# EECS 16A Designing Information Devices and Systems I
# Fall 2020 Homework 7

## This homework is due October 16, 2020, at 23:59.
## Self-grades are due October 19, 2020, at 23:59.

**Submission Format**
Your homework submission should consist of **one** file.

- `hw7.pdf`: A single PDF file that contains all of your answers (any handwritten answers should be scanned)

Submit the file to the appropriate assignment on Gradescope.

1. **Reading Assignment**

   For this homework, please read Notes 12 through 14. Note 12 will provide an overview of voltage dividers and resistors. Note 13 will refresh you on how simple 1-D resistive touchscreens work, as well as the notion of power in electric circuits. Note 14 will cover a slightly more complicated 2-D resistive touchscreens and how to analyze them from a circuits perspective.

   (a) Describe the key ideas behind how the 1D touchscreen works. In general, why is it useful to be able to convert a “physical" quantity like the position of your finger to an electronic signal (i.e. voltage)?

2. **It’s a Triforce!** *(Contributors: Ava Tan, Elena Herbold, Ryan Tsang, Taehwan Kim, Urmita Sikder, Wahid Rahman)*

   ***Learning Goal:*** *This problem explores passive sign convention and nodal analysis in a slightly more complicated circuit.*

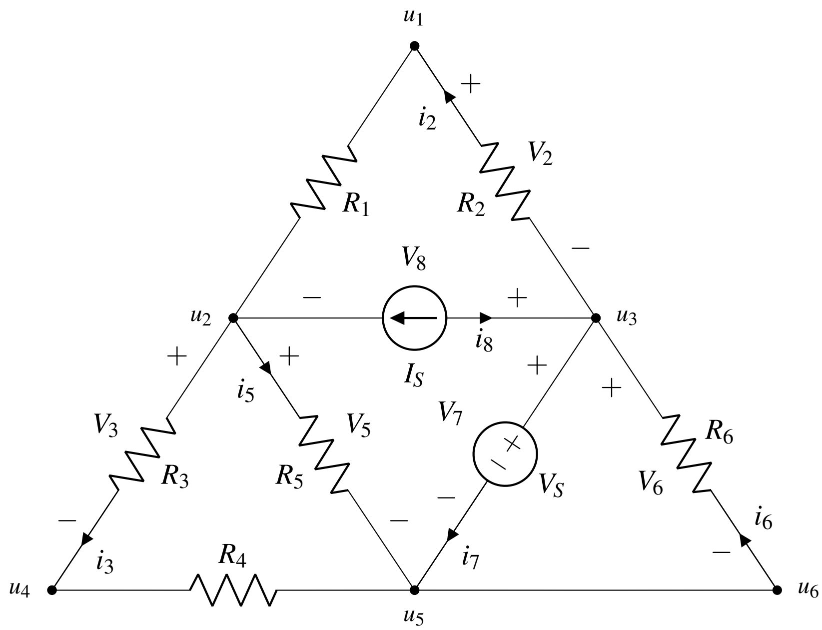



Figure 1: A triangular circuit consisting of a voltage source $V_s$, current source $I_s$, and resistors $R_1$ to $R_6$.

(a) Which of the elements $I_S$, $V_S$, $R_2$, $R_3$, $R_5$, or $R_6$ in Figure 1 have current-voltage labeling that violates Passive Sign Convention? There could be more than one possible element which violates Passive Sign Convention. Explain your reasoning.

**Solution:**

The correct answers are $I_S$, $R_2$, and $R_6$ since the current directions disagree with the signs chosen and depict the current leaving the positive terminal (or entering the negative terminal, equivalently). You would simply swap the signs for all components to fix them.

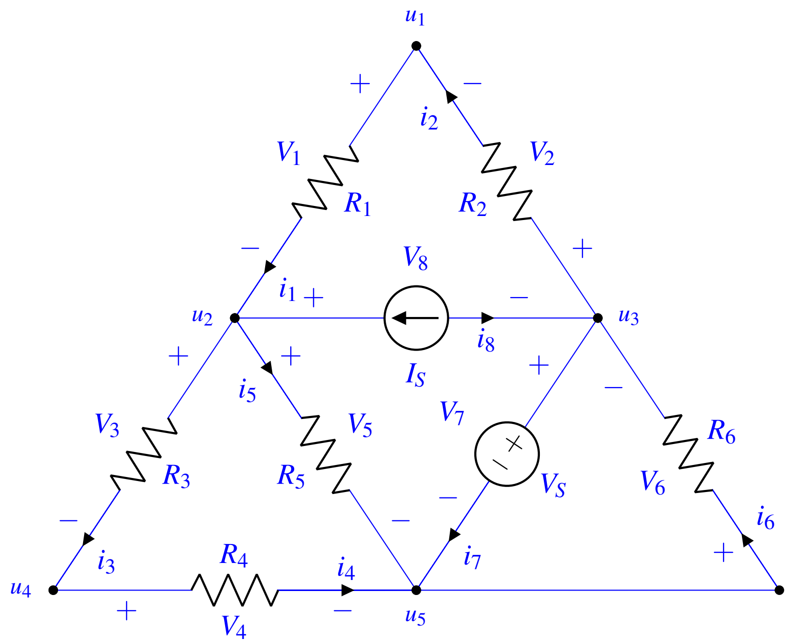
(b) In Figure 1, the nodes are labeled with $u_1$, $u_2$, ... etc. There is a subset of $u_i$'s in the given circuit that are redundant, i.e. there might be more than one label for the same node. Which node(s)? Justify your answer.

**Solution:**

The correct answer is $u_5$ and $u_6$ since there is a short connecting them.

(c) Redraw the circuit diagram by correctly labeling all the element voltages and element currents according to passive sign convention. (The component labels that were violating Passive Sign Convention in part (a), should be corrected by swapping the element voltage polarity. Also, the elements that have not been labeled yet, should be labeled following Passive Sign Convention.)

**Solution:**




(d) Write an equation to describe the current-voltage relationship for element $R_4$ in terms of the relevant $i$'s, $R$'s, and node voltages in this circuit.

**Solution:**

The resulting equation should look like:

$$R_4 = \frac{u_4 - u_5}{i_4}$$

Or any mathematically equivalent expression.

(e) Write the KCL equation for node $u_2$ in terms of the node voltages and other circuit elements.

**Solution:**

KCL at $u_2$ gives us:

$$-i_1 + i_8 + i_3 + i_5 = 0$$

Note that in this solution, we have assumed that the current $i_8$ is flowing out of node $u_2$. You could have also assumed the current flows into the node if you corrected it from the previous part. So long as you were consistent with your polarity and current direction, you should have gotten the correct answer.

