EECS 16A Designing Information Devices and Systems I
Spring 2021 Homework 7

This homework is due March 12, 2021, at 23:59.
Self-grades are due March 15, 2021, at 23:59.

Submission Format
Your homework submission should consist of **one** file.

- `hw7.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned)

Submit the file to the appropriate assignment on Gradescope.

1. **Reading Assignment**

   For this homework, please read Notes 13 and 14. Note 13 will refresh you on how simple 1-D resistive touchscreens work, as well as the notion of power in electric circuits. Note 14 will cover a slightly more complicated 2-D resistive touchscreens and how to analyze them from a circuits perspective.

   (a) Describe the key ideas behind how the 1D touchscreen works. In general, why is it useful to be able to convert a “physical" quantity like the position of your finger to an electronic signal (i.e. voltage)?

   Solution: You should give yourself full-credit for any reasonable answers.

2. **[PRACTICE] It’s a Triforce!**

   Learning Goal: *This problem explores passive sign convention and nodal analysis in a slightly more complicated circuit.*

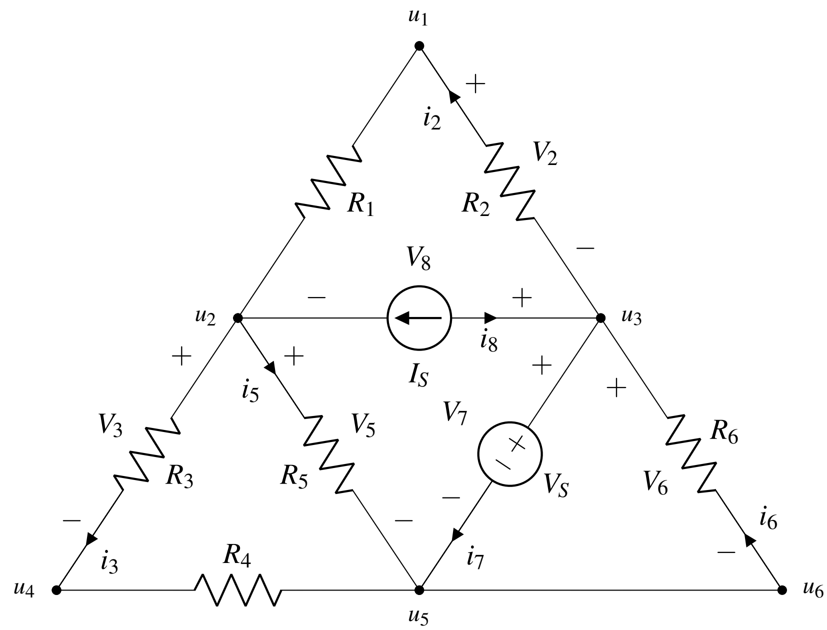


Figure 1: A triangular circuit consisting of a voltage source V_s, current source I_s, and resistors R_1 to R_6.

(a) Which of the elements I_S, V_S, R_2, R_3, R_5, or R_6 in Figure 1 have current-voltage labeling that violates Passive Sign Convention? There could be more than one possible element which violates Passive Sign Convention. Explain your reasoning.

Solution:

The correct answers are I_S, R_2, and R_6 since the current directions disagree with the signs chosen and depict the current leaving the positive terminal (or entering the negative terminal, equivalently). You would simply swap the signs for all components to fix them.

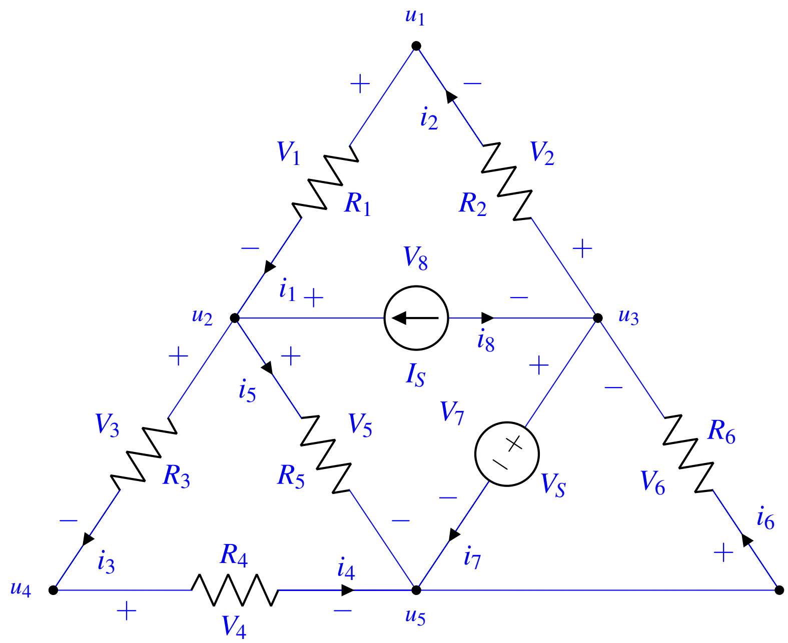
(b) In Figure 1, the nodes are labeled with u_1, u_2, ... etc. There is a subset of u_i's in the given circuit that are redundant, i.e. there might be more than one label for the same node. Which node(s)? Justify your answer.

Solution:

The correct answer is u_5 and u_6 since there is a short connecting them.

(c) Redraw the circuit diagram by correctly labeling all the element voltages and element currents according to passive sign convention. (The component labels that were violating Passive Sign Convention in part (a), should be corrected by swapping the element voltage polarity. Also, the elements that have not been labeled yet, should be labeled following Passive Sign Convention.)

Solution:



This is one of the possible correct labellings. There are other valid answers that obey Passive Sign Convention.

(d) Write an equation to describe the current-voltage relationship for element R_4 in terms of the relevant i's, R's, and node voltages in this circuit.

Solution:

The resulting equation should look like:

$$R_4 = \frac{u_4 - u_5}{i_4}$$

Alternatively, if you labelled the current i_4 going in the opposite direction (going from u_5 to u_4), the resulting equation should look like:

$$R_4 = \frac{u_5 - u_4}{i_4}$$

(e) Write the KCL equation for node u_2 in terms of the node voltages and other circuit elements.

Solution:

KCL at u_2 gives us:

$$-i_1 + i_8 + i_3 + i_5 = 0$$

Note that in this solution, we have assumed that the current i_8 is flowing out of node u_2. You could have also assumed the current flows into the node if you corrected it from the previous part. So long as you were consistent with your polarity and current direction, you should have gotten the correct answer.

