This homework is due November 12, 2021, at 23:59.
Self-grades are due November 15, 2021, at 23:59.

Submission Format
Your homework submission should consist of one file.

- hw11.pdf: A single PDF file that contains all of your answers (any handwritten answers should be scanned)

Submit each file to its respective assignment on Gradescope.

1. Reading Assignment
For this homework, please read Notes 18 and 19. They will provide an overview on operational amplifiers (op-amps), negative feedback, the "golden rules" of op-amps, and various op-amp configurations (non-inverting, inverting, buffers, etc). You are always encouraged to read beyond this as well.

(a) What are the two "golden rules" of ideal op-amps? When do these rules hold true?

**Solution:** The golden rules are the following:

- The error signal going into the op-amp must be zero, i.e. \( u_+ = u_- \). This rule only holds when there is negative feedback.
- The currents into the input terminals of the op-amp are zero, i.e. \( I_+ = I_- = 0 \). This rule holds regardless of whether there is negative feedback or not.

(b) What is the internal gain of an op-amp, \( A \)? What is its value for an ideal op-amp? For non-ideal?

**Solution:** The internal gain of an op-amp, \( A \) is the ratio of the output voltage to the error voltage, i.e. \( A \) is given by \( \frac{v_{out}}{u_+ - u_-} \). For ideal op-amps, \( A \to \infty \). For non-ideal op-amps, \( A \) is finite.

2. Basic Amplifier Building Blocks
The following amplifier stages are used often in many circuits and are well known as (a) the non-inverting amplifier and (b) the inverting amplifier.
(a) Label the input terminals of the op-amp with (+) and (−) signs in Figure (a), so that it is in negative feedback. Then derive the voltage gain \( G = \frac{v_o}{v_s} \) of the non-inverting amplifier in Figure (a) using the Golden Rules. Why do you think this circuit is called a non-inverting amplifier?

**Solution:**

![Non-inverting amplifier diagram]

The +, − should be labeled on the top and bottom of the op-amp, respectively. Now if we move the negative input of the op-amp \( u_− \) upward, \( v_o = A_{\text{error}} = A(u_+ - u_-) \) moves downward and as a result \( u_- = \frac{R_1}{R_1 + R_2} v_o \) moves downward. So the result of the initial stimulus goes in the opposite direction of the initial stimulus, which is the requirement for negative feedback.

By the Golden Rules, the voltage at the positive input terminal \( v_s \) must also set the voltage at the negative input terminal to be \( v_s \). Our Golden Rules also tell us that no current can flow into the input terminals of the op-amp. Therefore, we can write a single KCL equation at the input node of the negative terminal as follows:

\[
\begin{align*}
    u_- &= u_+ = v_s \\
    i_{R_1} &= i_{R_2} \\
    \implies v_s &= v_o - v_s \\
    \frac{v_s}{R_1} &= \frac{v_o}{R_2}
\end{align*}
\]

Rearranging and solving for \( v_o \), we therefore obtain:

\[
R_2 v_s = R_1 v_o - R_1 v_s
\]

\[
\implies v_o = \left(\frac{R_1 + R_2}{R_1}\right) v_s
\]

Note also that you may be familiar already with a faster way of solving this problem! Because no current flows into the negative input terminal of the op-amp, you may recognize \( R_1 \) and \( R_2 \) as simply forming a voltage divider over \( v_o \). Therefore, the potential at the negative terminal is:

\[
\begin{align*}
    u_- &= v_s = v_o \left(\frac{R_1}{R_1 + R_2}\right) \\
    \implies v_o &= \left(\frac{R_1 + R_2}{R_1}\right) v_s \\
    \implies G &= \frac{v_o}{v_s} = \left(\frac{R_1 + R_2}{R_1}\right)
\end{align*}
\]

This is called an *non-inverting amplifier* because the gain \( G \) is positive – it does not invert the input signal (in contrast to the amplifier in the next part of this problem).

(b) Label the input terminals of the op-amp with (+) and (−) signs in Figure (b), so that it is in negative feedback. Then derive the voltage gain \( G = \frac{v_o}{v_s} \) of the inverting amplifier using the Golden Rules. Can you explain why this circuit is called an inverting amplifier?