The equations for the current through each of the branches are:

$$i_1 = \frac{u_1 - u_2}{R_1}$$
$$i_3 = \frac{u_2 - u_4}{R_3}$$
$$i_8 = -I_S$$
$$i_5 = \frac{u_2 - u_5}{R_5}$$

The final expression is then:

$$-\frac{u_1 - u_2}{R_1} - I_S + \frac{u_2 - u_5}{R_5} + \frac{u_2 - u_4}{R_3} = 0$$

Or any equivalent equation.

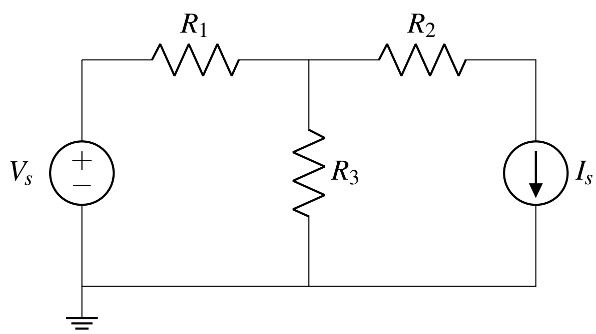
**3. Circuit Analysis**

*(Contributors: Ava Tan, Aviral Pandey, Christos Adamopoulos, Matthew McPhail, Moses Won, Panos Zarkos, Raghav Anand, Titan Yuan, Urmita Sikder)*

***Learning Goal:*** *This problem will help you practice circuit analysis using NVA method.*

Using the steps outlined in lecture or in Note 11B, analyze the following circuits to calculate the currents through each element and the voltages at each node. Use the ground node labelled for you. You may use a numerical tool such as IPython to solve the final system of linear equations.

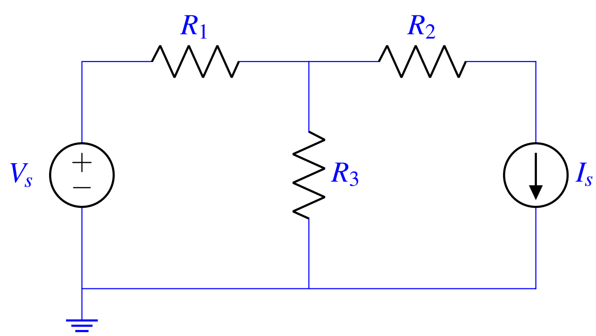
(a) $V_s = 5\,\text{V}, I_s = 2\,\text{A}, R_1 = R_2 = 2\,\Omega, R_3 = 4\,\Omega$




**Solution:**

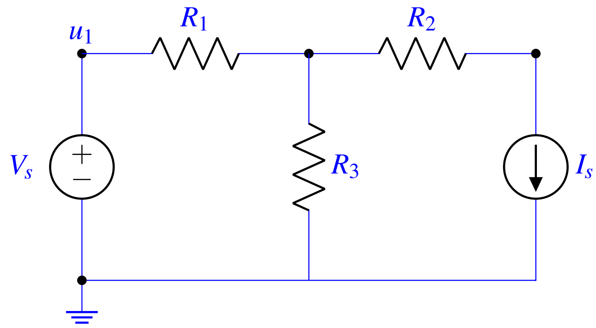
Step 1) Define Reference Node

Select a reference (ground) node. Any node can be chosen for this purpose. In this example, we choose the node at the bottom of the circuit diagram.

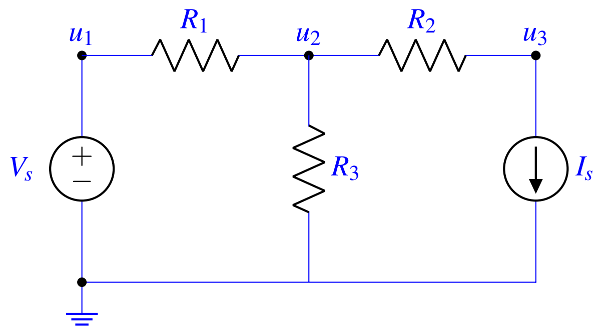



Step 2) Label Nodes with Voltage Set by Sources

Voltage sources set the voltage of the node they are connected to. In the example, there is only one source, $V_s$, and we label the corresponding source $u_1$ (names are arbitrary, but must be unique).

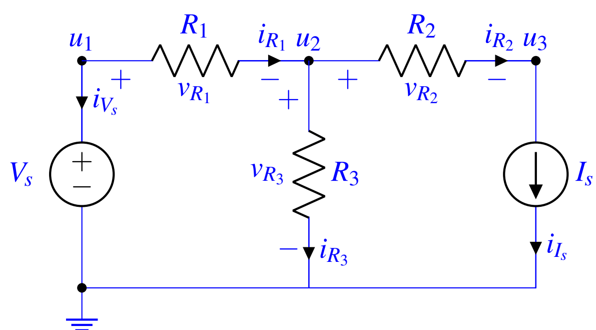



Step 3) Label Remaining Nodes
Now we label all remaining nodes in the circuit except the reference. In the example there are two, $u_2$ and $u_3$.




Step 4) Label Element Voltages and Currents

Next we mark all element voltages and currents.
Start with the current. The direction is arbitrary (top to bottom, bottom to top, it won't matter, but stick with your choice in subsequent steps). Then mark the element voltages following the passive sign convention, i.e. the voltage and current point in the "same" direction.




Step 5) KCL Equations
Write KCL equations for all nodes with unknown voltage, $u_2$ and $u_3$.

$$-i_{R_1} + i_{R_2} + i_{R_3} = 0$$
$$-i_{R_2} + i_{I_s} = 0$$

Step 6) Element Currents

Find expressions for all element currents in terms of voltage and element characteristics (e.g. Ohm's law) for all circuit elements except voltage sources. In this circuit there are four, $R_1$, $R_2$, $R_3$ and $I_s$.

$$\begin{aligned} i_{I_s} &= I_s \\ i_{R_1} &= \frac{V_{R_1}}{R_1} \\ i_{R_2} &= \frac{V_{R_2}}{R_2} \\ i_{R_3} &= \frac{V_{R_3}}{R_3} \end{aligned}$$

We also have

$$\begin{aligned} u_1 &= V_s \\ i_{I_s} &= I_s \\ i_{R_1} &= \frac{u_1 - u_2}{R_1} \\ i_{R_2} &= \frac{u_2 - u_3}{R_2} \\ i_{R_3} &= \frac{u_2}{R_3} \end{aligned}$$

Step 7) Substitute Element Currents in KCL Equations

Now we substitute the expressions derived in Step 6 into the KCL equations from Step 5.

$$\begin{aligned} -\frac{V_s - u_2}{R_1} + \frac{u_2 - u_3}{R_2} + \frac{u_2}{R_3} &= 0 \\ -\frac{u_2 - u_3}{R_2} + I_s &= 0 \end{aligned}$$