The equations for the current through each of the branches are:

$$i_1 = \frac{u_1 - u_2}{R_1}$$
$$i_3 = \frac{u_2 - u_4}{R_3}$$
$$i_8 = -I_S$$
$$i_5 = \frac{u_2 - u_5}{R_5}$$

The final expression is then:

$$-\frac{u_1 - u_2}{R_1} - I_S + \frac{u_2 - u_5}{R_5} + \frac{u_2 - u_4}{R_3} = 0$$

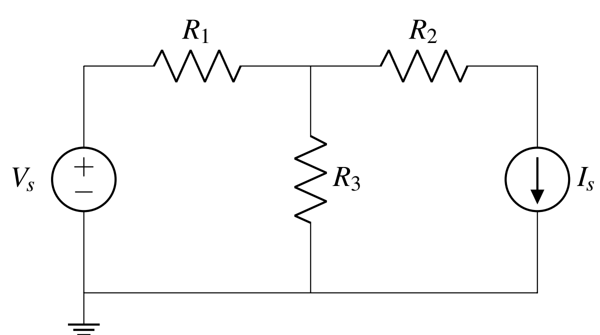
Or any equivalent equation.

3. Circuit Analysis

Learning Goal: *This problem will help you practice circuit analysis using NVA method.*

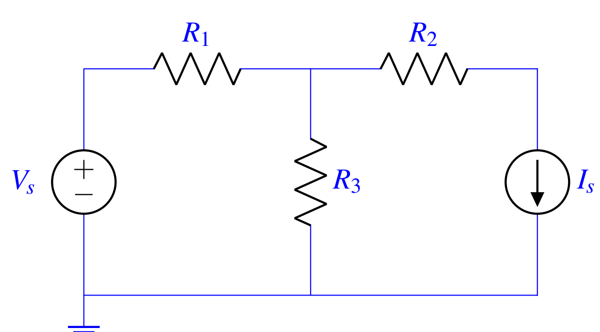
Using the steps outlined in lecture or in Note 11, analyze the following circuits to calculate the currents through each element and the voltages at each node. Use the ground node labelled for you. You may use a numerical tool such as IPython to solve the final system of linear equations.

$V_s = 5\,\text{V}, I_s = 2\,\text{A}, R_1 = R_2 = 2\,\Omega, R_3 = 4\,\Omega$

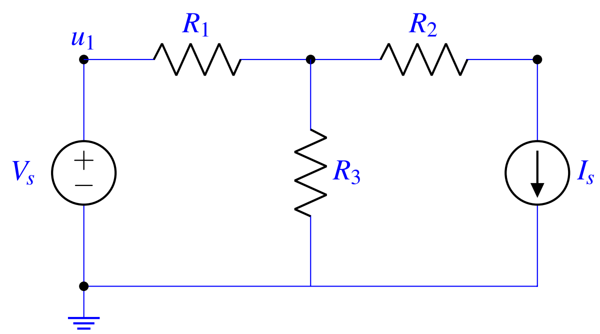


Solution:

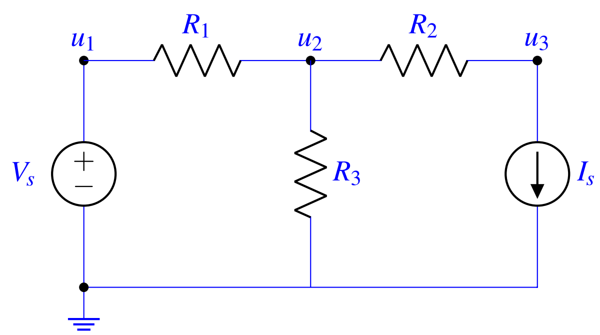
Step 1) Define Reference Node
Select a reference (ground) node. Any node can be chosen for this purpose. In this example, we choose the node at the bottom of the circuit diagram.



Step 2) Label Nodes with Voltage Set by Sources
Voltage sources set the voltage of the node they are connected to. In the example, there is only one source, V_s, and we label the corresponding source u_1 (names are arbitrary, but must be unique).

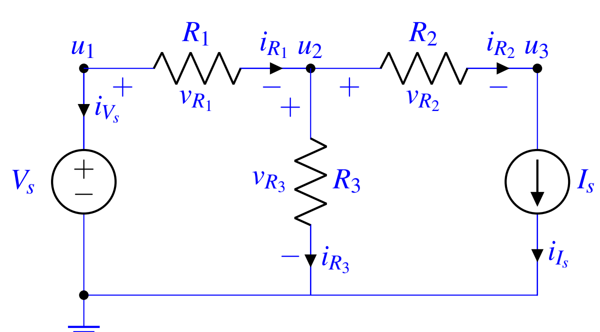


Step 3) Label Remaining Nodes
Now we label all remaining nodes in the circuit except the reference. In the example there are two, u_2 and u_3.



Step 4) Label Element Voltages and Currents

Next we mark all element voltages and currents.
Start with the current. The direction is arbitrary (top to bottom, bottom to top, it won't matter, but stick with your choice in subsequent steps). Then mark the element voltages following the passive sign convention, i.e. the voltage and current point in the "same" direction.



Step 5) KCL Equations
Write KCL equations for all nodes with unknown voltage, u_2 and u_3.

$$-i_{R_1} + i_{R_2} + i_{R_3} = 0$$
$$-i_{R_2} + i_{I_s} = 0$$

Step 6) Element Currents

Find expressions for all element currents in terms of voltage and element characteristics (e.g. Ohm's law) for all circuit elements except voltage sources. In this circuit there are four, R_1, R_2, R_3 and I_s.

$$i_{I_s} = I_s$$
$$i_{R_1} = \frac{V_{R_1}}{R_1}$$
$$i_{R_2} = \frac{V_{R_2}}{R_2}$$
$$i_{R_3} = \frac{V_{R_3}}{R_3}$$

We also have

$$u_1 = V_s$$
$$i_{I_s} = I_s$$
$$i_{R_1} = \frac{u_1 - u_2}{R_1}$$
$$i_{R_2} = \frac{u_2 - u_3}{R_2}$$
$$i_{R_3} = \frac{u_2}{R_3}$$

Step 7) Substitute Element Currents in KCL Equations

Now we substitute the expressions derived in Step 6 into the KCL equations from Step 5.

$$-\frac{V_s - u_2}{R_1} + \frac{u_2 - u_3}{R_2} + \frac{u_2}{R_3} = 0$$
$$-\frac{u_2 - u_3}{R_2} + I_s = 0$$

Let's make this a bit nicer by grouping the unknowns (u_2 and u_3) on the left side and the known terms on the right:

$$u_2\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) + u_3\left(-\frac{1}{R_2}\right) = \frac{V_s}{R_1}$$
$$u_2\left(-\frac{1}{R_2}\right) + u_3\left(\frac{1}{R_2}\right) = -I_s$$

Note that we now have 2 equations for 2 unknowns. Thus, we set up the following matrix relation:

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_1} \\ -I_s \end{bmatrix}$$

Finally, we plug in the values we were given into the matrix above and use Gaussian Elimination to find the vector of unknowns. We find that:

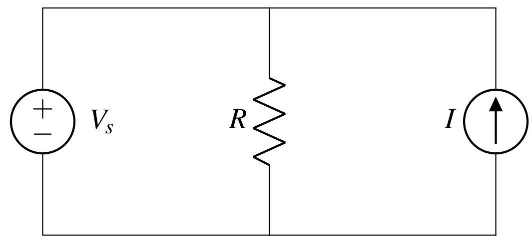
$$\begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0.667V \\ -3.33V \end{bmatrix}$$