**Solution:**

![Inverting amplifier diagram]
The $+, -$ should be labeled on the bottom and top of the op-amp, respectively. Now if we move the negative input of the op-amp $u_-$ upward, $v_o = A(v_{error} = A(u_+ - u_-)$ moves downward and as a result $u_-$ moves downward because of the following relationship:

$$\frac{v_o - u_-}{R_2} = \frac{u_- - v_s}{R_1}$$

$$\Rightarrow u_- = \frac{R_1}{R_1 + R_2} v_o + \frac{R_2}{R_1 + R_2} v_s$$

So the result of the initial stimulus goes in the opposite direction of the initial stimulus, which is the requirement for negative feedback.

Since the potential at the positive input terminal is $u_+ = 0$, the op-amp will act such that the potential at the negative input terminal is $u_- = 0$ as well (by the Golden Rules). Now, by KCL at the node with potential $u_-:

$$i_{R_1} = i_{R_2}$$

$$\frac{v_s - 0}{R_1} + \frac{v_o - 0}{R_2} = 0$$

Solving this yields:

$$v_o = -\left(\frac{R_2}{R_1}\right) v_s$$

Thus, the voltage gain of this amplifier circuit is:

$$G = \frac{v_o}{v_s} = -\frac{R_2}{R_1}$$

This is called an *inverting amplifier* because the voltage gain $G$ is negative, meaning it “inverts” its input signal.

(c) Using your toolkit of circuit topologies, design blocks that implement the following equations:

i. $v_o = 2v_s$

ii. $v_o = -3v_s + 8$

**Solution:**

i. We can use a non-inverting amplifier with gain 2.

$$\frac{v_o}{v_s} = \left(1 + \frac{R_{top}}{R_{bottom}}\right) = 2$$

$$\frac{R_{top}}{R_{bottom}} = 1$$

Any value for $R_{top}$ and $R_{bottom}$ is correct as long as they have the same resistance.
ii. We can use an inverting amplifier with reference voltage source. We need to determine values for \( R_f, R_s \) and \( V_{REF} \).

\[
v_o = v_s \left( -\frac{R_f}{R_s} \right) + V_{REF} \left( \frac{R_f}{R_s} + 1 \right) = -3v_s + 8
\]

Matching the coefficients in this equation:

\[
\frac{R_f}{R_s} = 3
\]

\[
V_{REF} \left( \frac{R_f}{R_s} + 1 \right) = 8
\]

\[
V_{REF} (3 + 1) = 8
\]

\[
V_{REF} = 2V
\]

Any values for \( R_f \) and \( R_s \) are correct as long as \( \frac{R_f}{R_s} = 3 \).

3. Amplifier with Multiple Inputs

(a) Use the Golden Rules to find \( v_{o1} \) for the circuit below.
Solution:
Applying the Golden Rules, we know that the positive and negative terminals must be at the same voltage. Thus, the voltage at the negative terminal of the op-amp is 0. The voltage drop across $R_1$ is 0 and no current flows through it. In addition, no current flows into the op-amp from the negative terminal due to its infinite input resistance (the negative terminal is connected to an “open” circuit). By KCL at the negative terminal of the op-amp, this means that the current going through $R_3$ and $R_2$ is $i_s$. Taking the positive terminal of $R_2$ to be on the right, the voltage drop across $R_2$ is $v_{o1}$. By Ohm’s law, we conclude:

$$\frac{v_{o1}}{R_2} = i_s$$

Rearranging we get:

$$v_{o1} = i_s \cdot R_2$$

(b) Use the Golden Rules to find $v_{o2}$ for the circuit below.

Solution:
Applying the Golden Rules, we know that the positive and negative terminals must be at the same voltage. Thus, the voltage at the negative terminal of the op-amp is $V^- = v_{x2}$. In addition, since no current can enter into the negative terminal of the op-amp, $R_1$ and $R_2$ are in series. This means that
the voltage at the negative terminal of the op-amp can be expressed in terms of $v_{o2}$ using the voltage divider formula:

$$v^- = v_{o2} \left( \frac{R_1}{R_1 + R_2} \right)$$

We also know that $v^- = v_{s2}$ and conclude:

$$v_{s2} = v_{o2} \left( \frac{R_1}{R_1 + R_2} \right)$$

After rearranging, we have:

$$v_{o2} = v_{s2} \left( \frac{R_2}{R_1} + 1 \right)$$

(c) Use the Golden Rules to find the output voltage $v_o$ for the circuit shown below.

