EECS 16A Designing Information Devices and Systems I
Fall 2020 Homework 10

This homework is due November 6, 2020, at 23:59, however it is not graded and no submission is required.

No self-grades are due and everyone will be granted full credit.

Submission Format
Your homework submission should consist of **one** file.

- `hw10.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned).

Submit the file to the appropriate assignment on Gradescope.

1. **Reading Assignment**

   No submission is required for this problem, however we ask that you read and understand the notes.

   For this homework, please read Note 17 (in particular, section 17.3) and Note 17C to learn about comparators and op-amps. You are always encouraged to read beyond this as well.

   (a) If the op-amp supply voltages are $V_{DD} = 5\,\text{V}$ and $V_{SS} = 0\,\text{V}$, then what is the minimum/maximum value of V_{out}?

   (b) What is the purpose of a comparator? How can we use a comparator circuit to detect a touch for a capacitive touchscreen?

2. **Maglev Train Height Control System**

   No submission is required for this problem, however we ask that you read and understand the provided solution.

   The solution to this problem is being released along with the question. If you wish, you may rewrite and submit your solution, however, it will not be graded.

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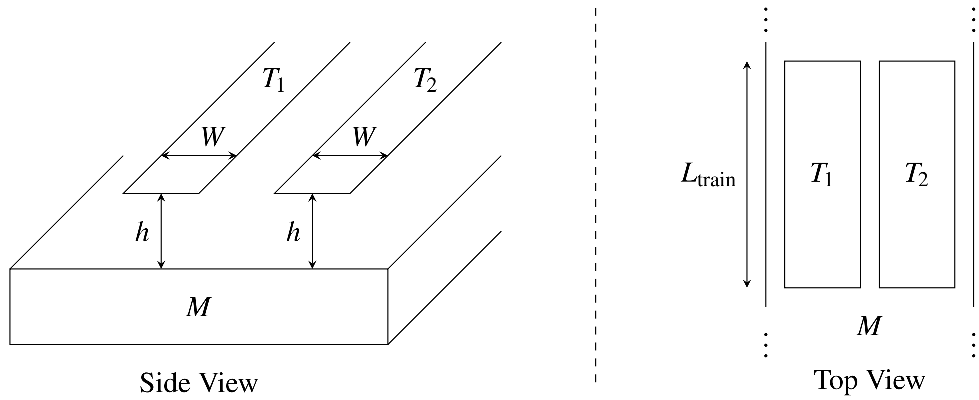
   One of the fastest forms of land transportation are trains that actually travel slightly elevated from the ground using magnetic levitation (or "maglev" for short). Ensuring that the train stays at a relatively constant height above its "tracks" (the tracks in this case are what provide the force to levitate the train and propel it forward) is critical to both the safety and fuel efficiency of the train. In this problem, we'll explore how maglev trains use capacitors to stay elevated. (Note that real maglev trains may use completely different and much more sophisticated techniques to perform this function, so if you get a contract to build such a train, you'll probably want to do more research on the subject.)

   (a) As shown below, we put two parallel strips of metal (T_1, T_2) along the bottom of the train and we have one solid piece of metal (M) on the ground below the train (perhaps as part of the track).



We assume that the entire train is at a uniform height above the track, so we can use the simple equations developed in lecture to model the capacitance.

As a function of L_{train} (the length of the train), W (the width of T_1 and T_2), and h (the height of the train above the track), determine the capacitances between T_1 and M and between T_2 and M. *Hint: Note that the area of a capacitor is given by the overlap area between its two plates.*

Solution:

The distance between the plates (T_1 & M or T_2 & M) is h. The area of the parallel plate capacitor is $A = WL_{\text{train}}$. Using the formula for capacitance of a parallel plate capacitor, we get:

$$C = \frac{\varepsilon A}{d}$$

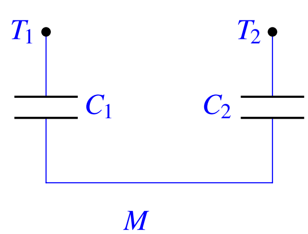
$$C_1 = \frac{\varepsilon W L_{\text{train}}}{h} \text{ (Capacitance between } \mathrm{T}_1 \text{ and M)}$$

$$C_2 = \frac{\varepsilon W L_{\text{train}}}{h} \text{ (Capacitance between } \mathrm{T}_2 \text{ and M)}$$

(b) Any circuit on the train can only make direct contact at T_1 and T_2. Thus, you can only measure the equivalent capacitance between T_1 and T_2. Draw a circuit model showing how the capacitors between T_1 and M and between T_2 and M are connected to each other.

Solution:

The capacitors C_1 and C_2 are in series. To realize this, let's consider the train circuit that is in contact with T_1 and T_2. If there is current entering plate T_1, the same current has to exit plate T_2. Thus, the circuit can be modeled as follows:



(c) Using the same parameters as in part (a), provide an expression for the equivalent capacitance measured between T_1 and T_2.