The branch currents can be obtained from the node voltages and element equations. Therefore, we can write:

$$\begin{aligned} i_{V_s} &= -2.167A \\ i_{I_s} &= I_s = 2A \\ i_{R_1} &= \frac{u_1-u_2}{R_1} = 2.167A \\ i_{R_2} &= \frac{u_2-u_3}{R_2} = 2A \\ i_{R_3} &= \frac{u_2}{R_3} = 0.167A \end{aligned}$$

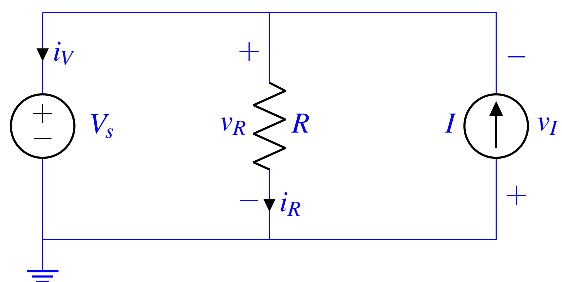
4. Power Analysis

Learning Goal: *This problem aims to help you practice calculating power dissipation in different circuit elements. It will also give you insights into how power is conserved in a circuit.*



(a) Find the expressions of power dissipated by each element in the circuit above. Remember to label voltage-current pairs using passive sign convention.

Solution: We label a ground node, and then solve for the currents i_V, i_R and the voltages V_R, V_I.



Solving the above circuit using nodal analysis, we get

$$i_R = \frac{V_s}{R}$$

$$i_V = I - \frac{V_s}{R}$$

$$v_I = -V_s$$

$$v_R = V_s$$

Using this we can calculate

$$P_{V_s} = V_s i_V = IV_s - \frac{{V_s}^2}{R}$$

$$P_I = Iv_I = -IV_s$$

$$P_R = i_R v_R = \frac{{V_s}^2}{R}$$

Note that $P_{V_s} + P_I + P_R = 0$, i.e. energy provided is energy dissipated, which verifies our intuition about conservation of energy.

(b) Use $R = 5\text{k}\Omega$, $V_s = 5\text{V}$, and $I = 5\text{mA}$. Calculate the power dissipated by the voltage source (P_{V_s}), the current source (P_I), and the resistor (P_R).

Solution:

$P_{V_S} = (0.005\text{A})(5\text{V}) - \frac{(5\text{V})^2}{5000\Omega} = 0.02W$
$P_I = -(0.005\text{A})(5\text{V}) = -0.025\text{W}$
$P_R = \frac{(5\text{V})^2}{5000\Omega} = 0.005\text{W}$

Note that $P_{V_S} + P_I + P_R = 0$.

(c) Once again, let $R = 5\text{k}\Omega$, $V_s = 5\text{V}$. What does the value I of the current source have to be such that the current source **dissipates** 40mW? Note that it is possible for a current source to *dissipate* power, i.e. under passive sign convention, $P_I = 40mW$. For this value of I, compute P_{V_s}, P_I, and P_R as well.

As an aside: If the current source were delivering power *it would have been $P_I = -40mW$, under passive sign convention, but this is NOT what the question is asking about.*

Solution:

Remember that using passive sign convention, an element whose power is negative is supplying power, and an element whose power is positive is dissipating power. Therefore, we want $P_I = 40\text{mW}$. We know that $P_I = -IV_S$. Therefore, $I = -\frac{0.04\text{W}}{5\text{V}} = -0.008\text{A}$.

$P_{V_S} = (-0.008\text{A})(5\text{V}) - \frac{(5\text{V})^2}{5000\Omega} = -0.045W$
$P_I = -(-0.008\text{A})(5\text{V}) = 0.04\text{W}$
$P_R = \frac{(5\text{V})^2}{5000\Omega} = 0.005\text{W}$

Note that $P_{V_S} + P_I + P_R = 0$.

5. Volt and ammeter

Learning Goal: *This problem helps you explore what happens to voltages and currents in a circuit when you connect voltmeters and ammeters in different configurations.*

Use the following numerical values in your calculations: $R_1 = 1\,\text{k}\Omega$, $R_2 = 2\,\text{k}\Omega$, $R_3 = 3\,\text{k}\Omega$, $R_4 = 4\,\text{k}\Omega$, $R_5 = 5\,\text{k}\Omega$, $V_s = 10V$.

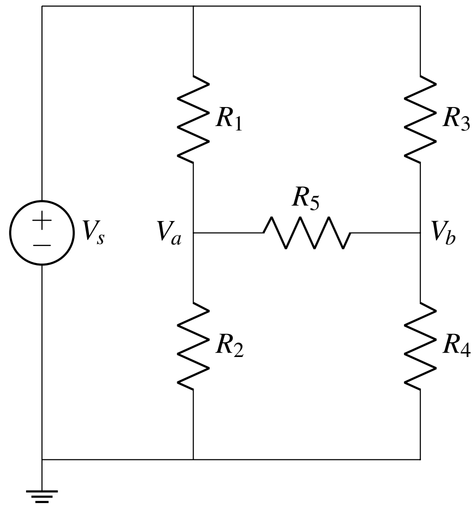
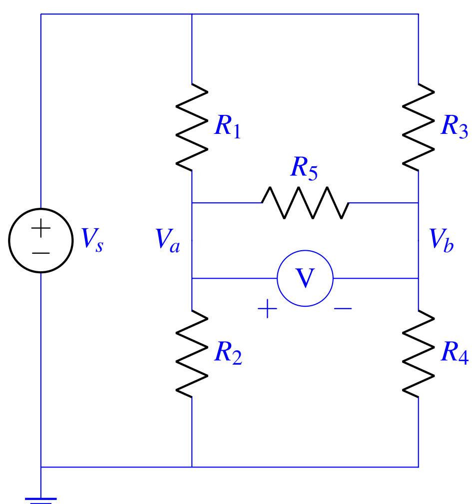


Figure 2: Circuit consisting of a voltage source V_s and five resistors R_1 to R_5

(a) Redraw the circuit diagram shown in Figure 2 by adding a voltmeter (letter V in a circle and plus and minus signs indicating direction) to measure voltage V_{ab} from node V_a (positive) to node V_b (negative). Calculate the value of V_{ab}. You may use a numerical tool such as IPython to solve the final system of linear equations.

Solution: Below is the redrawn circuit with the voltmeter. Note that it is also correct to have the voltmeter above R_5, as it will still be connected to the same nodes.



