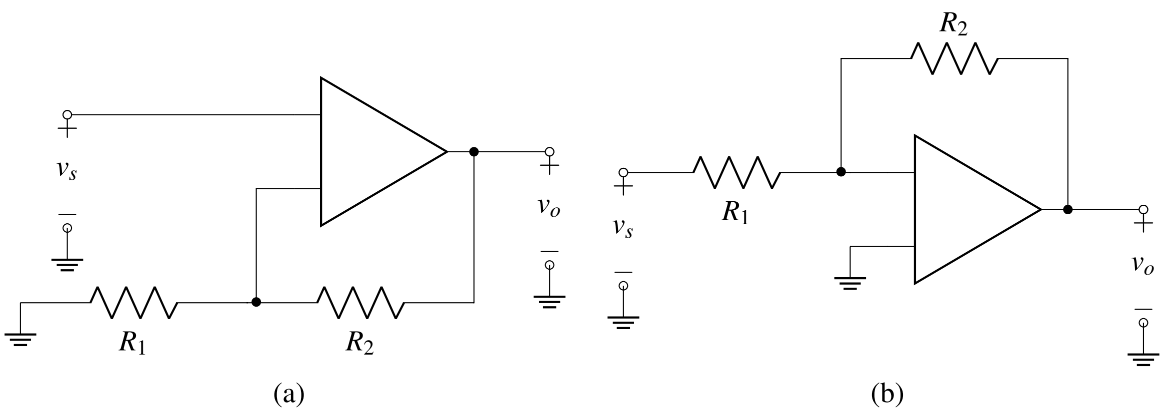
(c) Using the results from parts (a) and (b), find an expression of v_{out} in terms of V_{REF}, i_{in} and other relevant parameters.

(d) If we set $V_{\text{REF}} = 0\,\text{V}$, calculate the gain of the overall circuit $G = \frac{v_{\text{out}}}{i_{\text{in}}}$. Note that in this configuration, the input signal is current i_{in} (*aside: contrast this with other op-amp circuit examples that you have seen in which the input is typically a voltage*), and the output signal is voltage v_{out}. Therefore, in this case, you will want to report the gain of this circuit as $\frac{v_{\text{out}}}{i_{\text{in}}}$.

4. Basic Amplifier Building Blocks

(Contributors: Ava Tan, Aviral Pandey, Lam Nguyen, Michael Kellman, Titan Yuan, Vijay Govindarajan, Urmita Sikder)

The following amplifier stages are used often in many circuits and are well known as (a) the non-inverting amplifier and (b) the inverting amplifier.



(a) Label the input terminals of the op-amp with $(+)$ and $(-)$ signs in Figure (a), so that it is in negative feedback. Then derive the voltage gain ($G = \frac{v_o}{v_s}$) of the non-inverting amplifier in Figure (a) using the Golden Rules. Why do you think this circuit is called a non-inverting amplifier?

(b) Label the input terminals of the op-amp with $(+)$ and $(-)$ signs in Figure (b), so that it is in negative feedback. Then derive the voltage gain ($G = \frac{v_o}{v_s}$) of the inverting amplifier using the Golden Rules. Can you explain why this circuit is called an inverting amplifier?

5. Cool For The Summer

(Contributors: Ava Tan, Aviral Pandey, Lam Nguyen, Lydia Lee, Titan Yuan, Urmita Sikder, Vijay Govindarajan)

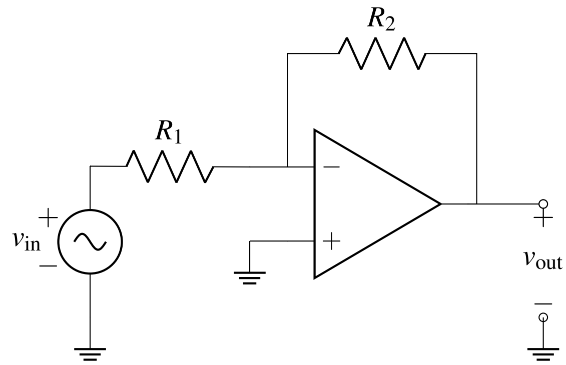
You and a friend want to make a box that helps control an air conditioning unit based on both your inputs. You both have individual dials which you can each use to set a control voltage. An input of 0 V means that you want to leave the temperature as is. A **negative voltage input** means that you want to **reduce** the temperature. (It's hot out, so we will assume that you never want to increase the temperature – so no, we're not talking about a Berkeley summer...)

Your air conditioning unit, however, responds only to **positive voltages**. The higher the magnitude of the voltage, the stronger it runs. At zero, it is off. You also need a system that **sums up** both you and your friend's control inputs.