Solution:

Since the two capacitors are in series, the equivalent capacitance between T_1 and T_2 is given by:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2}$$

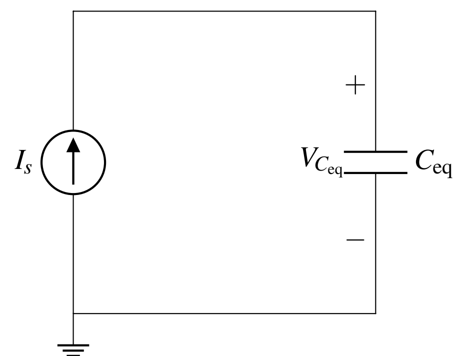
Thus, we get

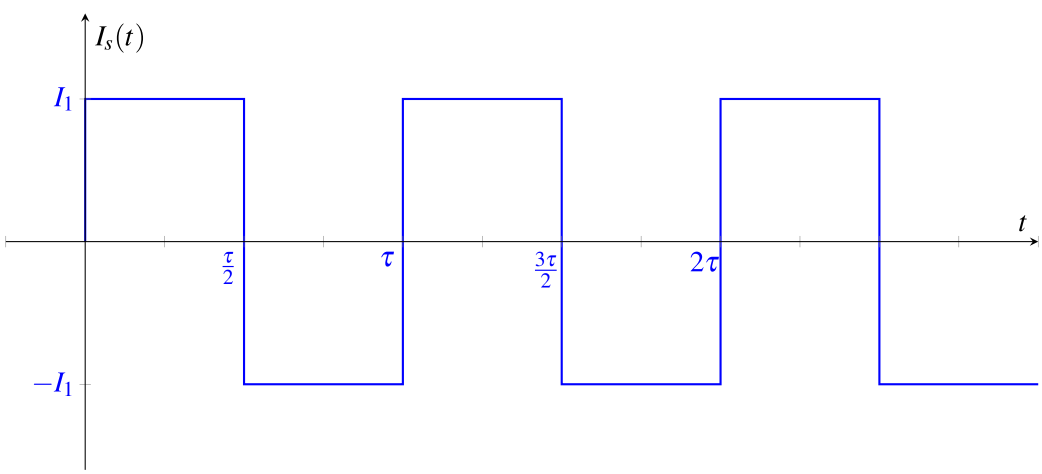
$$\frac{1}{C_{\text{eq}}} = \frac{h}{\varepsilon W L_{\text{train}}} + \frac{h}{\varepsilon W L_{\text{train}}}$$

$$C_{\text{eq}} = \frac{\varepsilon W L_{\text{train}}}{2h}$$

(d) We want to build a circuit that creates a voltage waveform with an amplitude that changes based on the height of the train. Your colleague recommends you start with the circuit as shown below, where I_s is a periodic current source, and C_{eq} is the equivalent capacitance between T_1 and T_2. The graph below shows I_s, a square wave with period τ and amplitude I_1, as a function of time t.

Find an equation for and draw the voltage $V_{C_{\text{eq}}}(t)$ as a function of time t. Assume the capacitor C_{eq} is discharged at time $t = 0$, so $V_{C_{\text{eq}}}(0) = 0\,\text{V}$.





Solution: We know the rate of change of voltage across a capacitor is related to the the current into the capacitor. That is:

$$I_{C_{\text{eq}}}(t) = C_{\text{eq}} \frac{dV_{C_{\text{eq}}}}{dt}$$

From KCL, we know $I_{C_{\text{eq}}}(t) = I_s(t)$. Then:

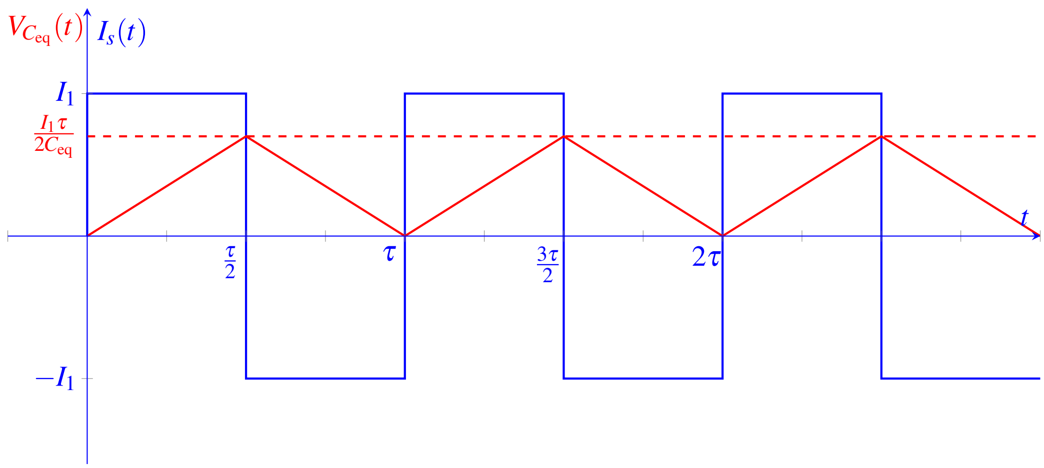
$$I_{C_{\text{eq}}}(t) = I_s(t) = C_{\text{eq}} \frac{dV_{C_{\text{eq}}}}{dt} \implies \frac{dV_{C_{\text{eq}}}}{dt} = \frac{I_s(t)}{C_{\text{eq}}}$$