Using KCL at node V_a, we observe that

$$\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5}\right)V_a-\left(\frac{1}{R_5}\right)V_b=\frac{V_s}{R_1}.$$

Likewise, at node V_b, we observe that

$$-\left(\frac{1}{R_5}\right)V_a+\left(\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}\right)V_b=\frac{V_s}{R_3}.$$

We can then represent this in matrix form as:

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5} & -\frac{1}{R_5} \\ -\frac{1}{R_5} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_1} \\ \frac{V_s}{R_3} \end{bmatrix}$$

Plugging in the values we were given into the matrix above and using Gaussian elimination we can find the vector of unknowns.

$$\begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} 6.58V \\ 5.936V \end{bmatrix}$$

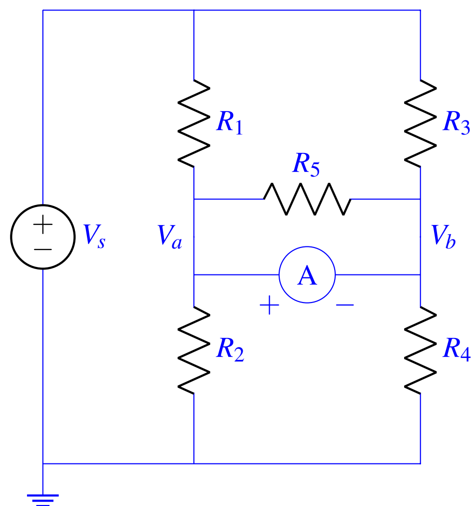
From these node voltages, the voltage V_{ab} can be calculated.

$$V_{ab} = V_a - V_b = 0.644V$$

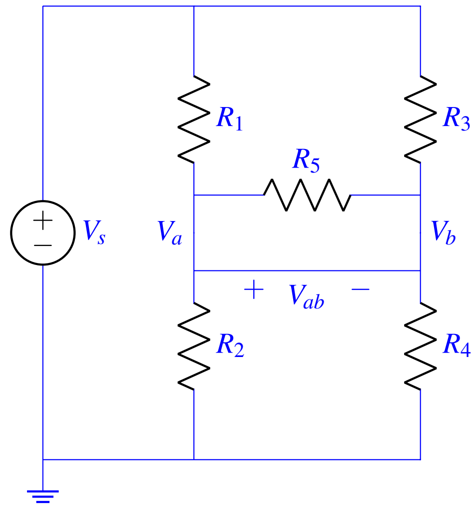
You should give yourself full-credit if your answer is off by a rounding error.

(b) Suppose you accidentally connect an ammeter in part (a) instead of a voltmeter. Calculate the value of V_{ab} with the ammeter connected.

Solution: While you did not have to redraw the circuit, it is depicted below.

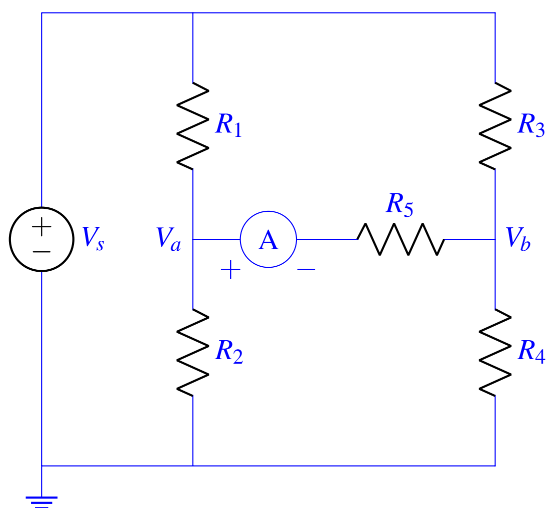


If we assume that the internal resistance of an ammeter is ideally zero, placing it across the nodes V_a and V_b will short them. So $V_a = V_b$. Thus $V_{ab} = 0$. The circuit below shows how the ammeter behaves as a short that unifies the previously separate nodes.



(c) Redraw the circuit diagram shown in Figure 2 by adding an ammeter (letter *A* in a circle and plus and minus signs indicating direction) in series with resistor R_5. This will measure the current I_{R_5} through R_5. Calculate the value of I_{R_5}.

Solution: The redrawn circuit with the ammeter measuring the current through R_5 is shown in the following circuit. It is also correct to draw the ammeter to the right of R_5 with the orientation of the meter remaining the same: the plus sign should be most proximal to the node labeled V_a, and the minus sign is most proximal to the node labeled V_b.



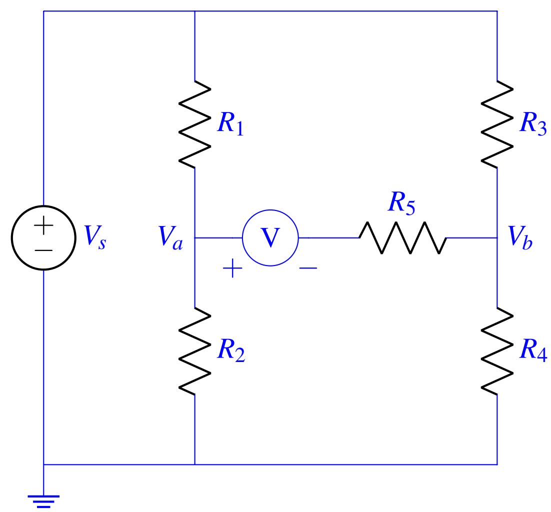
After calculating the node voltages V_a and V_b from part a, we can write:

$$I_{R_5} = \frac{V_a - V_b}{R_5} = 128.8\mu A$$

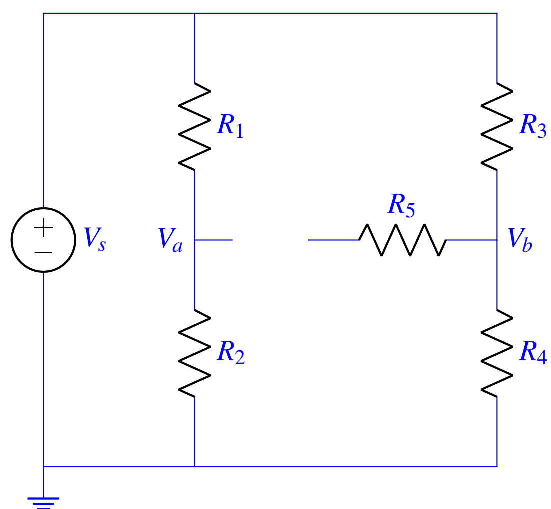
You should give yourself full-credit if your answer is off by a rounding error.

(d) Your friend accidentally connects a voltmeter in part (c) above, rather than an ammeter. Calculate the value of I_{R_5} with the voltmeter connected.

Solution: While you were not required to redraw the new circuit, the circuit is shown below.



The resistance of a voltmeter is infinite and it behaves as an open circuit. There will be no current flowing through R_5. Therefore, $I_{R_5} = 0$. The circuit below depicts how the voltmeter behaves as an open that prevents any current through R_5.