![Circuit Diagram]

**Solution:**

Applying the Golden Rules we know that the positive and negative terminals must be at the same voltage. Thus, the voltage at the negative terminal of the op-amp is $v^- = v_{s2}$. Then we write a KCL equation at the node connected to the minus terminal of the op-amp (recalling that no current flows into or out of the op-amp’s terminals). All currents are defined as flowing out of the node:

$$i_{R_1} + i_{R_2} + i_{R_3} = 0$$

Because of the independent current source, we know:

$$i_{R_3} = i_s$$

By Ohm’s law, we know:

$$i_{R_1} = \frac{v^- - v_{s1}}{R_1}$$

and

$$i_{R_2} = \frac{v^- - v_o}{R_2}$$
Then, substituting back into the original KCL equation, we have:

$$\frac{v^- - v_{s1}}{R_1} + \frac{v^- - v_o}{R_2} + i_s = 0$$

and substituting $v^- = v_{s2}$, we have:

$$\frac{v_{s2} - v_{s1}}{R_1} + \frac{v_{s2} - v_o}{R_2} + i_s = 0$$

which we rearrange to find $v_o$, giving:

$$v_o = v_{s2} \left( 1 + \frac{R_2}{R_1} \right) + i_s R_2 - \frac{R_2}{R_1} v_{s1}$$

(d) Use superposition and the answers to the first few parts of this problem to check your work.

**Solution:** Using superposition we can analyze the circuit leaving only one source on at a time. If we leave on $v_{s1}$ and turn off $v_{s2}$ and $i_s$, then we have an inverting amplifier. If we leave on $i_s$ and turn off $v_{s1}$ and $v_{s2}$, then we have the circuit in (a). If we leave on $v_{s2}$ and turn off $v_{s1}$ and $i_s$, then we have the circuit in (b). From this we can see that $v_o$ is the sum from the solutions.

$$v_o = -\frac{R_2}{R_1} v_{s1} + i_s R_2 + v_{s2} \frac{R_2 + R_1}{R_1}$$

(e) Now add a second stage as shown below. What is $v_o,_{\text{new}}$? Does $v_o$ change between part (c) and this part? Does the voltage $v_o,_{\text{new}}$ depend on $R_L$?

**Solution:**
Adding the second stage does not change the voltages in the first stage. This is because the circuit connected to the positive and negative terminals of the first stage op-amp “sees” an open circuit/infinite input resistance in the op-amp.
Hence $v_o$ remains unchanged from part (c).

$$v_o = - \left( \frac{R_2}{R_1} \right) v_{s1} + i_s \cdot R_2 + v_{s2} \left( \frac{R_2 + R_1}{R_1} \right)$$

By the Golden Rules, the negative terminal of the second op-amp must have the same voltage as the plus terminal, which is $v_o$. No current can flow into the negative terminal, so $R_3$ and $R_4$ are in series and have the same current, so we know:

$$\frac{v_o}{R_4} = \frac{v_{o,\text{new}} - v_o}{R_3}$$

Therefore:

$$v_{o,\text{new}} = \left( \frac{R_3 + R_4}{R_4} \right) v_o = \frac{R_3 + R_4}{R_4} \left( - \frac{R_2}{R_1} \cdot v_{s1} + i_s \cdot R_2 + v_{s2} \cdot R_2 + R_1 \right)$$

Note that you could have directly used the non-inverting amplifier gain formula $(1 + \frac{R_3}{R_4})$ for this extra stage.

The output voltage does not depend on the load resistance $R_L$, since it is set by the dependent voltage source inside the op-amp. Remember that a voltage source will provide any amount of current necessary while maintaining its voltage constant. That is the beauty of op-amps: they provide isolation between stages because of the open circuit at the input and they get rid of the loading effect, since they can maintain the output voltage constant regardless of the load value.

### 4. Op-Amps and State Transition Matrices

Consider the following circuit where $v_{\text{ref}}$ is dependent on $v_{\text{adj}}$ and $v_{\text{ref}} = v_{\text{adj}} + 1.25V$.

![Circuit Diagram](image)

(a) Express $v_{\text{out}}$ in terms of the other voltages and resistor values. Then express $v_{\text{adj}}$ in terms of $v_{\text{out}}$.