Let's make this a bit nicer by grouping the unknowns ($u_2$ and $u_3$) on the left side and the known terms on the right:

$$\begin{aligned} u_2\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) + u_3\left(-\frac{1}{R_2}\right) &= \frac{V_s}{R_1} \\ u_2\left(-\frac{1}{R_2}\right) + u_3\left(\frac{1}{R_2}\right) &= -I_s \end{aligned}$$

Note that we now have 2 equations for 2 unknowns. Thus, we set up the following matrix relation:

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_1} \\ -I_s \end{bmatrix}$$

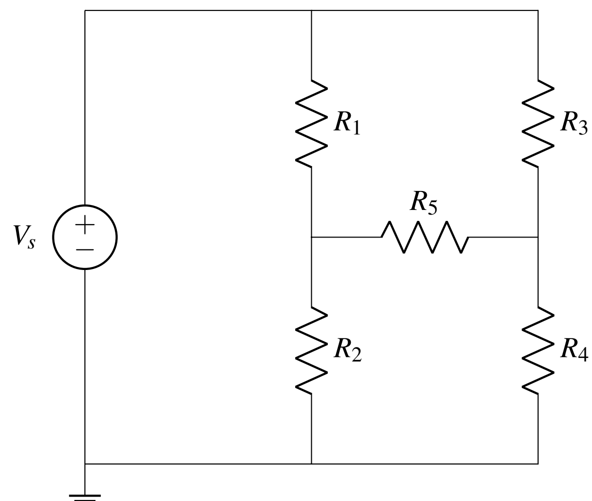
Finally, we plug in the values we were given into the matrix above and use Gaussian Elimination to find the vector of unknowns. We find that:

$$\begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 0.667V \\ -3.33V \end{bmatrix}$$

The branch currents can be obtained from the node voltages and element equations. Therefore, we can write:

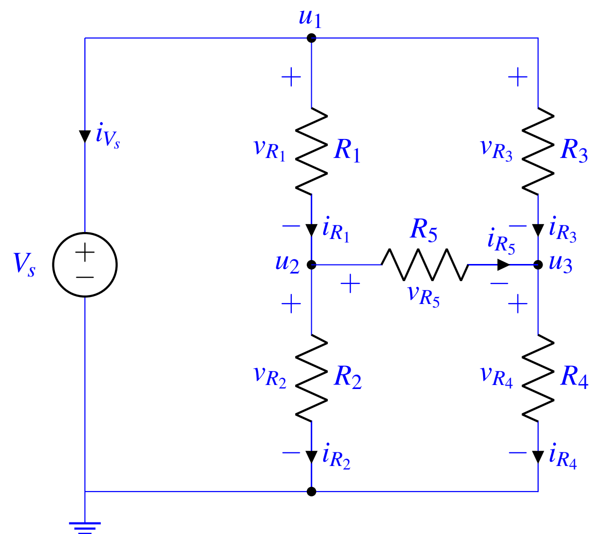
$$\begin{aligned} i_{V_s} &= -2.167A \\ i_{I_s} &= I_s = 2A \\ i_{R_1} &= \frac{u_1 - u_2}{R_1} = 2.167A \\ i_{R_2} &= \frac{u_2 - u_3}{R_2} = 2A \\ i_{R_3} &= \frac{u_2}{R_3} = 0.167A \end{aligned}$$

(b) $V_s = 5\,\text{V}, R_1 = 1\,\Omega, R_2 = 2\,\Omega, R_3 = 3\,\Omega, R_4 = 4\,\Omega, R_5 = 5\,\Omega$




**Solution:**

Here, we will skip showing all of the individual labeling steps. Below is the circuit with our choice of ground and current directions.




From the above circuit, we get the following KCL equations:

$$-i_{R_1} + i_{R_2} + i_{R_5} = 0$$
$$-i_{R_3} + i_{R_4} - i_{R_5} = 0$$

Using the *IV* relations for each element, we have:

$$u_1 - 0 = V_s$$
$$i_{R_1} = \frac{u_1 - u_2}{R_1}$$
$$i_{R_2} = \frac{u_2 - 0}{R_2}$$
$$i_{R_3} = \frac{u_1 - u_3}{R_3}$$
$$i_{R_4} = \frac{u_3}{R_4}$$
$$i_{R_5} = \frac{u_2 - u_3}{R_5}$$

We also know that $u_1 = V_S$

Now we substitute these expressions into the KCL equations we previously derived.

$$-\frac{V_s - u_2}{R_1} + \frac{u_2 - 0}{R_2} + \frac{u_2 - u_3}{R_5} = 0$$
$$-\frac{V_s - u_3}{R_3} + \frac{u_3}{R_4} - \frac{u_2 - u_3}{R_5} = 0$$

Let's make this a bit nicer by grouping the unknowns ($u_2$ and $u_3$) on the left side and the known terms on the right:

$$u_2\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5}\right) + u_3\left(-\frac{1}{R_5}\right) = \frac{V_s}{R_1}$$
$$u_2\left(-\frac{1}{R_5}\right) + u_3\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right) = \frac{V_s}{R_3}$$

Note that we now have 2 equations for 2 unknowns. Thus, we set up the following matrix relation:

$$\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_5} & -\frac{1}{R_5} \\ -\frac{1}{R_5} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5} \end{bmatrix} \begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} \frac{V_s}{R_1} \\ \frac{V_s}{R_3} \end{bmatrix}$$

Finally, we plug in the values we were given into the matrix above and use Gaussian Elimination to find the vector of unknowns.

$$\begin{bmatrix} u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} 3.29V \\ 2.968V \end{bmatrix}$$

The branch currents can be obtained from the node voltages and element equations. Therefore, we can write:

$$i_{R_1} = \frac{u_1 - u_2}{R_1} = 1.709A$$
$$i_{R_2} = \frac{u_2 - 0}{R_2} = 1.645A$$
$$i_{R_3} = \frac{u_1 - u_3}{R_3} = 0.677A$$
$$i_{R_4} = \frac{u_3}{R_4} = 0.741A$$
$$i_{R_5} = \frac{u_2 - u_3}{R_5} = 0.0644A$$
$$i_{V_s} = -(i_{R_1} + i_{R_3}) = -2.38A$$