Therefore, you need a box that acts as an **an inverting summer** – *it outputs a weighted sum of two voltages where the weights are both negative*. The sum is weighted because each of you has your own subjective sense of how much to turn the dial down, so you need to compensate for this.

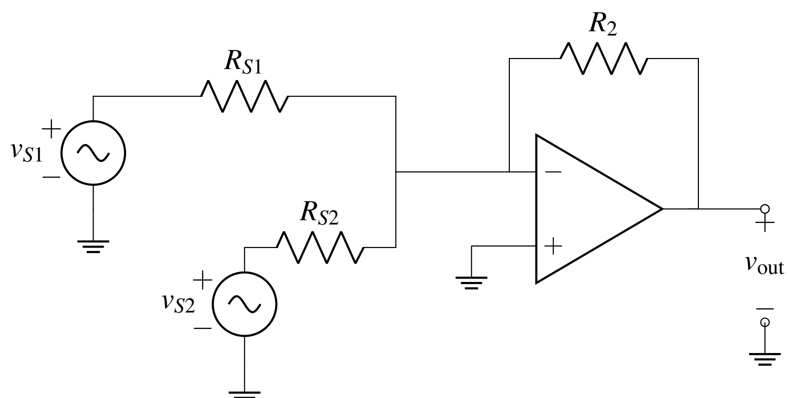
This problem walks you through designing this inverting summer using an op-amp.

(a) As a first step, derive v_{out} in terms of R_2, R_1, and v_{in}.
Hint: Have you solved for this particular amplifier configuration before? You can use your answer from the time you did this earlier.



(b) Now we will add a second input to this circuit as shown below. Find v_{out} in terms of v_{S1}, v_{S2}, R_{S1}, R_{S2} and R_2.

Hint: You can solve this problem using either superposition or our tried-and-true KCL analysis.



(c) Let's suppose that you want $v_{out} = -\left(\frac{1}{4}v_{S1} + 2v_{S2}\right)$ where v_{S1} and v_{S2} represent the input voltages from you and your friend. Select resistor values such that the circuit from part (b) implements this desired relationship.

(d) Now suppose that you have a new AC unit that you want to use with your control inputs v_{S1} and v_{S2}. **This unit, however, responds only to negative voltages – the opposite of your previous air conditioning unit, which only responded to positive input voltages.** For this unit, the higher the magnitude of the negative voltage, the stronger the AC runs.

You want to reuse your older design to now design a circuit for the new AC unit. So your circuit takes in as input two control voltages but outputs a weighted sum, such that the weighted sum becomes more negative as you increase your input voltages.

Hint: Specifically, you could consider adding another op-amp circuit to the output of your circuit from part (b), such that you invert the output of the op-amp circuit of part (b) ***without*** *adding additional gain.*

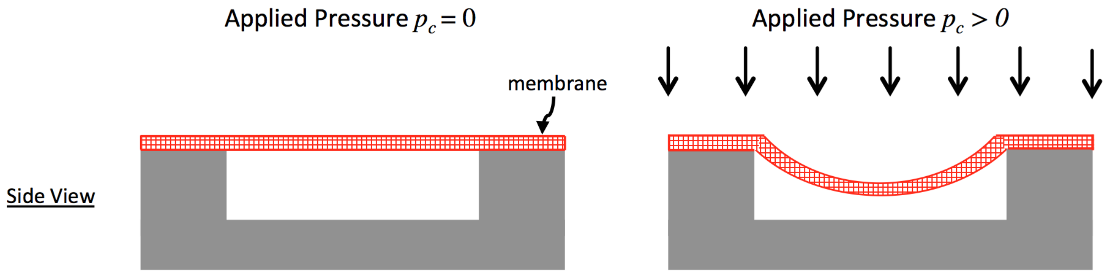
6. Putting on the Pressure: Build Your Own InstantPot

(Contributors: Craig Schindler, Ava Tan, Wahid Rahman, Urmita Sikder)

Prof. Ranade had a great experience with her automatic pressure cooker, so she was inspired to try and build her own. She's enlisting your help! The design of the pressure cooker uses a pressure sensor and a heating element. Whenever the pressure is below a set target value, an electronic circuit turns on the heating element.