**Solution:**

$$v_{\text{out}} = v_{\text{ref}}$$

$$v_{\text{adj}} = v_{\text{out}} \frac{R_2}{R_1 + R_2}$$
(b) To find out how the steady state of the circuit in (a) behaves, we model the nodal voltages as a function of time $t$. $dt$ represents one timestep. Use the state vector given below and construct a state transition matrix for the circuit. More precisely, find the matrix $A$ such that $\vec{s}(t + dt) = A\vec{s}(t)$.

$$\vec{s} = \begin{bmatrix} v_{\text{out}} \\ v_{\text{adj}} \\ v_{\text{ref}} \\ 1.25 \end{bmatrix}$$

**Solution:** Using the equations from part a and the expression for $v_{\text{ref}}$ in the question, we get

$$\begin{bmatrix} v_{\text{out}}(t + dt) \\ v_{\text{adj}}(t + dt) \\ v_{\text{ref}}(t + dt) \\ 1.25 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ \frac{R_2}{R_1 + R_2} & 0 & 0 & v_{\text{adj}}(t) \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_{\text{out}}(t) \\ v_{\text{adj}}(t) \\ v_{\text{ref}}(t) \\ 1.25 \end{bmatrix}$$

(c) Now find the eigenvalues of the matrix. What do these eigenvalues say about the existence of a steady state of the system? If there exists a steady state, write it down. The following equation might help you find the eigenvalues:

If matrix $A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ k & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, then $\det(A - \lambda I) = \lambda^4 - \lambda^3 - \lambda R + R = (\lambda - 1)(\lambda^3 - k)$.

**Solution:** The eigenvalues of this system are $1, k^{\frac{1}{3}}$ where $k = \frac{R_2}{R_1 + R_2}$. Note that the latter root is repeated 3 times. An eigenvalue of 1 is present, which means that the system has a steady state. Moreover, the other eigenvalues are always smaller than 1, which means the system converges to its steady state. The eigenvector corresponding to eigenvalue 1 is

$$\vec{v} = \begin{bmatrix} \frac{R_1 + R_2}{R_1} \\ \frac{R_1}{R_1 + R_2} \\ \frac{R_2}{R_1 + R_2} \\ 1 \end{bmatrix}$$

The eigenvector associated with the steady state must have the value of 1.25 in the last entry. So we multiply by the appropriate constant to get the steady state as

$$\vec{s}' = 1.25 \begin{bmatrix} \frac{R_1 + R_2}{R_1} \\ \frac{R_1}{R_1 + R_2} \\ \frac{R_2}{R_1 + R_2} \\ 1 \end{bmatrix}$$

5. **Cool For The Summer**

You and a friend want to make a box that helps control an air conditioning unit based on both your inputs. You both have individual dials which you can use to control the voltage. An input of 0 V means that you want to leave the temperature as is. A **negative voltage input** means that you want to reduce the temperature. (It’s hot out, so we will assume that you never want to increase the temperature – so no, we’re not talking about a Berkeley summer...)
Your air conditioning unit, however, responds only to **positive voltages**. The higher the magnitude of the voltage, the stronger it runs. At zero, it is off. You also need a system that **sums up** both you and your friend’s control inputs.

Therefore, you need a box that acts as an **inverting summer** — it outputs a weighted sum of two voltages where the weights are both negative. The sum is weighted because one room is bigger, so you need to compensate for this.

(a) As a first step, derive $v_{\text{out}}$ in terms of $R_2$, $R_1$, and $v_{\text{in}}$.

*Hint: Have you solved for this particular amplifier configuration before? You can use your answer from the time you did this earlier.*

![Amplifier Diagram](attachment:image.png)

**Solution:**

We will first need to check that the amplifier is in negative feedback in order to apply the Golden Rules. The amplifier is configured in negative feedback if when the negative input terminal voltage is elevated, the feedback moves it back downward. Going around the loop:

- We move the negative input voltage of the op amp upward
- The output voltage of the amplifier moves downward
- The negative input voltage moves downward with it

Thus, we’ve confirmed that the amplifier is in negative feedback.