**4. Fruity Fred**

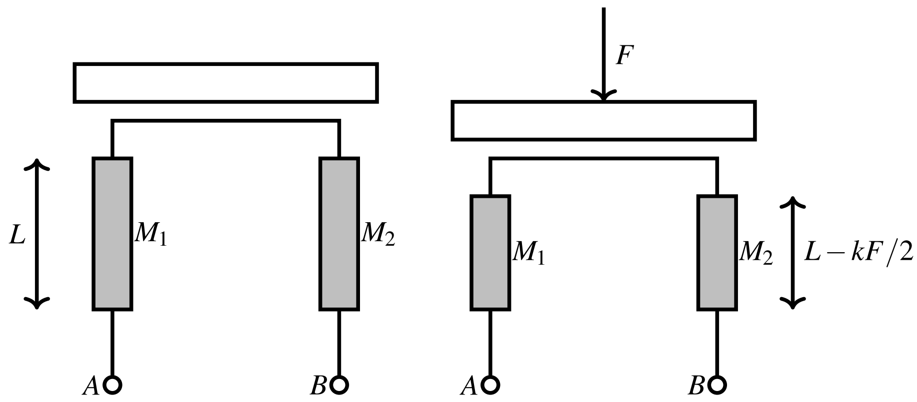
*(Contributors: Ava Tan, Christos Adamopoulos, Matthew McPahil, Moses Won, Panos Zarkos, Urmita Sikder, Wahid Rahman)*

***Learning Goal:*** *This problem will introduce the process of designing a sensing circuit for the purpose of measuring a physical quantity. This will also help to build your intuition for modeling physical elements.*

Fruity Fred just got back from Berkeley Bowl with a bunch of mangoes, pineapples, and coconuts. He wants to sort his mangoes in order of weight, so he decides to use his knowledge from EECS16A to build a scale.

He finds two identical bars of material ($M_1$ and $M_2$) of length $L$ (in meters) and a cross-sectional area (i.e. width $\times$ thickness) of $A_c$ (in meters$^2$). The bars are made of a material with resistivity $\rho$. He knows that the **length of these bars decreases** by $k$ meters per Newton of force applied, while the **cross-sectional area remains constant**.

He builds his scale as shown below, where the top of the vertical bars are connected with an ideal electrical wire. The left side of the diagram shows the scale at rest (with no object placed on it), and the right side shows it when the applied force is $F$ (Newtons). The force $F$ is equally distributed between two bars, causing the length of each bar to decrease by $kF/2$ meters. Fred's mangoes are not very heavy, so the change in each bar's length is very small compared to the total length (mathematically, we can write this by using the "much smaller than" symbol $\ll$, i.e. $kF/2 \ll L$).

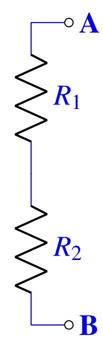



(a) Let $R_{AB}$ be the resistance between nodes $A$ and $B$ with the weights on the scale. Write an expression for $R_{AB}$ as a function of $A_c$, $L$, $\rho$, $F$, and $k$. *Hint: You can start by representing each bar as a resistor, then find how they are connected.*

**Solution:**

Redrawn in circuit representation, we note that Fred's scale design looks like the following. Note that the values of the two resistors $R_1$ and $R_2$ are *variable*, and their lengths are changed by the same amount (given in the problem statement as $kF/2$) upon the application of a force $F$.

The key observation to make is firstly that these two variable resistors are connected in *series*.




Because the length of each bar decreases by $k$ meters per Newton of force applied it, and there are two bars supporting the scale, each bar can be thought to have half the force, $\frac{F}{2}$, on it. Because of this, each bar's length diminishes by $\frac{kF}{2}$, to lengths $L - \frac{kF}{2}$, when a force $F$ is applied to Fred's scale.

Therefore, the combination of $R_1$ and $R_2$ has a resistance $R_{AB} = R_1 + R_2$. From the problem statement, we note that because the bars $M_1$ and $M_2$ have identical properties of $L, A_C$, and $\rho$, $R_1 = R_2$ is calculated as $R_1 = R_2 = \rho \frac{L-kF/2}{A_c}$.

Therefore $R_{AB} = 2 \cdot \rho \frac{L-kF/2}{A_c}$.

(b) Fred wants to measure a voltage that changes based on how much weight is placed on his scale. He knows that $R_{AB}$ will change with the weight on the scale.

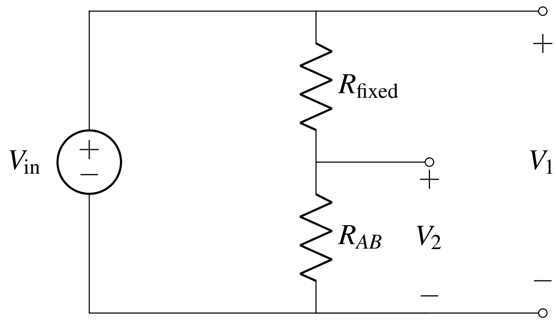
Design a circuit for Fred that outputs a voltage that is some function of the weight $F$. **Your circuit should include $R_{AB}$, and you may use any number of voltage sources and resistors in your design.** Be sure to **label** where the voltage should be measured in your circuit.

Also provide an **expression** relating the output voltage of your circuit to the force applied on the scale. This expression can contain any necessary parameters.

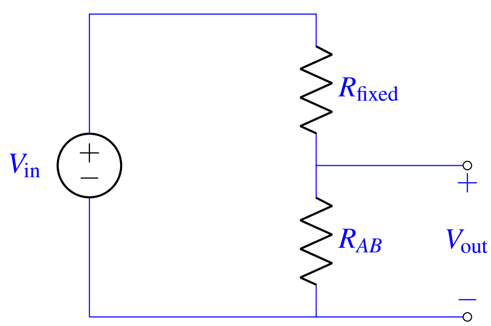
*Hint: If you connected only a voltage source across A and B and measured the voltage ($V_{AB}$) between A and B, would $V_{AB}$ change based on the value of $R_{AB}$? It turns out it wouldn't. Why?*

*Hint: Consider the following circuit, where $R_{fixed}$ is a value you know and choose. Which voltage would you choose to measure in the circuit to help you determine the weight on the scale: $V_1$ or $V_2$?*




**Solution:**

If your circuit solution also measures the voltage across just one of the resistances $R_1$ or $R_2$ rather than the series combination $R_{AB}$, your solution is also acceptable. Our solution here chooses the elemental voltage across $R_{AB}$ in a simple voltage divider circuit to measure as $V_{\text{out}}$.




$$V_{\text{out}} = \frac{R_{AB}}{R_{\text{fixed}} + R_{AB}} V_{\text{in}}$$

$$V_{\text{out}} = \frac{\frac{2\rho(L - \frac{kF}{2})}{A_c}}{R_{\text{fixed}} + \frac{2\rho(L - \frac{kF}{2})}{A_c}} V_{\text{in}} = \frac{2\rho(L - \frac{kF}{2})}{R_{\text{fixed}} A_c + 2\rho(L - \frac{kF}{2})} V_{\text{in}}$$

## 5. Resistive Touchscreen

*(Contributors: Ava Tan, Ben Osoba, Christos Adamopoulos, Matthew McPhail, Moses Won, Panos Zarkos, Urmita Sikder, Wahid Rahman)*

***Learning Goal:*** *The objective of this problem is to provide insight into modeling of resistive elements. This will also help to apply the concepts from 1D resistive touchscreen.*

In this problem, we will investigate how a 1D resistive touchscreen with a defined thickness, width, and length can actually be modeled as a series combination of resistors. As we know the value of a resistor depends on its length.