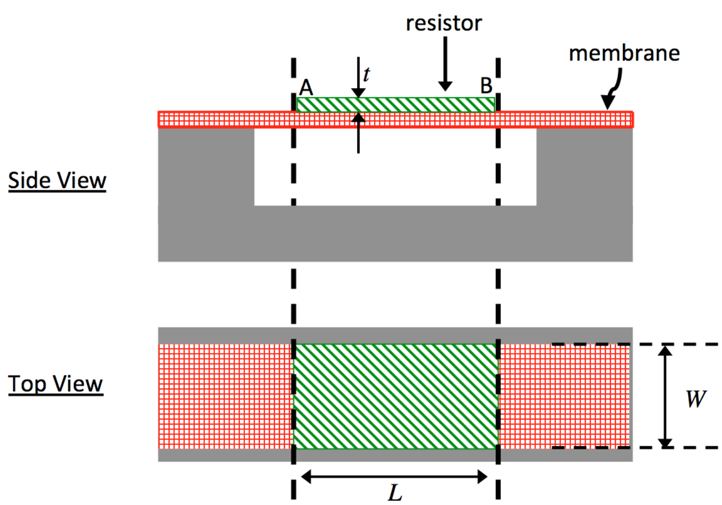
Pressure Sensor Resistance

The first step is designing a pressure sensor. The figure below shows your design. As pressure p_c is applied, the flexible membrane stretches.



(a) You attach a resistor layer R_p with resistivity $\rho = 0.1\Omega\text{m}$, width W, length L, and thickness t to the pressure sensor membrane, as illustrated in the figure below. When the pressure $p_c = 0\,\text{Pa}$ (i.e. there is no applied pressure), $W = 1\,\text{mm}$, $L = L_0 = 1\,\text{cm}$, $t = 100\,\mu\text{m} = 100 \times 10^{-6}$ m.

R_{p0} **is the value of** R_p **when there is no applied pressure. Calculate** R_{p0}**.** Note that direction of current flow in the resistor is from A to B as marked in the diagram.



(b) When pressure is applied, the length of the resistor L changes from L_0 and is a function of applied pressure p_c, and is given by

$$L = L_0 + \beta p_c,$$

where L_0 is the nominal length of the resistor with no pressure applied, and β is a constant with units m/Pa. As a result of the length change, the value of resistance R_p also changes from its nominal value R_{p0} (the value of R_p with no pressure applied).

Derive an expression for R_p **as a function of resistivity** ρ**, width** W**, thickness** t**, nominal length** L_0**, constant** β**, and applied pressure** p_c**, when pressure is applied.**

Note: The width and thickness of the resistor will also change with applied pressure. However, we ignore this to keep the math simple.

(c) **Pressure Sensor Circuit Design**

For this sub-part and the following sub-parts, we will use a new model for pressure-sensitive resistance R_p. Assume that the resistance R_p is a function of applied pressure p_c according to the relationship $R_p = R_o \times \frac{p_c}{p_{\text{ref}}}$, where $R_o = 1\text{k}\Omega$, and $p_{\text{ref}} = 100\text{kPa}$.

To complete our sensor circuit, we would like to generate a voltage V_p that is a function of the pressure p_c.

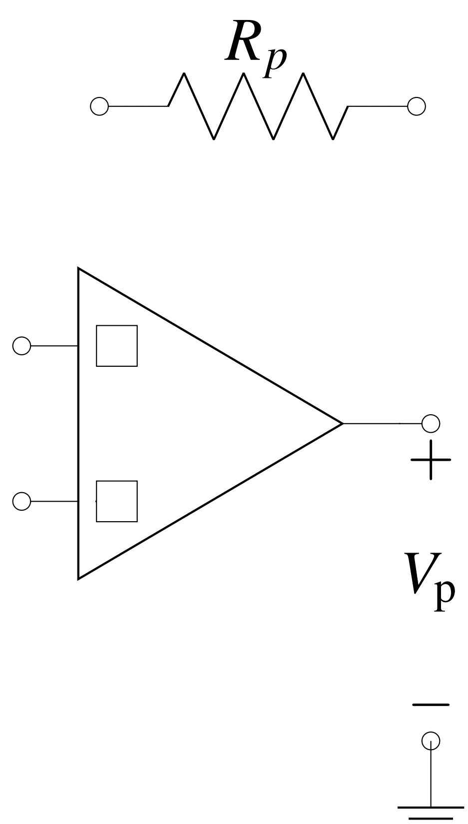
Complete the circuit below so that the output voltage V_p depends on the pressure p_c as:

$$V_p = -V_o \times \frac{p_c}{p_{\text{ref}}}, \text{ where } V_o = 1\,\text{V}.$$

Restrictions on your pressure sensor circuit design are as follows:

- **You may add at most one ideal voltage source and one additional resistor to the circuit, but you must calculate their values and mark them in the diagram.**
- **Mark the positive and negative inputs of the operational amplifier with "+" and "-" symbols, respectively, in the boxes provided.**
- **Assume op-amp supply voltages V_{DD} and $V_{SS} = -V_{DD}$ are already provided.**

You may assume that the operational amplifier is ideal.