Second, we perform KCL at the $u_-$ terminal.

$$
\frac{v_{\text{in}} - u_-}{R_1} + \frac{v_{\text{out}} - u_-}{R_2} = 0
$$

Since we’re in negative feedback, we can apply the Golden Rules. The voltages at the input terminals of the amplifier, $u_-$ and $u_+$, respectively, must be held at the same voltage. In other words, $u_+ = u_- = 0V$.

$$
\frac{v_{\text{in}}}{R_1} + \frac{v_{\text{out}}}{R_2} = 0
$$

$$
v_{\text{out}} = v_{\text{in}} \left( -\frac{R_2}{R_1} \right)
$$

The general inverting amplifier shown above has a voltage gain $G = \frac{v_{\text{out}}}{v_{\text{in}}} = -\frac{R_2}{R_1}$.

(b) Now we will add a second input to this circuit as shown below. Find $v_{\text{out}}$ in terms of $v_{S1}$, $v_{S2}$, $R_{S1}$, $R_{S2}$ and $R_2$.

*Hint: You can solve this problem using either superposition or our tried-and-true KCL analysis.*
Solution:

Method 1: Superposition
First, when considering $v_{S1}$, we zero out $v_{S2}$, and therefore we can disregard $R_{S2}$. The reason why we can disregard $R_{S2}$ is because by the Golden Rules, we know that the voltage at the $-$ terminal of the op-amp must be equal to the voltage at the $+$ terminal. Therefore, both terminals of $R_{S2}$ are at 0V, and no current flows through $R_{S2}$. With this insight, we recognize that it becomes identical to the circuit in part (a), except with $V_{in} \rightarrow V_{S1}$ and $R_1 \rightarrow R_{S1}$.

Now apply the equation from part (a): $v_{out} = -\frac{R_2}{R_{S1}}v_{S1}$.

Similarly, when $v_{S2}$ is on and $v_{S1}$ is zeroed out, we disregard $R_{S1}$ by the same argument leading to $v_{out} = -\frac{R_2}{R_{S2}}v_{S2}$.

Combining the two $v_{out}$ equations from superposition, we get $v_{out} = -R_2 \left( \frac{v_{S1}}{R_{S1}} + \frac{v_{S2}}{R_{S2}} \right)$.

Method 2: KCL without superposition
According to the Golden Rules, $u_- = u_+ = 0V$, so we can write a single KCL equation at the $u_-$ node and solve:

$$\frac{v_{S1}}{R_{S1}} + \frac{v_{S2}}{R_{S2}} + \frac{v_{out}}{R_2} = 0$$

$$v_{out} = -v_{S1} \left( \frac{R_2}{R_{S1}} \right) - v_{S2} \left( \frac{R_2}{R_{S2}} \right)$$

(c) Let’s suppose that you want $v_{out} = -\left( \frac{1}{4}v_{S1} + 2v_{S2} \right)$ where again $v_{S1}$ and $v_{S2}$ represent the input voltages from you and your friend’s control knobs. Select resistor values such that the circuit from part (b) implements this desired relationship.

Solution: Using the configuration from the previous part, the conditions which need to be satisfied are:

- $\frac{R_2}{R_{S1}} = \frac{1}{4}$
- $\frac{R_2}{R_{S2}} = 2$

One possible set of values is $R_2 = 2\,\text{k}\Omega$, $R_{S1} = 8\,\text{k}\Omega$, and $R_{S2} = 1\,\text{k}\Omega$, but any combination of resistors which satisfies $R_{S1} = 4R_2 = 8R_{S2}$ are valid solutions.

(d) Suppose that you have a new AC unit that you want to use with your original control inputs $v_{S1}$ and $v_{S2}$. This unit, however, responds only to negative voltages – the opposite of your previous air
conditioning unit, which only responded to positive input voltages. The higher the magnitude of the negative voltage, the stronger the AC runs.

You want to modify your prior circuit for the new AC unit. Your circuit takes in two control voltages and outputs a weighted sum, but the sum should now become more negative as you increase your input voltages.

Hint: Consider adding another op-amp circuit to the output of your circuit from part (b), such that you invert the output of the op-amp circuit of part (b) without adding additional gain.

Solution:

Here, we add another inverting op-amp stage with unity voltage gain, and we can pick any equal-valued resistors for $R_3$ and $R_4$.

6. Putting on the Pressure: Build Your Own InstantPot

Prof. Arias had a great experience with her automatic pressure cooker, so she was inspired to try and build her own. She’s enlisting your help! The design of the pressure cooker uses a pressure sensor and a heating element. Whenever the pressure is below a set target value, an electronic circuit turns on the heating element.