Figure 2 shows the top view of a resistive touchscreen consisting of a conductive layer with resistivity $\rho_1$, thickness $t$, width $W$, and length $L$. At the top and bottom it is connected through good conductors ($\rho = 0$) to the rest of the circuit. The touchscreen is wired to voltage source $V_s$.

Use the following numerical values in your calculations: $W = 50$ mm, $L = 80$ mm, $t = 1$ mm, $\rho_1 = 0.5\,\Omega\,\text{m}$, $V_s = 5\text{V}$, $x_1 = 20$ mm, $x_2 = 45$ mm, $y_1 = 30$ mm, $y_2 = 60$ mm.

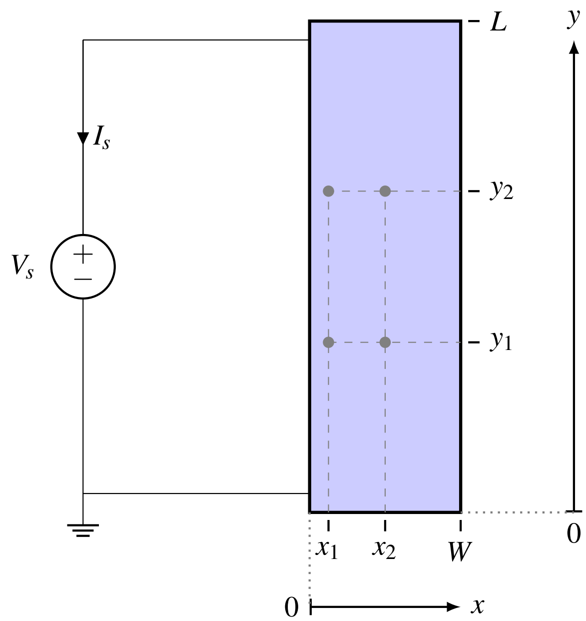
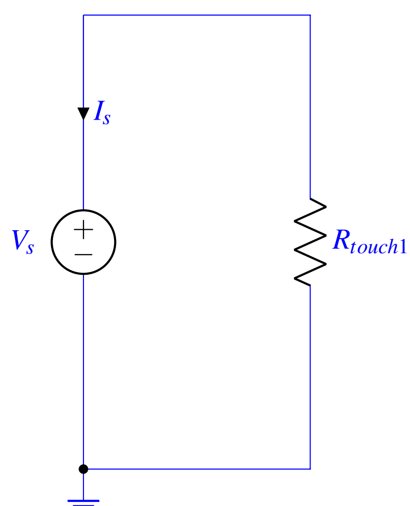



Figure 2: Top view of resistive touchscreen (not to scale). $z$ axis i.e. the thickness not shown (into the page).

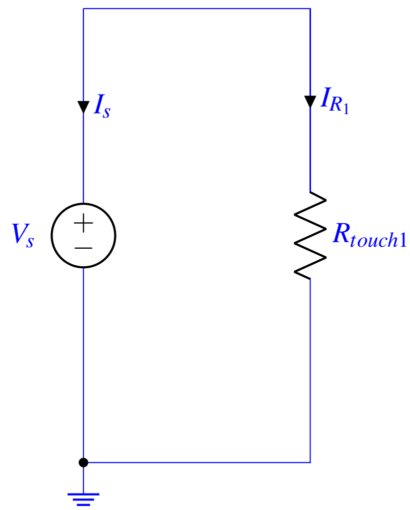
(a) Draw a circuit diagram representing Figure 2, where the touchscreen is represented as *a resistor*. **Note that no touch is occurring in this scenario.** Remember that circuit diagrams in general consist of only circuit elements (resistors, sources, etc) represented by symbols, connecting wires, and the reference/ground symbol. Calculate the value of current $I_S$ based on the circuit diagram you drew. Do not forget to specify the correct unit as always.

**Solution:**



The touchscreen resistance can be found from the following expression:

$$\begin{aligned} R_{touch1} &= \rho_1 \cdot \frac{L}{A} \\ &= \rho_1 \cdot \frac{L}{W \cdot t} \\ &= 0.5\,\Omega\,\mathrm{m}\left(\frac{80 \times 10^{-3}\,\mathrm{m}}{50 \times 10^{-3}\,\mathrm{m} \cdot 1 \times 10^{-3}\,\mathrm{m}}\right) \\ R_{touch1} &= 800\,\Omega \end{aligned}$$




From KCL, we can write:

$$I_s + I_{R_1} = 0 \quad (1)$$

$$I_s = -I_{R_1} \quad (2)$$

Therefore, the current $I_{R_1}$ is equal to:

$$I_{R_1} = \frac{V_s}{R_{touch1}} = \frac{5}{800}A = 6.25mA$$

And the current $I_s$ is equal to:

$$I_s = -I_{R_1} = -6.25mA$$

(b) Let us assume $u_{12}$ is the node voltage at the node represented by coordinates $(x_1, y_2)$ of the touchscreen, as shown in Figure 3. What is the value of $u_{12}$? You should first draw a circuit diagram representing Figure 3, which includes node $u_{12}$. Specify all resistance values in the diagram. Does the value of $u_{12}$ change based on the value of the x-coordinate $x_1$?

*Hint: You will need more than one resistor to represent this scenario.*

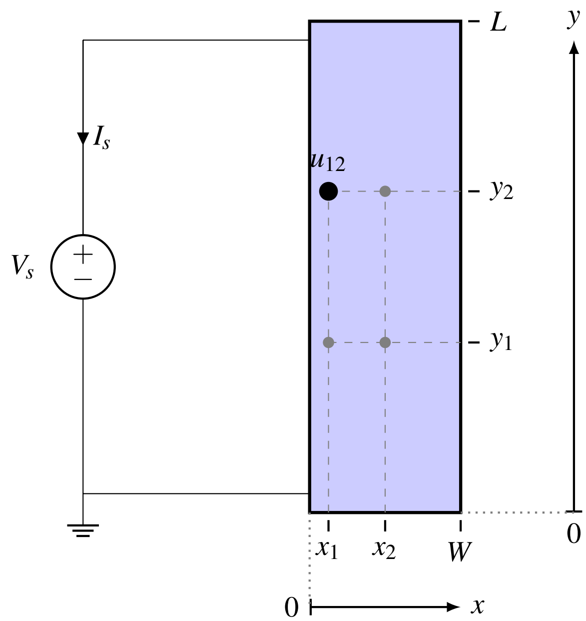
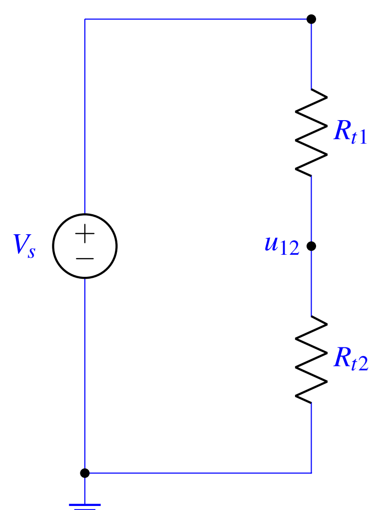



Figure 3: Top view of resistive touchscreen showing node $u_{12}$.