Since $I_s(t)$ is periodic and piecewise-constant, we can examine what happens in the first period. We recall that the capacitor is uncharged at $t = 0$ so that $V_{C_{\text{eq}}}(0) = 0\,\text{V}$.

$$V_{C_{\text{eq}}}(t) = \begin{cases} \frac{I_1}{C_{\text{eq}}} t & \text{when } 0 \leq t \leq \frac{\tau}{2} \\ \frac{-I_1}{C_{\text{eq}}} \left(t - \frac{\tau}{2}\right) + \frac{I_1 \tau}{2C_{\text{eq}}} & \text{when } \frac{\tau}{2} < t \leq \tau \end{cases}$$

Since $V_{C_{\text{eq}}}(\tau) = V_{C_{\text{eq}}}(0) = 0$, we notice this equation for $V_{C_{\text{eq}}}(t)$ repeats in subsequent periods (i.e. $[k\tau, (k+1)\tau], k = 1, 2, \cdots$).

Given this equation for the output voltage, $V_{C_{\text{eq}}}(t)$, as a function of the current, I_s, we can draw what the output waveform should look like.



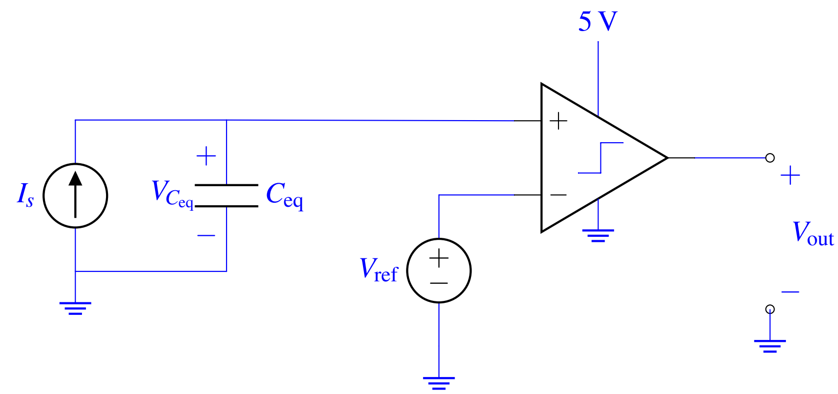
(e) We now want to develop an indicator that alerts us when the train is too high above the tracks. We want to have an output of 5 V (to trigger the alert) when the height of the train h is above 1 cm, and an output of 0 V when h is below 1 cm.

We will assume the train has length $L_{\text{train}} = 100\,\text{m}$ and that the metals, T_1 and T_2, have width $W = 1\,\text{cm}$ and permitivity $\varepsilon = 8.85 \times 10^{-12} \frac{\text{F}}{\text{m}}$.

Design a circuit using **a square wave current source (i.e. I_s in part (d)) with period $\tau = 1\,\mu\text{s}$ and pulses of amplitude $I_1 = 1\,\text{mA}$, a comparator, and any number of voltage sources** to implement this function. *Hint: You should use the circuit you analyzed in part (d).*

Solution:

The circuit is shown below:



From the choice of supply voltages, we see that $V_{out} = 5\,\text{V}$ when $V_+ > V_-$.
We know the amplitude of $V_{C_{eq}}(t)$ when $h = 1\,\text{cm}$ is:

$$\frac{I_1\tau}{2C_{eq}} = \frac{1\,\text{mA}\cdot 1\,\mu\text{s}}{2\cdot C_{eq}} = \frac{1\,\text{mA}\cdot 1\,\mu\text{s}}{2\cdot\frac{\varepsilon W L_{train}}{2h}} = \frac{1\,\text{mA}\cdot 1\,\mu\text{s}\cdot h}{\varepsilon W L_{train}} = \frac{1\,\text{mA}\cdot 1\,\mu\text{s}\cdot 1\,\text{cm}}{8.85\times 10^{-12}\,\frac{\text{F}}{\text{m}}\cdot 1\,\text{cm}\cdot 100\,\text{m}} = 1.13\,\text{V}$$

Thus we can set V_{ref} to this peak value assuming the train is 1 cm above the ground. If the train's height is larger than 1 cm, the peak voltage rises, and we continue to get pulses. If the train's height is below 1 cm, the peak value is less than 1.13 V preventing any pulses from the output of the circuit.

As an example, let's suppose the train's height is 2 cm. Then we would observe the following output for V_{out}. Note that the x-axis is in μs, that $V_{ref} = 1.13\,\text{V}$ as we found before, and that $V_{peak} = 2\cdot V_{ref} = 2.26\,\text{V}$. The cyan waveform is what we measure at V_+, and the red waveform is the 5 V pulse generated at V_{out}.

Circuit Behavior at $h = 2\,\text{cm}$