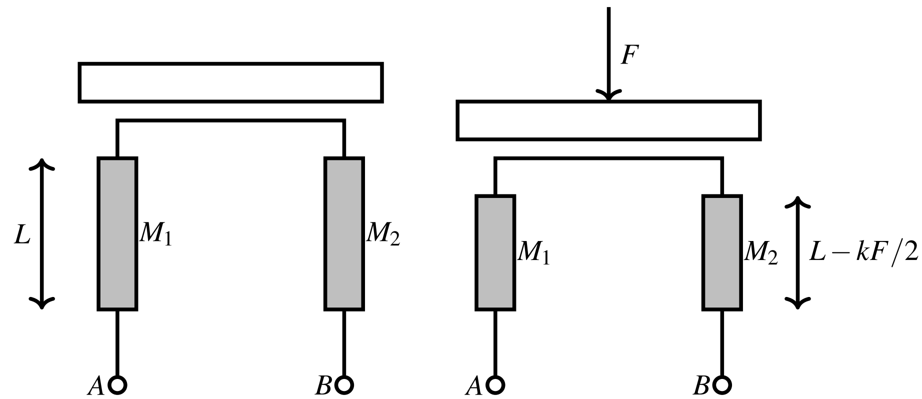
6. Fruity Fred

Learning Goal: *This problem will introduce the process of designing a sensing circuit for the purpose of measuring a physical quantity. This will also help to build your intuition for modeling physical elements.*

Fruity Fred just got back from Berkeley Bowl with a bunch of mangoes, pineapples, and coconuts. He wants to sort his mangoes in order of weight, so he decides to use his knowledge from EECS16A to build a scale.

He finds two identical bars of material (M_1 and M_2) of length L (in meters) and a cross-sectional area (i.e. width $\times$ thickness) of A_c (in meters^2). The bars are made of a material with resistivity ρ. He knows that the **length of these bars decreases** by k meters per Newton of force applied, while the **cross-sectional area remains constant**.

He builds his scale as shown below, where the top of the vertical bars are connected with an ideal electrical wire. The left side of the diagram shows the scale at rest (with no object placed on it), and the right side shows it when the applied force is F (Newtons). The force F is equally distributed between two bars, causing the length of each bar to decrease by $kF/2$ meters. Fred's mangoes are not very heavy, so the change in each bar's length is very small compared to the total length (mathematically, we can write this by using the "much smaller than" symbol $\ll$, i.e. $kF/2 \ll L$).

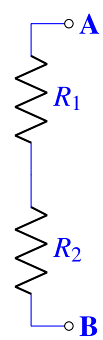


(a) Let R_{AB} be the resistance between nodes A and B with the weights on the scale. Write an expression for R_{AB} as a function of A_c, L, ρ, F, and k. *Hint: You can start by representing each bar as a resistor, then find how they are connected.*

Solution:

Redrawn in circuit representation, we note that Fred's scale design looks like the following. Note that the values of the two resistors R_1 and R_2 are *variable*, and their lengths are changed by the same amount (given in the problem statement as $kF/2$) upon the application of a force F.

The key observation to make is firstly that these two variable resistors are connected in *series*.



Because the length of each bar decreases by k meters per Newton of force applied it, and there are two bars supporting the scale, each bar can be thought to have half the force, $\frac{F}{2}$, on it. Because of this, each bar's length diminishes by $\frac{kF}{2}$, to lengths $L - \frac{kF}{2}$, when a force F is applied to Fred's scale.

Therefore, the combination of R_1 and R_2 has a resistance $R_{AB} = R_1 + R_2$. From the problem statement, we note that because the bars M_1 and M_2 have identical properties of L, A_C, and ρ, $R_1 = R_2$ is calculated as $R_1 = R_2 = \rho \frac{L-kF/2}{A_c}$.

Therefore $R_{AB} = 2 \cdot \rho \frac{L-kF/2}{A_c}$.

(b) Fred wants to measure a voltage that changes based on how much weight is placed on his scale. He knows that R_{AB} will change with the weight on the scale.

Design a circuit for Fred that outputs a voltage that is some function of the weight F. This function does *not* have to be linear. **Your circuit should include R_{AB}, and you may use any number of voltage**

sources and resistors in your design. Be sure to **label** where the voltage should be measured in your circuit.

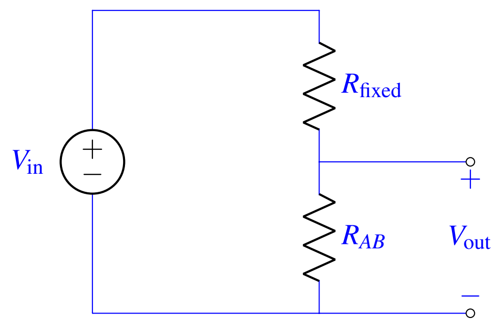
Also provide an **expression** relating the output voltage of your circuit to the force applied on the scale. This expression can contain any necessary parameters.

Hint: If you connected only a voltage source across A and B and measured the voltage (V_{AB}) between A and B, would V_{AB} change based on the value of R_{AB}? It turns out it wouldn't. Why?

Hint: Consider tools we have learned in this class such as voltage dividers. Can you build a circuit using R_{AB} and R_{fixed}, where R_{fixed} is a value you know and choose?

Solution:

If your circuit solution also measures the voltage across just one of the resistances R_1 or R_2 rather than the series combination R_{AB}, your solution is also acceptable. Our solution here chooses the elemental voltage across R_{AB} in a simple voltage divider circuit to measure as V_{out}.



$$V_{\text{out}} = \frac{R_{AB}}{R_{\text{fixed}} + R_{AB}} V_{\text{in}}$$

$$V_{\text{out}} = \frac{\frac{2\rho(L-\frac{kF}{2})}{A_c}}{R_{\text{fixed}} + \frac{2\rho(L-\frac{kF}{2})}{A_c}} V_{\text{in}} = \frac{2\rho(L-\frac{kF}{2})}{R_{\text{fixed}}A_c + 2\rho(L-\frac{kF}{2})} V_{\text{in}}$$

7. Resistive Touchscreen

Learning Goal: *The objective of this problem is to provide insight into modeling of resistive elements. This will also help to apply the concepts from resistive touchscreen.*

In this problem, we will investigate how a resistive touchscreen with a defined thickness, width, and length can actually be modeled as a series combination of resistors. As we know the value of a resistor depends on its length.

Figure 3 shows the top view of a resistive touchscreen consisting of a conductive layer with resistivity ρ_1, thickness t, width W, and length L. At the top and bottom it is connected through good conductors ($\rho = 0$) to the rest of the circuit. The touchscreen is wired to voltage source V_s.

Use the following numerical values in your calculations: $W = 50$ mm, $L = 80$ mm, $t = 1$ mm, $\rho_1 = 0.5\,\Omega\,\text{m}$, $V_s = 5\text{V}$, $x_1 = 20$ mm, $x_2 = 45$ mm, $y_1 = 30$ mm, $y_2 = 60$ mm.