**Solution:**

We can represent this setup with the circuit shown below.




Using voltage division, $u_{12}$ can be found from the following expression:

$$u_{12} = V_s \frac{R_{t2}}{R_{t1} + R_{t2}} = V_s \frac{y_2}{L} = 5 \cdot \frac{3}{4} V = 3.75V$$

where $R_{t1} = \rho_1 \cdot \frac{L-y_2}{W \cdot t}$ and $R_{t2} = \rho_1 \cdot \frac{y_2}{W \cdot t}$.

(c) Assume $V_{ab}$ is the voltage measured between the nodes represented by touchscreen coordinates $(x_1, y_1)$ and coordinates $(x_1, y_2)$, as shown in Figure 4. Calculate the absolute value of $V_{ab}$. As with the previous part, you should first draw the circuit diagram representing Figure 4, which includes $V_{ab}$. Calculate all resistor values in the circuit. *Hint: Try representing the segment of the touchscreen between these two coordinates as a separate resistor itself.*

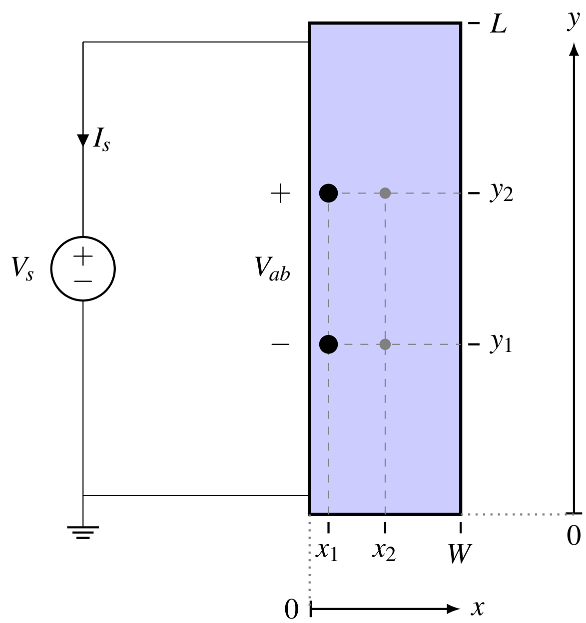
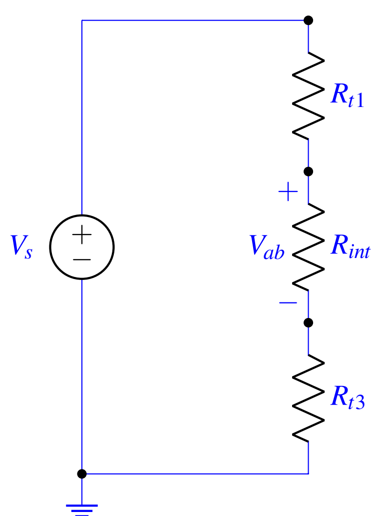


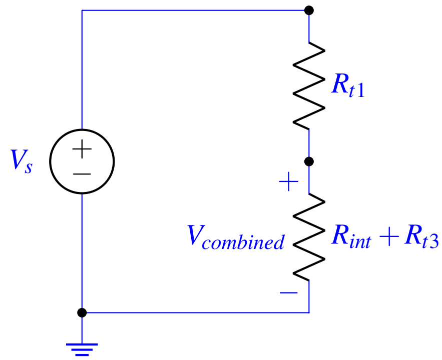

Figure 4: Top view of resistive touchscreen showing voltage $V_{ab}$.

**Solution:**




One method to find the voltage is by using node voltage analysis. Presented is an alternative approach by using resistor equivalence. To find the voltage, $V_{ab}$, first, find the voltage over $R_{int}$ and $R_{t3}$ together. We can represent them as an equivalent resistance as follows:




As this circuit is a voltage divider, we can find $V_{combined}$ by voltage division.

$$V_{combined} = \frac{R_{int} + R_{t3}}{R_{t1} + R_{int} + R_{t3}} V_s$$

Once we know $V_{combined}$, this will be the voltage over the two resistors, $R_{int}$ and $R_{t3}$. We can apply voltage division again to get $V_{ab}$:

$$V_{ab} = \frac{R_{int}}{R_{int} + R_{t3}} V_{combined}$$

By substituting, we get that $V_{ab}$ is:

$$\begin{aligned} V_{ab} &= \frac{R_{int}}{R_{int} + R_{t3}} \frac{R_{int} + R_{t3}}{R_{t1} + R_{int} + R_{t3}} V_s \\ &= \frac{R_{int}}{R_{t1} + R_{int} + R_{t3}} V_s \end{aligned}$$

Each of the resistances can be calculated as $R_{t1} = \rho_1 \cdot \frac{L - y_2}{W \cdot t}$, $R_{int} = \rho_1 \cdot \frac{y_2 - y_1}{W \cdot t}$ and $R_{t3} = \rho_1 \cdot \frac{y_1}{W \cdot t}$. This gives for $V_{ab}$:

$$V_{ab} = \frac{y_2 - y_1}{L} V_s = \frac{3}{8} \cdot 5V = 1.875V$$

(d) Calculate (the absolute value of) the voltage between the nodes represented by touchscreen coordinates $(x_1, y_1)$ and coordinates $(x_2, y_1)$.

**Solution:**

The two points have the same y coordinate, therefore they have the same potential in our 1-D touchscreen model. Recall that the touchscreen is effectively being modeled as a single resistor which can be considered as several resistors of varying lengths, all totaling to $L$. Hence, we do not consider the effect of the $x$-coordinate on potential – we just need to consider the difference in the $y$-coordinate between two points. Thus, $\Delta V = 0$.

(e) Calculate (the absolute value of) the voltage between the nodes represented by touchscreen coordinates $(x_1, y_1)$ and coordinates $(x_2, y_2)$.