(d) **Resistive Heating Element**

To heat the pressure cooker, you use a heating element with resistance R_{heat}. Calculate the value of R_{heat} such that the power dissipated is $P_{\text{heat}} = 1000\,\text{W}$ with $V_{\text{heat}} = 100\,\text{V}$ applied across the heating element.

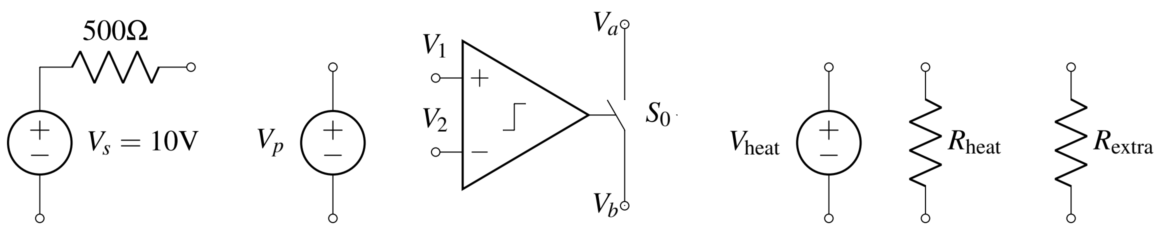
(e) **Pressure Regulation**

You are finally ready to complete the design of your pressure cooker.

Using all of the circuit elements below, make a circuit that will turn the heater on (i.e. will cause a current to flow through R_{heat}) when the pressure is less than 500 kPa, and off (i.e. will cause no current to flow through R_{heat}) when the pressure is greater than 500 kPa.

The elements are:

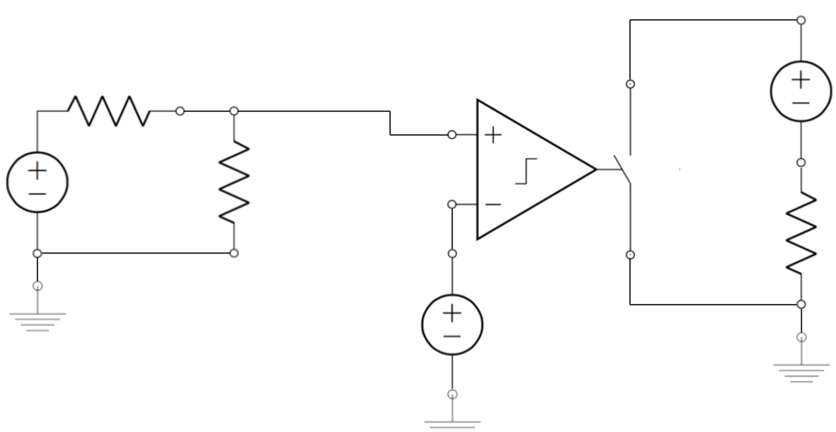
- A voltage source $V_s = 10\text{V}$ in series with a resistance of 500Ω.
- A voltage source $V_p = V_o \times \frac{p_c}{p_{\text{ref}}}$, with V_o = 1V and $p_{\text{ref}} = 100\text{kPa}$. (This is a voltage source whose voltage is a function of pressure p_c, unrelated to any previous parts of the question.)
- A comparator that controls switch S_0. The switch is normally opened (i.e. an open circuit between nodes V_a and V_b), and is closed only when $V_1 > V_2$ (i.e. a short circuit between nodes V_a and V_b).
- The heater supply ($V_{\text{heat}} = 100\text{V}$).
- The heater resistor R_{heat}.
- One additional resistor R_{extra} that can have **any value**.
- You may assume you have access to a ground node.
- Assume comparator supply voltages V_{DD} and $V_{SS} = -V_{DD}$ are already provided.



(i) Since you are looking to *compare* the change in voltage associated with a change in pressure, you decide to assign the **variable voltage** source V_p as one of the inputs to your comparator.
What is the value of the **variable voltage source** V_p for $p_c = 500\text{kPa}$?

(ii) For your comparator inputs, you also need to generate a **reference voltage** which can be used to compare against the value of the **variable voltage** source which you calculated above.
Combine the voltage source $V_s = 10\text{V}$ with an associated resistance of 500Ω with an an additional resistor R_{extra} to **generate a reference voltage equal to the voltage V_p you calculated above.** What would you choose the value of R_{extra} to be?

(iii) Label the circuit elements in the schematic below with the **circuit elements presented above and your calculated R_{extra} value** to turn the heater on when the pressure is less than 500 kPa.



7. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.