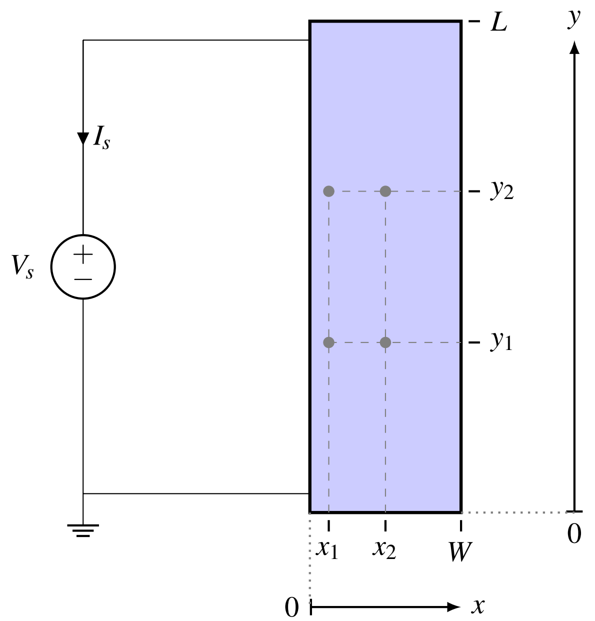
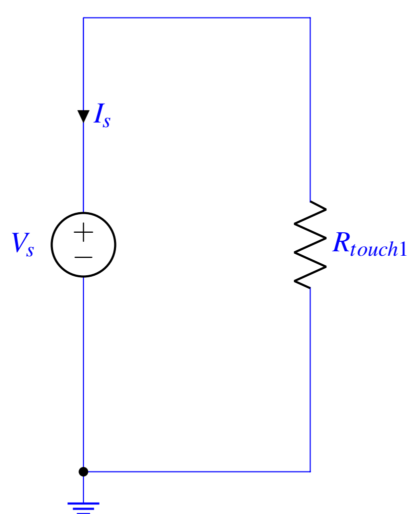


Figure 3: Top view of resistive touchscreen (not to scale). z axis i.e. the thickness not shown (into the page).

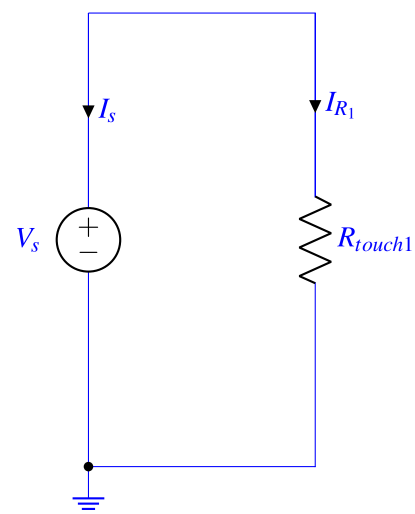
(a) Draw a circuit diagram representing Figure 3, where the touchscreen is represented as *a resistor*. **Note that no touch is occurring in this scenario.** Remember that circuit diagrams in general consist of only circuit elements (resistors, sources, etc) represented by symbols, connecting wires, and the reference/ ground symbol. Calculate the value of current I_S based on the circuit diagram you drew. Do not forget to specify the correct unit as always.

Solution:



The touchscreen resistance can be found from the following expression:

$$\begin{aligned}
R_{touch1} &= \rho_1 \cdot \frac{L}{A} \\
&= \rho_1 \cdot \frac{L}{W \cdot t} \\
&= 0.5\,\Omega\,\mathrm{m}\left(\frac{80\times10^{-3}\,\mathrm{m}}{50\times10^{-3}\,\mathrm{m}\cdot1\times10^{-3}\,\mathrm{m}}\right) \\
R_{touch1} &= 800\,\Omega
\end{aligned}$$



From KCL, we can write:

$$I_s + I_{R_1} = 0 \tag{1}$$
$$I_s = -I_{R_1} \tag{2}$$

Therefore, the current I_{R_1} is equal to:

$$I_{R_1} = \frac{V_s}{R_{touch1}} = \frac{5}{800}A = 6.25mA$$

And the current I_s is equal to:

$$I_s = -I_{R_1} = -6.25mA$$

(b) Let us assume u_{12} is the node voltage at the node represented by coordinates (x_1, y_2) of the touchscreen, as shown in Figure 4. What is the value of u_{12}? You should first draw a circuit diagram representing Figure 4, which includes node u_{12}. Specify all resistance values in the diagram. Does the value of u_{12} change based on the value of the x-coordinate x_1?

Hint: You will need more than one resistor to represent this scenario.

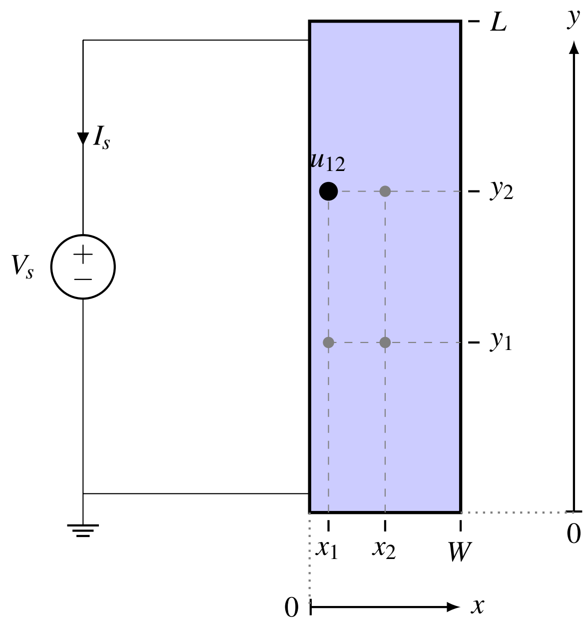
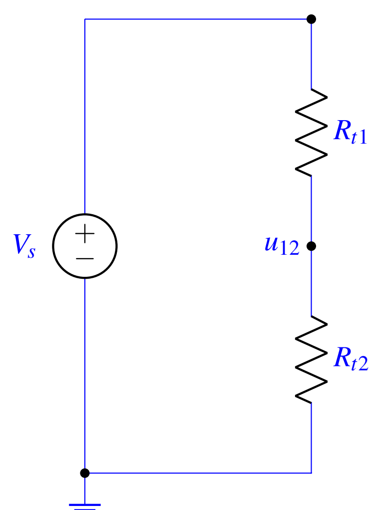


Figure 4: Top view of resistive touchscreen showing node u_{12}.

Solution:
We can represent this setup with the circuit shown below.



Using voltage division, u_{12} can be found from the following expression:

$$u_{12} = V_s \frac{R_{t2}}{R_{t1} + R_{t2}} = V_s \frac{y_2}{L} = 5 \cdot \frac{3}{4} V = 3.75V$$

where $R_{t1} = \rho_1 \cdot \frac{L - y_2}{W \cdot t}$ and $R_{t2} = \rho_1 \cdot \frac{y_2}{W \cdot t}$.