Pressure Sensor Resistance

The first step is designing a pressure sensor. The figure below shows your design. As pressure $p_c$ is applied, the flexible membrane stretches.

(a) You attach a resistor layer $R_p$ with resistivity $\rho = 0.1\,\Omega\,m$, width $W$, length $L$, and thickness $t$ to the pressure sensor membrane, as illustrated in the figure below. When the pressure $p_c = 0\,\text{Pa}$ (i.e. there is no applied pressure), $W = 1\,\text{mm}$, $L = L_0 = 1\,\text{cm}$, $t = 100\,\mu\text{m} = 100 \times 10^{-6}\,\text{m}$.

$R_{p0}$ is the value of $R_p$ when there is no applied pressure. Calculate $R_{p0}$. Note that direction of current flow in the resistor is from A to B as marked in the diagram.
Solution: Resistance \( R_{p0} = \frac{\rho \times L}{A} = \frac{\rho \times L_0}{W \times t} = \frac{0.1 \Omega \times 0.01 m}{0.001 m \times 100 \times 10^{-6} m} = 10k\Omega \).

(b) When pressure is applied, the length of the resistor \( L \) changes from \( L_0 \) and is a function of applied pressure \( p_c \), and is given by

\[
L = L_0 + \beta p_c,
\]

where \( L_0 \) is the nominal length of the resistor with no pressure applied, and \( \beta \) is a constant with units m/Pa. As a result of the length change, the value of resistance \( R_p \) also changes from its nominal value \( R_{p0} \) (the value of \( R_p \) with no pressure applied).

Derive an expression for \( R_p \) as a function of resistivity \( \rho \), width \( W \), thickness \( t \), nominal length \( L_0 \), constant \( \beta \), and applied pressure \( p_c \), when pressure is applied.

Note: The width and thickness of the resistor will also change with applied pressure. However, we ignore this to keep the math simple.

Solution: Resistance \( R_p = \frac{\rho \times L}{A} = \frac{\rho \times L_0}{W \times t} \). Now when pressure is applied the length is given by

\[
L = L_0 + \beta p_c
\]

Plugging in the value of \( L_{pc} \) we have:

\[
R_p = \frac{\rho \times (L_0 + \beta p_c)}{W \times t}
\]

(c) Pressure Sensor Circuit Design

For this sub-part and the following sub-parts, we will use a new model for pressure-sensitive resistance \( R_p \). Assume that the resistance \( R_p \) is a function of applied pressure \( p_c \) according to the relationship \( R_p = R_o \times \frac{p_c}{p_{ref}} \), where \( R_o = 1k\Omega \), and \( p_{ref} = 100kPa \).

To complete our sensor circuit, we would like to generate a voltage \( V_p \) that is a function of the pressure \( p_c \).

Complete the circuit below so that the output voltage \( V_p \) depends on the pressure \( p_c \) as:

\[
V_p = -V_o \times \frac{p_c}{p_{ref}} \quad \text{where} \quad V_o = 1 \text{ V}.
\]

Restrictions on your pressure sensor circuit design are as follows:
• You may add at most one ideal voltage source and one additional resistor besides $R_p$ to the circuit, but you must calculate their values and mark them in the diagram.
• Mark the positive and negative inputs of the operational amplifier with “+” and “-” symbols, respectively, in the boxes provided.
• Assume op-amp supply voltages $V_{DD}$ and $V_{SS} = -V_{DD}$ are already provided.

You may assume that the operational amplifier is ideal.

\[
R_p
\]

\[
\text{Solution:} \quad \text{We can combine the relationships } R_p = R_o \times \frac{P_c}{p_c} \text{ and } V_p = -V_o \times \frac{P_c}{p_c} \text{ to get the relationship:}
\]

\[
V_p = -V_o \times \frac{R_p}{R_o}
\]

We can then use an inverting amplifier op-amp configuration to realize the circuit, where $R_o = 1k \Omega$ and $R_p = \frac{1k \Omega}{100k \Omega} \times p_c$.

\[
\text{(d) Resistive Heating Element}
\]

To heat the pressure cooker, you use a heating element with resistance $R_{heat}$. Calculate the value of $R_{heat}$ such that the power dissipated is $P_{heat} = 1000W$ with $V_{heat} = 100V$ applied across the heating element.