**Solution:**

The two points have different $x$ and $y$ coordinates. However, the potential is the same across the $x$-axis for a fixed $y$ coordinate, as was explained in the solution for part (d). Therefore, the problem is similar to part (c), since the potential is only determined by the $y$-coordinate of a point. Using the same equivalent circuit in part (c) we have:

$$\Delta V = V_s \frac{R_{int}}{R_{t1} + R_{int} + R_{t3}} = 1.875V$$

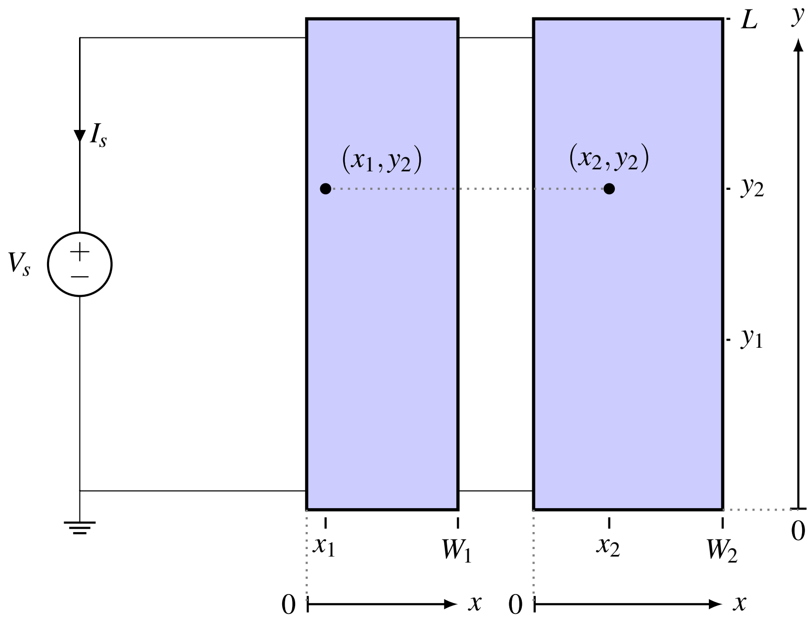


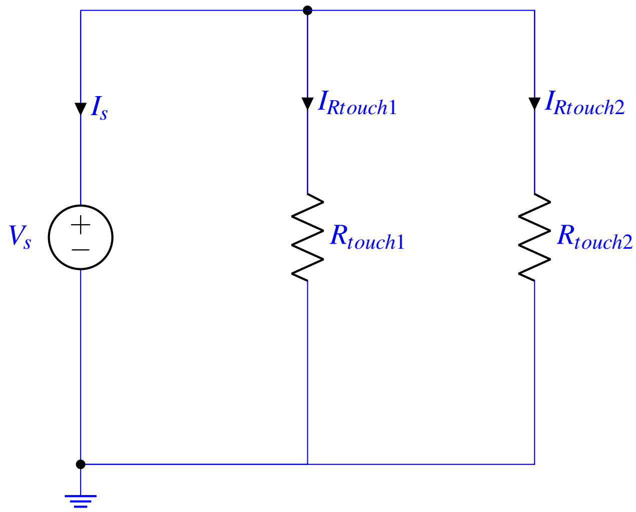

Figure 5: Top view of two touchscreens wired in parallel (not to scale). $z$ axis not shown (into the page).

(f) Figure 5 shows a new arrangement with two touchscreens. The second touchscreen (the one on the right) is identical to the one shown in Figure 2, except for different width, $W_2$, and resistivity, $\rho_2$.

Use the following numerical values in your calculations: $W_1 = 50$ mm, $L = 80$ mm, $t = 1$ mm, $\rho_1 = 0.5\,\Omega\,\text{m}$, $V_s = 5\text{V}$, $x_1 = 20$ mm, $x_2 = 45$ mm, $y_1 = 30$ mm, $y_2 = 60$ mm, which are the same values as before. The new touchscreen has the following numerical values which are different: $W_2 = 85$ mm, $\rho_2 = 0.6\,\Omega\,\text{m}$.

Draw a circuit diagram representing Figure 5, where the two touchscreens are represented as *two separate resistors*. **Note that no touch is occurring in this scenario.**

**Solution:**




(g) Calculate the value of current $I_s$ for the two touchscreen arrangement based on the circuit diagram you drew in the last part.

**Solution:**

From KCL, we can write:

$$I_s + I_{Rtouch1} + I_{Rtouch2} = 0 \quad (3)$$

$$I_s = -(I_{Rtouch1} + I_{Rtouch2}) \quad (4)$$

Using Ohm's Law for each element:

$$I_s = -\left(\frac{V_s}{R_{touch1}} + \frac{V_s}{R_{touch2}}\right)$$

However, the resistance of the second touchscreen can be given by:

$$R_{touch2} = \rho_2 \cdot \frac{L}{W_2 \cdot t} = 0.6\,\Omega\,\mathrm{m}\left(\frac{80 \times 10^{-3}\,\mathrm{m}}{85 \times 10^{-3}\,\mathrm{m} \cdot 1 \times 10^{-3}\,\mathrm{m}}\right)$$

$$R_{touch2} = 564.705\,882\,4\,\Omega$$

Therefore, using the resistance values for the first and second touchscreen and the applied voltage source, we have:

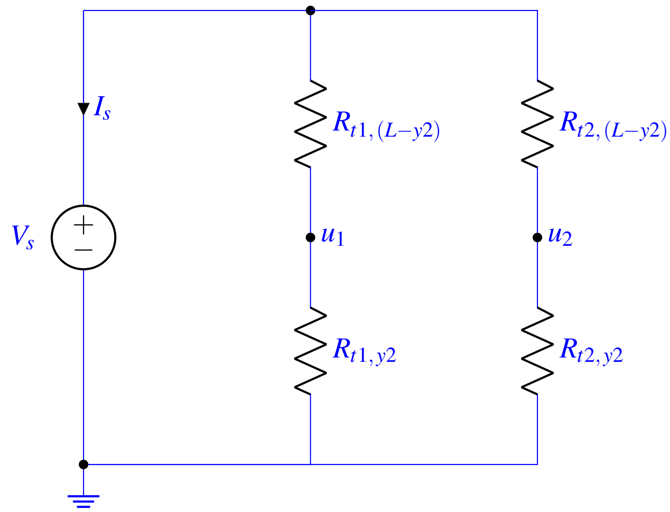
$$I_s \approx -(6.25\text{mA} + 8.85\text{mA}) = -15.1\text{mA}$$

(h) Consider the two points: $(x_1, y_2)$ in the touchscreen on the left, and $(x_2, y_2)$ in the touchscreen on the right in Figure 5. Show that the node voltage at $(x_1, y_2)$ is the same that at $(x_2, y_2)$, i.e. the potential difference between the two points is 0. You can show this without explicitly calculating the node voltages at the two points.