(c) Assume V_{ab} is the voltage measured between the nodes represented by touchscreen coordinates (x_1, y_1) and coordinates (x_1, y_2), as shown in Figure 5. Calculate the absolute value of V_{ab}. As with the previous part, you should first draw the circuit diagram representing Figure 5, which includes V_{ab}. Calculate all resistor values in the circuit. *Hint: Try representing the segment of the touchscreen between these two coordinates as a separate resistor itself.*

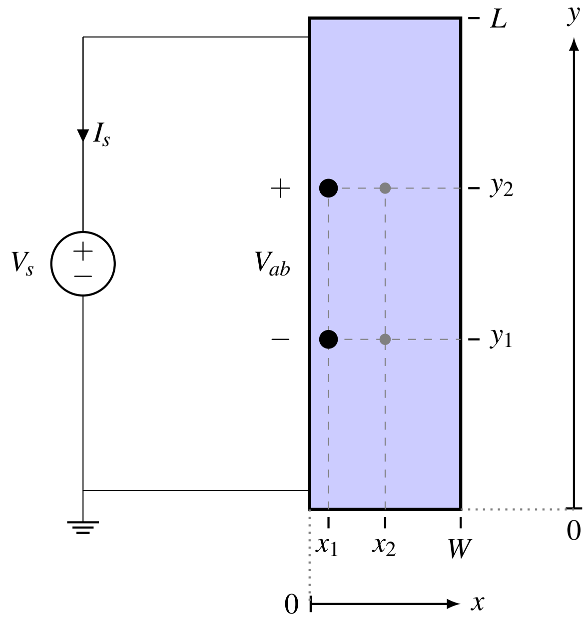
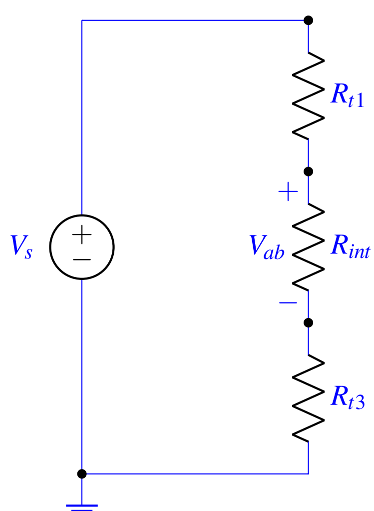
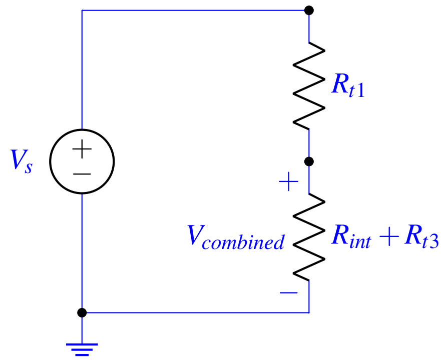


Figure 5: Top view of resistive touchscreen showing voltage V_{ab}.

Solution:



One method to find the voltage is by using node voltage analysis. Presented is an alternative approach by using resistor equivalence. To find the voltage, V_{ab}, first, find the voltage over R_{int} and R_{t3} together. We can represent them as an equivalent resistance as follows:



As this circuit is a voltage divider, we can find $V_{combined}$ by voltage division.

$$V_{combined} = \frac{R_{int}+R_{t3}}{R_{t1}+R_{int}+R_{t3}}V_s$$

Once we know $V_{combined}$, this will be the voltage over the two resistors, R_{int} and R_{t3}. We can apply voltage division again to get V_{ab}:

$$V_{ab} = \frac{R_{int}}{R_{int}+R_{t3}}V_{combined}$$

By substituting, we get that V_{ab} is:

$$\begin{aligned} V_{ab} &= \frac{R_{int}}{R_{int}+R_{t3}}\frac{R_{int}+R_{t3}}{R_{t1}+R_{int}+R_{t3}}V_s \\ &= \frac{R_{int}}{R_{t1}+R_{int}+R_{t3}}V_s \end{aligned}$$

Each of the resistances can be calculated as $R_{t1} = \rho_1 \cdot \frac{L-y_2}{W \cdot t}$, $R_{int} = \rho_1 \cdot \frac{y_2-y_1}{W \cdot t}$ and $R_{t3} = \rho_1 \cdot \frac{y_1}{W \cdot t}$. This gives for V_{ab}:

$$V_{ab} = \frac{y_2-y_1}{L}V_s = \frac{3}{8} \cdot 5V = 1.875V$$

(d) Calculate (the absolute value of) the voltage between the nodes represented by touchscreen coordinates (x_1, y_1) and coordinates (x_2, y_1).

Solution:

The two points have the same y coordinate, therefore they have the same potential in our touchscreen model. Recall that the touchscreen is effectively being modeled as a single resistor which can be considered as several resistors of varying lengths, all totaling to L. Hence, we do not consider the effect of the x-coordinate on potential – we just need to consider the difference in the y-coordinate between two points. Thus, $\Delta V = 0$.

(e) Calculate (the absolute value of) the voltage between the nodes represented by touchscreen coordinates (x_1, y_1) and coordinates (x_2, y_2).

Solution:

The two points have different x and y coordinates. However, the potential is the same across the x-axis for a fixed y coordinate, as was explained in the solution for part (d). Therefore, the problem is similar to part (c), since the potential is only determined by the y-coordinate of a point. Using the same equivalent circuit in part (c) we have:

$$\Delta V = V_s \frac{R_{int}}{R_{t1} + R_{int} + R_{t3}} = 1.875V$$

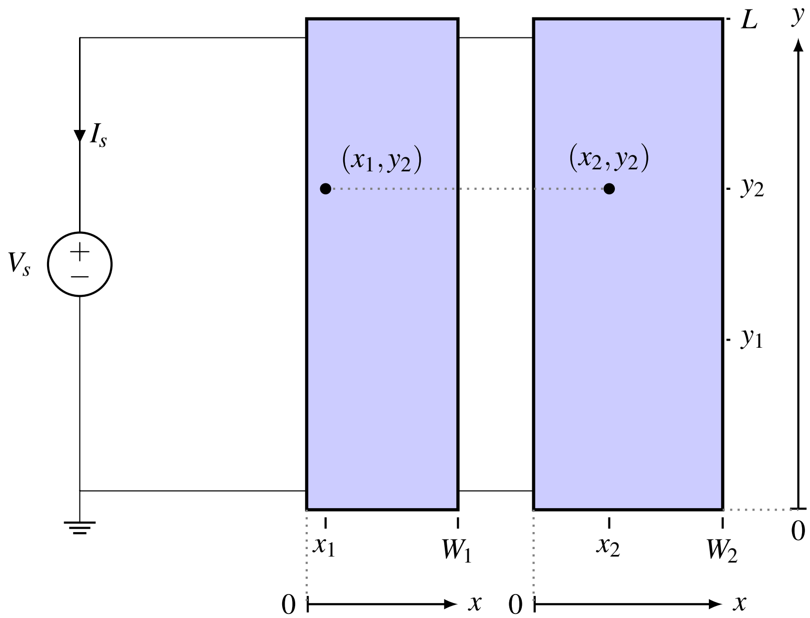


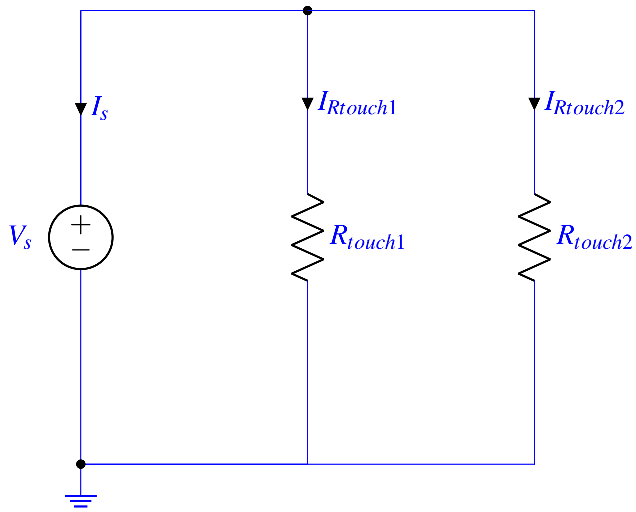
Figure 6: Top view of two touchscreens wired in parallel (not to scale). z axis not shown (into the page).