\[
\text{Solution:} \quad P_{heat} = V_{heat}I_{heat} = V_{heat} \times \frac{V_{heat}}{R_{heat}}. \text{ Therefore, } R_{heat} = \frac{V_{heat}^2}{P_{heat}} = \frac{(100V)^2}{1000W} = 10 \Omega.
\]

\[
\text{(e) Pressure Regulation}
\]

You are finally ready to complete the design of your pressure cooker.
Using all of the circuit elements below, make a circuit that will turn the heater on (i.e. will cause a current to flow through \( R_{\text{heat}} \)) when the pressure is less than 500 kPa, and off (i.e. will cause no current to flow through \( R_{\text{heat}} \)) when the pressure is greater than 500 kPa.

The elements are:

- A voltage source \( V_s = 10V \) in series with a resistance of 500\( \Omega \).
- A voltage source \( V_p = V_o \times \frac{p_c}{p_{\text{ref}}} \), with \( V_o = 1V \) and \( p_{\text{ref}} = 100\text{kPa} \). (This is a voltage source whose voltage is a function of pressure \( p_c \), unrelated to any previous parts of the question.)
- A comparator that controls switch \( S_0 \). The switch is normally opened (i.e. an open circuit between nodes \( V_a \) and \( V_b \)), and is closed only when \( V_1 > V_2 \) (i.e. a short circuit between nodes \( V_a \) and \( V_b \)).
- The heater supply (\( V_{\text{heat}} = 100V \)).
- The heater resistor \( R_{\text{heat}} \).
- One additional resistor \( R_{\text{extra}} \) that can have any value.
- You may assume you have access to a ground node.
- Assume comparator supply voltages \( V_{DD} \) and \( V_{SS} = -V_{DD} \) are already provided.

(i) Since you are looking to compare the change in voltage associated with a change in pressure, you decide to assign the variable voltage source \( V_p \) as one of the inputs to your comparator.

What is the value of the variable voltage source \( V_p \) for \( p_c = 500\text{kPa} \)?

**Solution:** For \( p_c = 500\text{kPa} \),

\[
V_p = 1V \times \frac{500\text{kPa}}{100\text{kPa}} = 5V
\]

(ii) For your comparator inputs, you also need to generate a reference voltage which can be used to compare against the value of the variable voltage source which you calculated above.

Combine the voltage source \( V_s = 10V \) with an associated resistance of 500\( \Omega \) with an additional resistor \( R_{\text{extra}} \) to generate a reference voltage equal to the voltage \( V_p \) you calculated above.

What would you choose the value of \( R_{\text{extra}} \) to be?

**Solution:** If you combine voltage source in series with a resistor \( R_{\text{extra}} \), you create a voltage divider. If you set \( R_{\text{extra}} = 500\Omega \), you will achieve a reference voltage output of 5V as shown below:

\[
V_{\text{out,REF}} = 10V \frac{500\Omega}{500\Omega + 500\Omega} = 5V
\]
(iii) Label the circuit elements in the schematic below with the circuit elements presented above and your calculated $R_{\text{extra}}$ value to turn the heater on when the pressure is less than 500 kPa.

**Solution:** We have already computed $V_p = 5V$ for $p_c = 500kPa$. $5V$ is the voltage that we want to compare against the output of $V_p$ in order to make sure the heater is on when the pressure $p_c$ is less than 500kPa and off when the pressure $p_c$ is greater than 500kPa.

We were able to generate a 5V reference voltage by using $R_{\text{extra}}$ to make a voltage divider with the voltage source $V_s$. By setting $R_{\text{extra}}$ equal to the voltage source’s associated resistance, the output of the voltage divider is 5V. We connect this to the positive input of the comparator. We then connect the source $V_p$ to the negative input of the comparator. Finally, we connect the voltage source $V_{\text{heat}}$ in series with the resistor $R_{\text{heat}}$ so that when $S_0$ is closed (i.e. when the pressure $p_c$ is less than 500kPa) power will be delivered to $R_{\text{heat}}$.

7. **Homework Process and Study Group**

Who did you work with on this homework? List names and student ID’s. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.

**Solution:**

I first worked by myself for 2 hours, but got stuck on problem 5. Then I met with my study group. XYZ played the role of facilitator ... etc. We were still stuck on problem 5 so we went to office hours to talk about the problem.
Then I went to homework party for a few hours, where I finished the homework.