If you were to connect a wire between the two coordinates $(x_1, y_2)$ in the touchscreen on the left, and $(x_2, y_2)$ in the touchscreen on the right, would any current flow through this wire?

**Solution:**

It will be helpful to first consider the circuit representation of this scenario to understand why the node voltages at the two points on each of the touch screens should be same (and therefore that the potential difference between points $(x_1, y_2)$ and $(x_2, y_2)$ is 0).




Without calculating the node voltages, note that the ratio of the value of $R_{t1,(L-y_2)}$ to $R_{t1,y2}$ is the same as the ratio of the value of $R_{t2,(L-y2)}$ to $R_{t2,y2}$:

$$\frac{R_{t1,y2}}{R_{t1,(L-y2)}} = \frac{\rho_1(y_2)/(W_1 \cdot t)}{\rho_1(L-y_2)/(W_1 \cdot t)} = \frac{y_2}{L-y_2}$$
$$\frac{R_{t2,y2}}{R_{t2,(L-y2)}} = \frac{\rho_2(y_2)/(W_2 \cdot t)}{\rho_2(L-y_2)/(W_2 \cdot t)} = \frac{y_2}{L-y_2}$$

Note also that the voltage across the entirety of each of the individual touchscreens is the same: it is $V_s - 0$ or just $V_s$. The voltage $V_s$ is therefore *divided* between $R_{t1,(L-y2)}$ and $R_{t1,y2}$ exactly the same as it is divided between $R_{t_2,(L-y2)}$ and $R_{t_2,y2}$ because of the ratio argument presented above.

Therefore, the potential difference between $u_1$ and $u_2$ will be 0, so long as the $y$-coordinate value is chosen to be the same.

This also means that there is no current flowing through the wire, since the points in the two touch-screens have the same potential. Therefore,

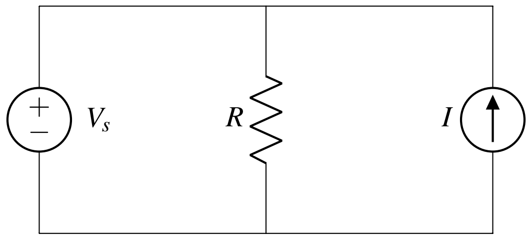
$$I_{12} = \frac{u_1 - u_2}{R_{wire}} = 0$$

We should note that what causes current to flow is a voltage difference. Here, we have a voltage difference of zero, so the current flowing through the wire is zero. Generally, in a wire when current is flowing we have a very small voltage difference $V_{wire} = IR_{wire}$. However, since $R_{wire}$ is super small we neglect $V_{wire}$.

## 6. Power Analysis

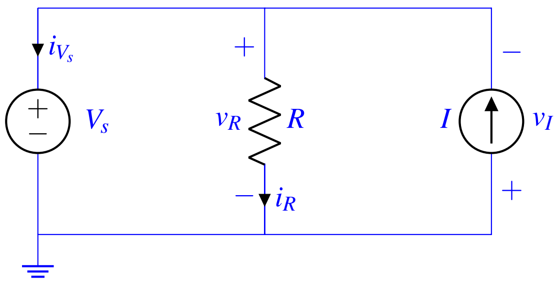
*(Contributors: Ava Tan, Craig Schindler, Moses Won, Raghav Anand, Urmita Sikder, Wahid Rahman)*

***Learning Goal:*** *This problem aims to help you practice calculating power dissipation in different circuit elements. It will also give you insights into how power is conserved in a circuit.*




(a) Find the expressions of power dissipated by each element in the circuit above. Remember to label voltage-current pairs using passive sign convention.

**Solution:** We label a ground node, and then solve for the currents $i_V, i_R$ and the voltages $V_R, V_I$.




Solving the above circuit using nodal analysis, we get

$$i_R = \frac{V_s}{R}$$

$$i_V = I - \frac{V_s}{R}$$

$$v_I = -V_s$$

$$v_R = V_s$$

Using this we can calculate

$$P_{V_s} = V_s i_V = IV_s - \frac{{V_s}^2}{R}$$

$$P_I = Iv_I = -IV_s$$

$$P_R = i_R v_R = \frac{{V_s}^2}{R}$$

Note that $P_{V_s} + P_I + P_R = 0$, i.e. energy provided is energy dissipated, which verifies our intuition about conservation of energy.

(b) Use $R = 5\text{k}\Omega$, $V_s = 5\text{V}$, and $I = 5\text{mA}$. Calculate the power dissipated by the voltage source ($P_{V_s}$), the current source ($P_I$), and the resistor ($P_R$).

**Solution:**

$P_{V_S} = (0.005\text{A})(5\text{V}) - \frac{(5\text{V})^2}{5000\Omega} = 0.02W$
$P_I = -(0.005\text{A})(5\text{V}) = -0.025\text{W}$
$P_R = \frac{(5\text{V})^2}{5000\Omega} = 0.005\text{W}$

Note that $P_{V_S} + P_I + P_R = 0$.

(c) Once again, let $R = 5\text{k}\Omega$, $V_s = 5\text{V}$. What does the value $I$ of the current source have to be such that the current source **dissipates** 40mW? Note that it is possible for a current source to *dissipate* power, i.e. under passive sign convention, $P_I = 40mW$. For this value of $I$, compute $I, P_{V_s}, P_I$, and $P_R$ as well.

*As an aside: If the current source were* delivering power *it would have been* $P_I = -40mW$*, under passive sign convention, but this is NOT what the question is asking about.*

**Solution:**

Remember that using passive sign convention, an element whose power is negative is supplying power, and an element whose power is positive is dissipating power. Therefore, we want $P_I = 40\text{mW}$. We know that $P_I = -IV_S$. Therefore, $I = -\frac{0.04\text{W}}{5\text{V}} = -0.008\text{A}$.

$P_{V_S} = (-0.008\text{A})(5\text{V}) - \frac{(5\text{V})^2}{5000\Omega} = -0.045W$
$P_I = -(-0.008\text{A})(5\text{V}) = 0.04\text{W}$
$P_R = \frac{(5\text{V})^2}{5000\Omega} = 0.005\text{W}$

Note that $P_{V_S} + P_I + P_R = 0$.

**7. Homework Process and Study Group**

Who did you work with on this homework? List names and student ID's. (In case you met people at homework party or in office hours, you can also just describe the group.) How did you work on this homework? If you worked in your study group, explain what role each student played for the meetings this week.

**Solution:**

I first worked by myself for 2 hours, but got stuck on problem 5. Then I met with my study group.

XYZ played the role of facilitator ... etc. We were still stuck on problem 5 so we went to office hours to talk about the problem.

Then I went to homework party for a few hours, where I finished the homework.