(f) Figure 6 shows a new arrangement with two touchscreens. The two touchscreens are next to each other and are connected to the voltage source in the same way. The second touchscreen (the one on the right) is identical to the one shown in Figure 3, except for different width, W_2, and resistivity, ρ_2.

Use the following numerical values in your calculations: $W_1 = 50$ mm, $L = 80$ mm, $t = 1$ mm, $\rho_1 = 0.5\,\Omega\,\text{m}$, $V_s = 5\text{V}$, $x_1 = 20$ mm, $x_2 = 45$ mm, $y_1 = 30$ mm, $y_2 = 60$ mm, which are the same values as before. The new touchscreen has the following numerical values which are different: $W_2 = 85$ mm, $\rho_2 = 0.6\,\Omega\,\text{m}$.

Draw a circuit diagram representing Figure 6, where the two touchscreens are represented as *two separate resistors*. **Note that no touch is occurring in this scenario.**

Solution:



(g) Calculate the value of current I_s for the two touchscreen arrangement based on the circuit diagram you drew in the last part.

Solution:

From KCL, we can write:

$$I_s + I_{Rtouch1} + I_{Rtouch2} = 0 \tag{3}$$
$$I_s = -(I_{Rtouch1} + I_{Rtouch2}) \tag{4}$$

Using Ohm's Law for each element:

$$I_s = -\left(\frac{V_s}{R_{touch1}} + \frac{V_s}{R_{touch2}}\right)$$

However, the resistance of the second touchscreen can be given by:

$$R_{touch2} = \rho_2 \cdot \frac{L}{W_2 \cdot t} = 0.6\,\Omega\,\mathrm{m}\left(\frac{80 \times 10^{-3}\,\mathrm{m}}{85 \times 10^{-3}\,\mathrm{m} \cdot 1 \times 10^{-3}\,\mathrm{m}}\right)$$
$$R_{touch2} = 564.705\,882\,4\,\Omega$$

Therefore, using the resistance values for the first and second touchscreen and the applied voltage source, we have:

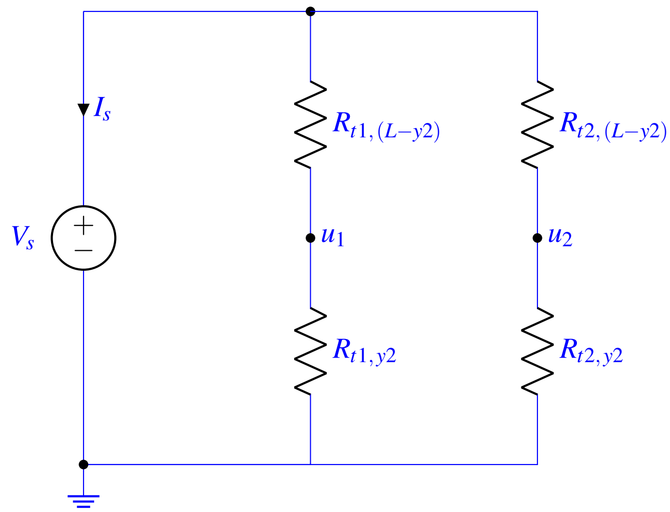
$$I_s \approx -(6.25\mathrm{mA} + 8.85\mathrm{mA}) = -15.1\mathrm{mA}$$

(h) Consider the two points: (x_1, y_2) in the touchscreen on the left, and (x_2, y_2) in the touchscreen on the right in Figure 6. Show that the node voltage at (x_1, y_2) is the same that at (x_2, y_2), i.e. the potential difference between the two points is 0. You can show this without explicitly calculating the node voltages at the two points.

If you were to connect a wire between the two coordinates (x_1, y_2) in the touchscreen on the left, and (x_2, y_2) in the touchscreen on the right, would any current flow through this wire?

Solution:

It will be helpful to first consider the circuit representation of this scenario to understand why the node voltages at the two points on each of the touch screens should be same (and therefore that the potential difference between points (x_1, y_2) and (x_2, y_2) is 0).



Without calculating the node voltages, note that the ratio of the value of $R_{t1,(L-y_2)}$ to $R_{t1,y2}$ is the same as the ratio of the value of $R_{t2,(L-y2)}$ to $R_{t2,y2}$:

$$\frac{R_{t1,y2}}{R_{t1,(L-y2)}} = \frac{\rho_1(y_2)/(W_1 \cdot t)}{\rho_1(L-y_2)/(W_1 \cdot t)} = \frac{y_2}{L-y_2}$$
$$\frac{R_{t2,y2}}{R_{t2,(L-y2)}} = \frac{\rho_2(y_2)/(W_2 \cdot t)}{\rho_2(L-y_2)/(W_2 \cdot t)} = \frac{y_2}{L-y_2}$$

Note also that the voltage across the entirety of each of the individual touchscreens is the same: it is $V_s - 0$ or just V_s. The voltage V_s is therefore *divided* between $R_{t1,(L-y2)}$ and $R_{t1,y2}$ exactly the same as it is divided between $R_{t_2,(L-y2)}$ and $R_{t_2,y2}$ because of the ratio argument presented above.

Therefore, the potential difference between u_1 and u_2 will be 0, so long as the y-coordinate value is chosen to be the same.

This also means that there is no current flowing through the wire, since the points in the two touchscreens have the same potential. Therefore,

$$I_{12} = \frac{u_1 - u_2}{R_{wire}} = 0$$

We should note that what causes current to flow is a voltage difference. Here, we have a voltage difference of zero, so the current flowing through the wire is zero. In a physical wire (as opposed to an ideal wire), there is some small amount of resistance so when current is flowing we have a very small voltage difference $V_{wire} = IR_{wire}$. However, since R_{wire} is super small we neglect V_{wire}.

8. Homework Process and Study Group

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.

Solution:

I first worked by myself for 2 hours, but got stuck on problem 5. Then I met with my study group.

XYZ played the role of facilitator ... etc. We were still stuck on problem 5 so we went to office hours to talk about the problem.

Then I went to homework party for a few hours, where I finished the homework.

